University of Twente
The Netherlands

Control Laboratory

PID-controllers

Job van Amerongen

Control Engineering, Dept. of Electrical Engineering
University of Twente, Netherlands

www.ce.utwente.nl/amn
J.vanAmerongen@utwente.nl

Contents

- Why PID-control
- When PID control
- Tuning rules (Ziegler-Nichols)
- Examples

PID controllers

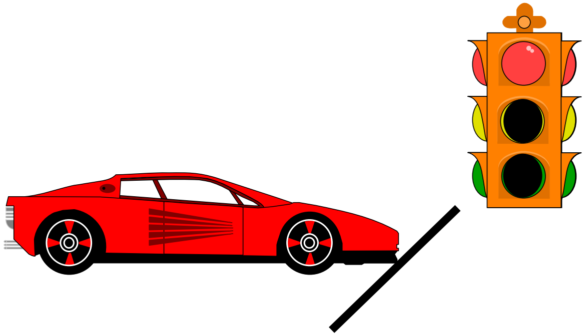
- Most widely applied type of controller
  - in industry probably 90% of all controllers
- Available as single controllers or implemented in computers
- Applicable to many processes
- Give a reasonable performance
- Fuzzy is often PID-like as well

First lecture

- Stopping a car at a traffic light
- Cruise control system

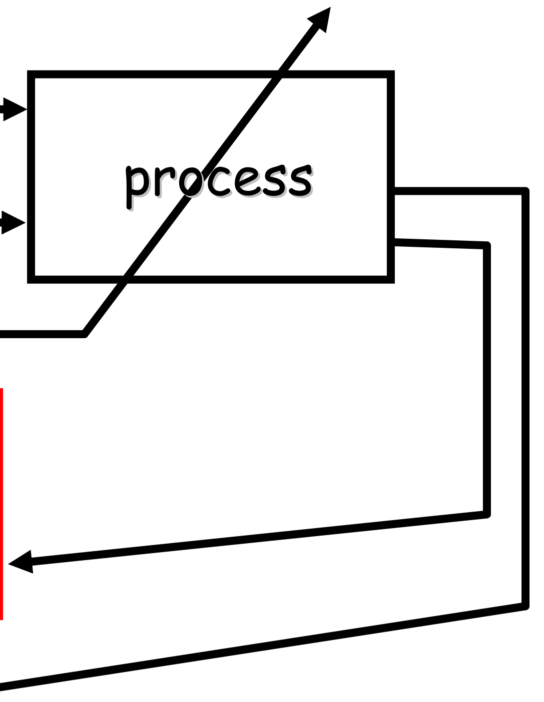
Controlling a car

- Stopping a car at a traffic light
- Goal
  - Stop in time at the white line



Variables

- Gas pedal
- Brakes
- Road condition

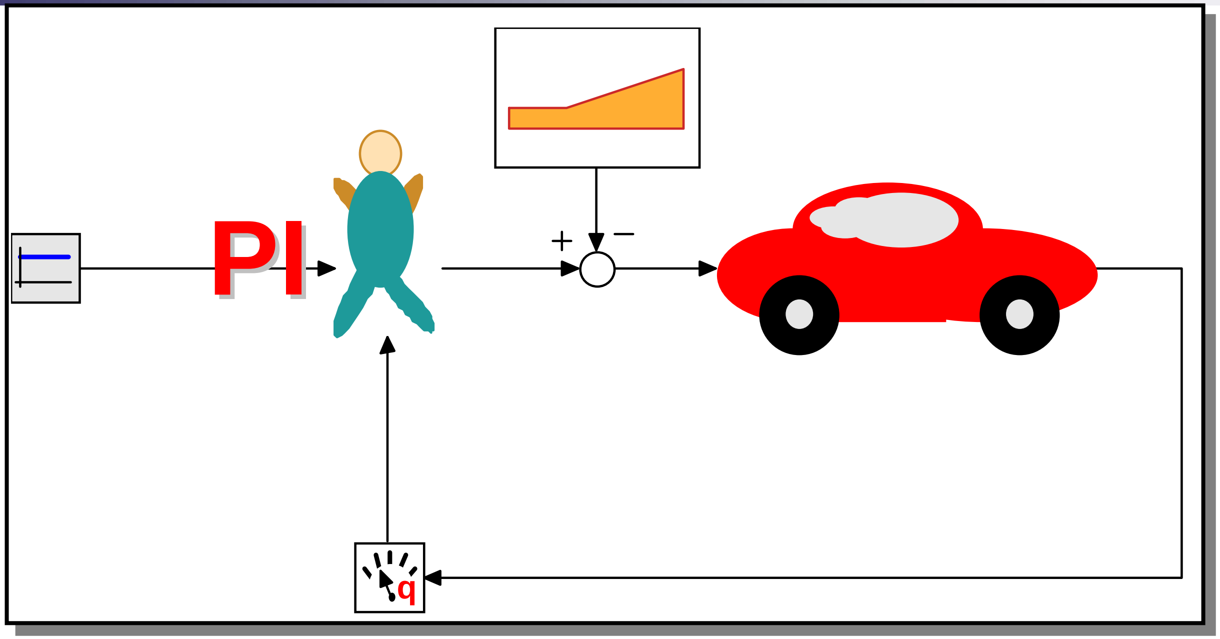


P

- Distance to white line
- Speed of the car

D

Cruise Control



I-action necessary in order to deal with constant disturbance (slope of a hill) at input of process

Proportional Control

- Limited accuracy for systems higher than first order (stability issues)
- Limited bandwidth

PD-control

- For a second order system:
  - PD-control ≈ state feedback
- Comparable with a lead network

PD-control is applicable as long as the system behaves more or less like a second-order system

PI-control

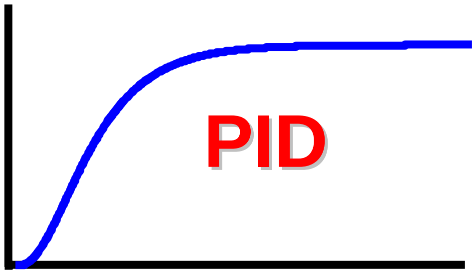
- For increased accuracy of type-0 systems
  - add an integration
- Comparable with a lag network
- If dead time is dominant:
  - pure I-control

PID-control

- For systems of type-0 and a dominant second-order behaviour
  - PID-control
- Can be seen as a combination of a lead and a lag network

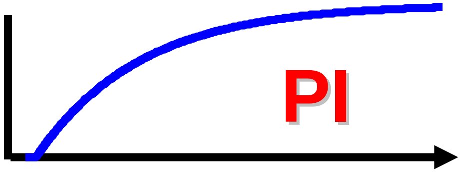
Types of systems

$$H(s) = \frac{K}{(s\tau_1 + 1)(s\tau_2 + 1)(s\tau_3 + 1)(\ldots)}$$

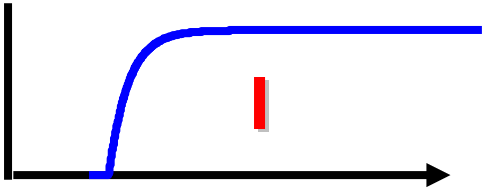


typical S-curve

$$H(s) = \frac{K\, e^{-sT_\mathrm{d}}}{(s\tau_1 + 1)}$$



$\tau \gg T_d$



$T_d \gg \tau$

Ziegler-Nichols rules

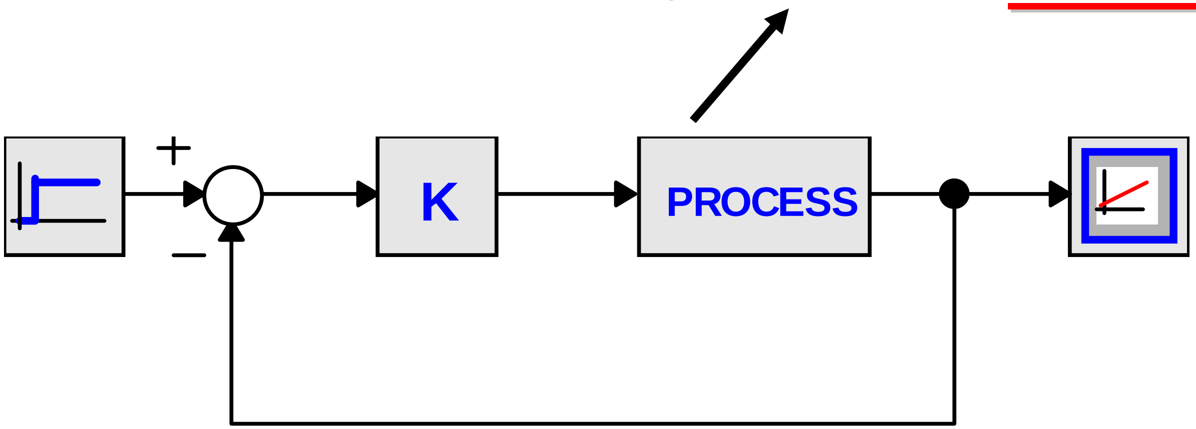
Bring feedback system
at the border of instability

P-control	$K_p = 0.5K_u$

Tuning

$$H_{\text{process}}(s) = \frac{K'}{(s+1)(s+2)(s+3)}$$

assumed to be unknown



20-sim: PID_tuning_P

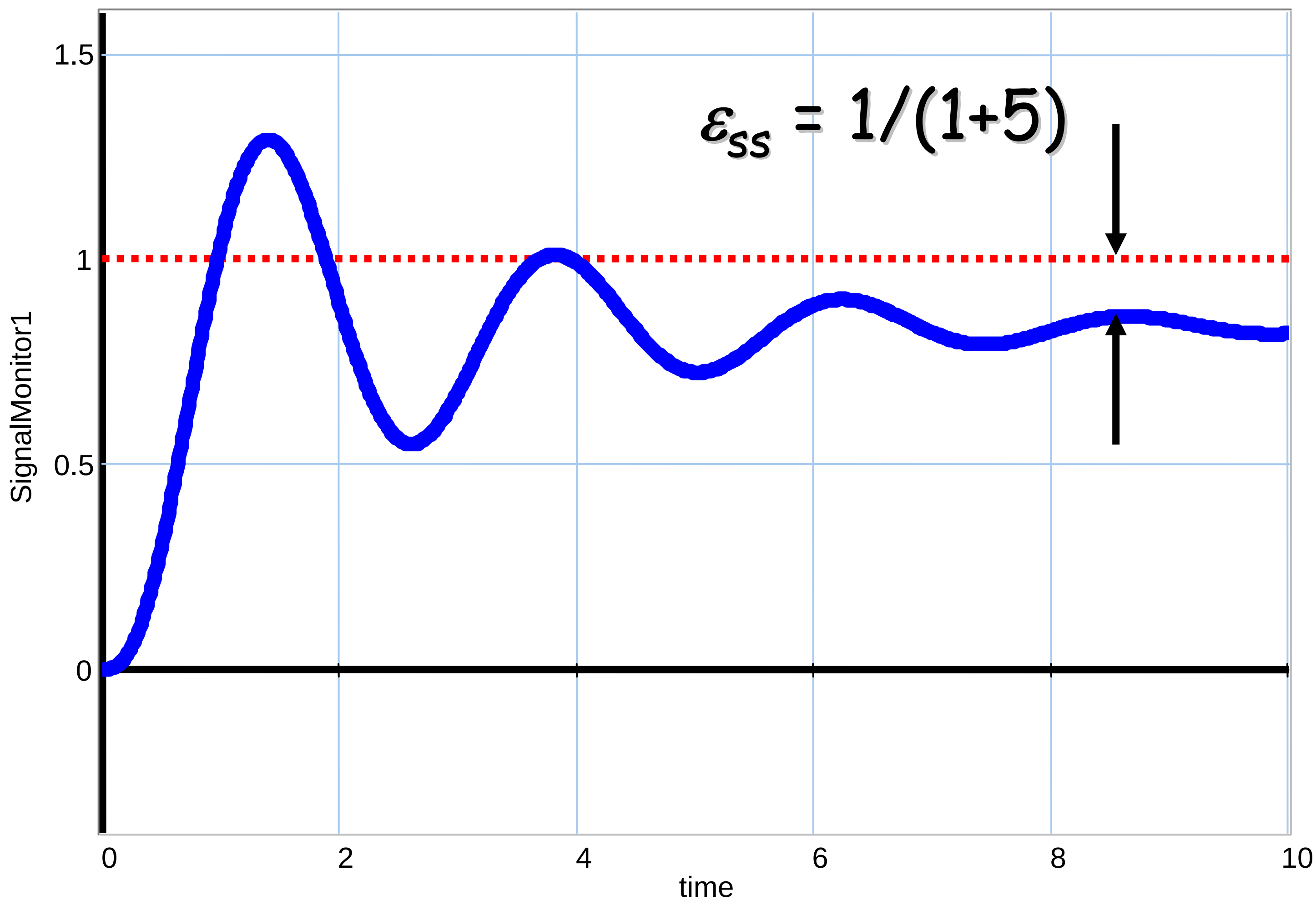
Bring feedback system at the border of instability

Border of instability

- After a few experiments we find
  - controller gain $K_u = 10$ brings the system at the border of instability

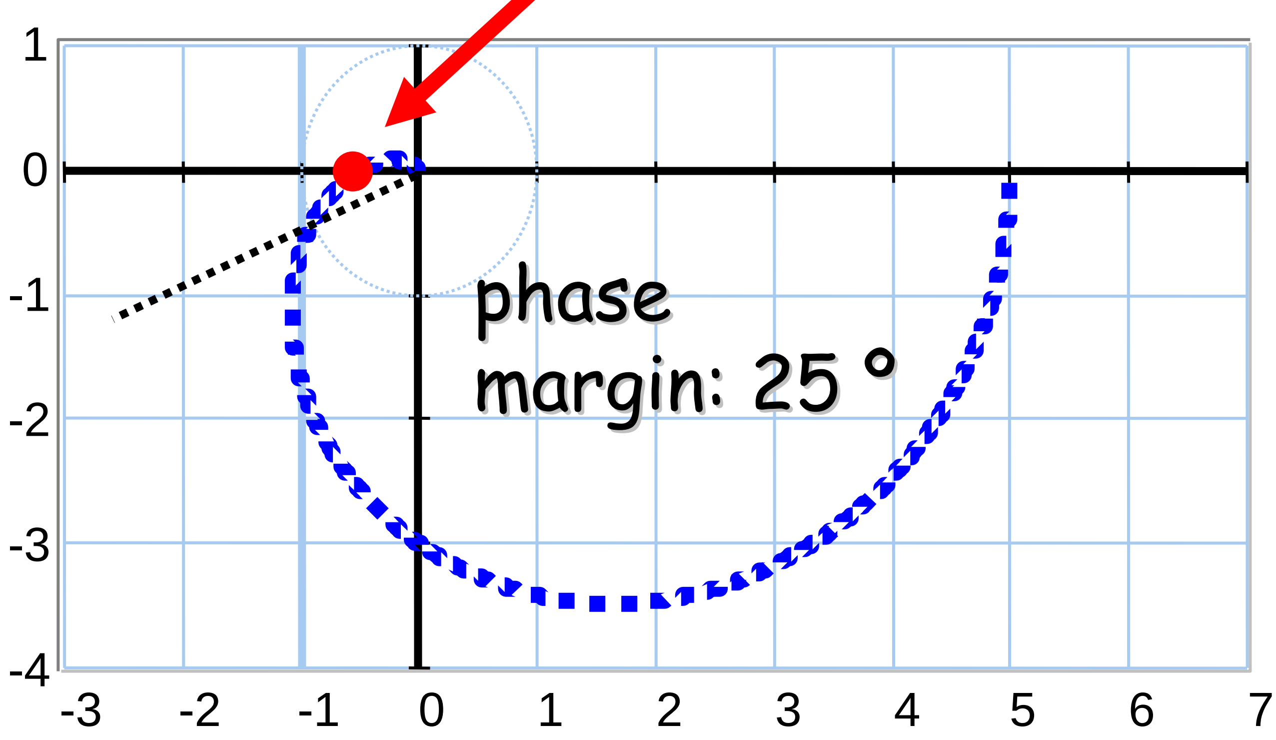
- A gain margin of 6 dB (a factor two) is a reasonable choice for the gain of a proportional controller

Response (GM = 6 dB)

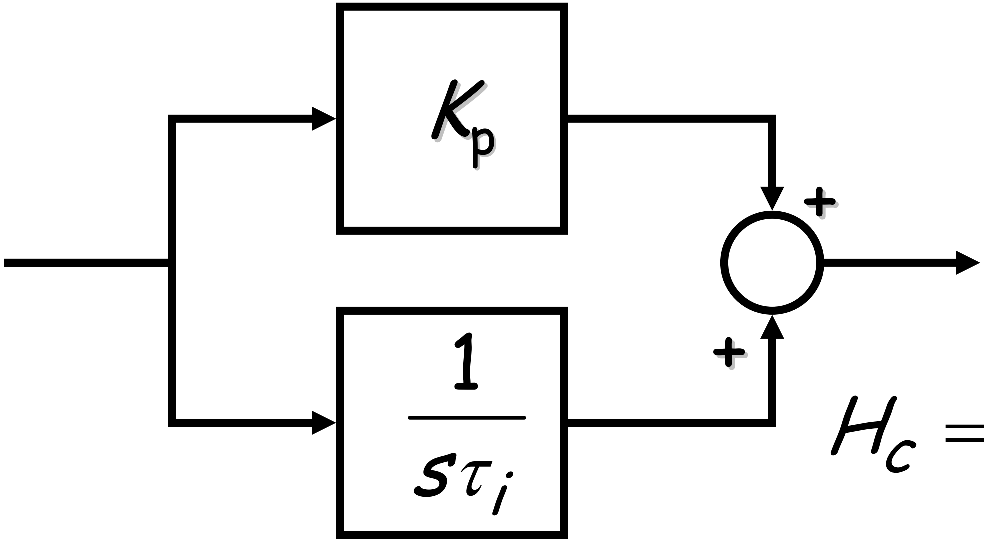


Nyquist plot

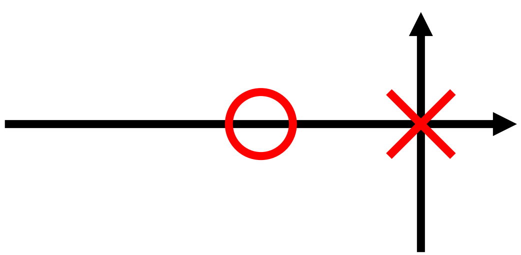
Identification of one single point of Nyquist plot



PI-controller (parallel)



$$H_c = K_p + \frac{1}{s\tau_i} = \frac{1}{\tau_i}\frac{K_p s\tau_i + 1}{s}$$

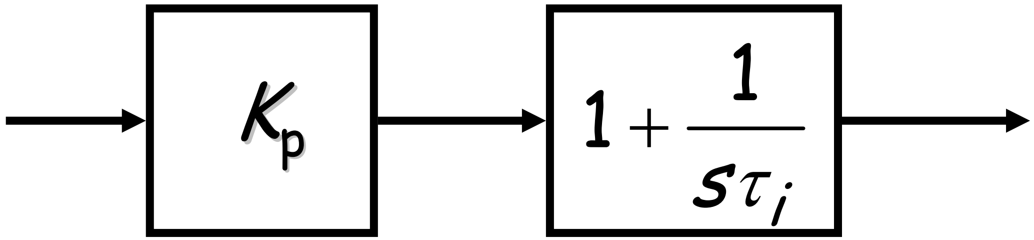


lag network
with pole in origin

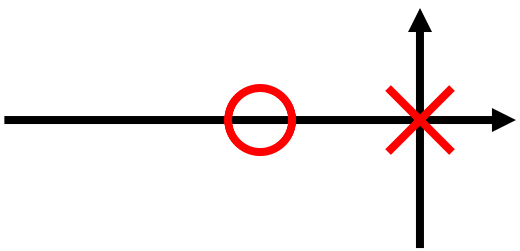
zero in: $s = -\frac{1}{K_p\tau_i}$

system gain: $\frac{1}{\tau_i}$

PI-controller (series)



$$H_c = K_p\left(1+\frac{1}{s\tau_i}\right) = \frac{K_p}{\tau_i}\,\frac{s\tau_i+1}{s}$$



zero in: $s = -\frac{1}{\tau_i}$

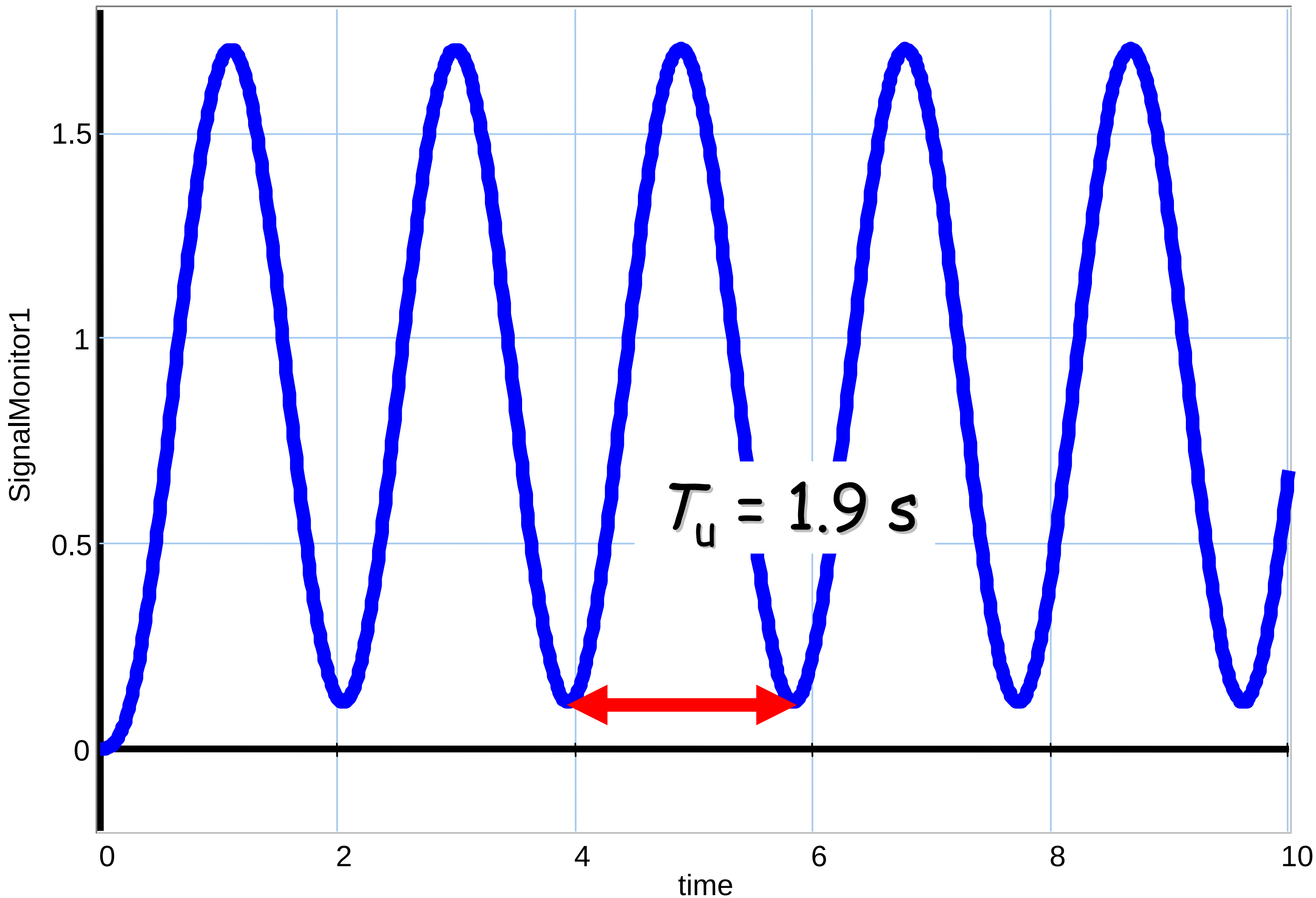
lag network
with pole in origin

system gain: $\frac{K_p}{\tau_i}$

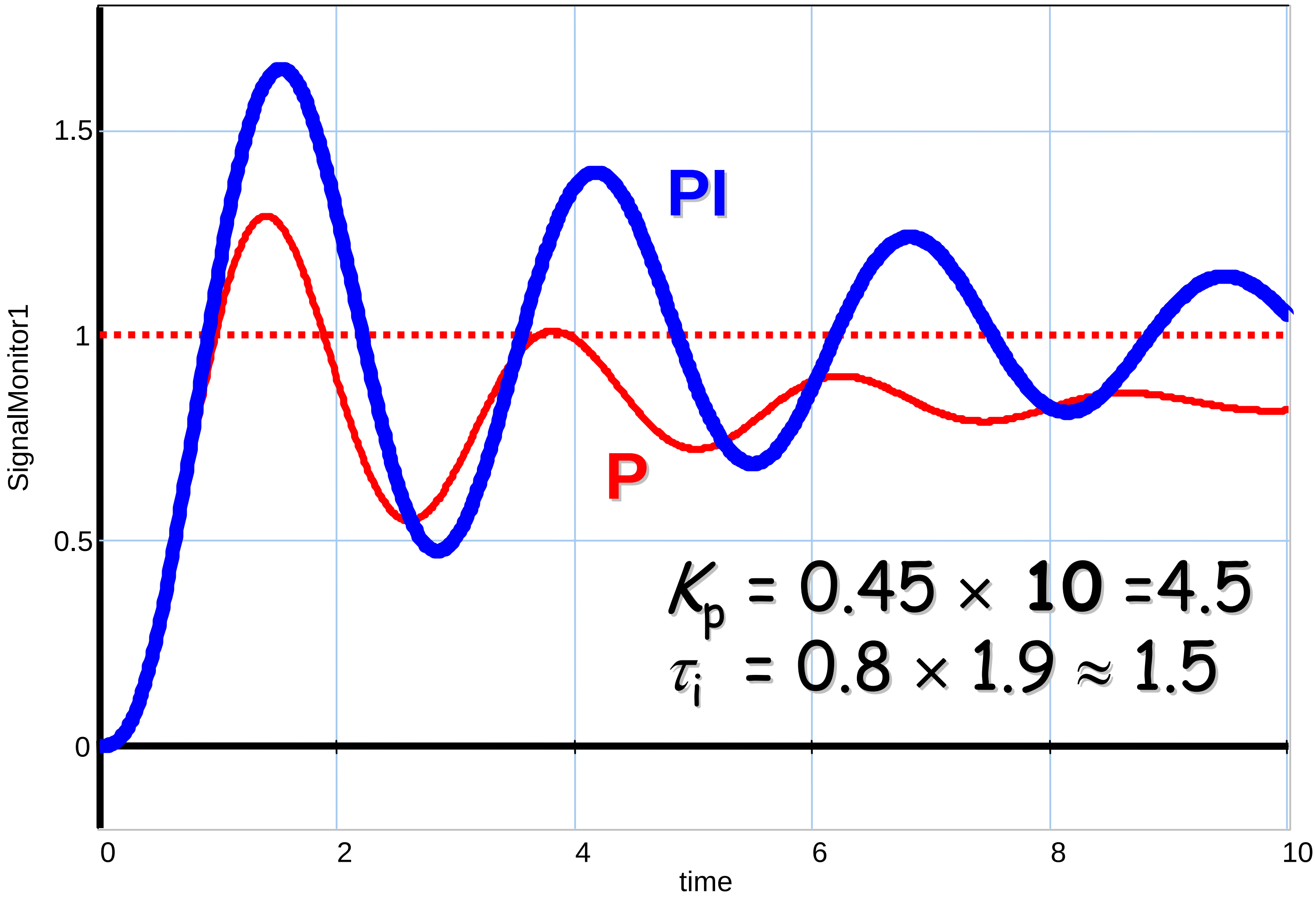
Ziegler-Nichols rules

P-control	$K_p = 0.5 K_u$
PI-control series form	$K_p = 0.45 K_u$ $\tau_i = 0.8 T_u$

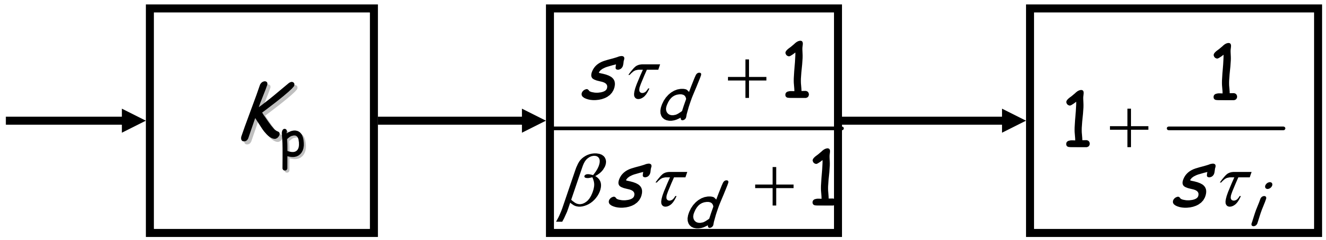
Oscillation period



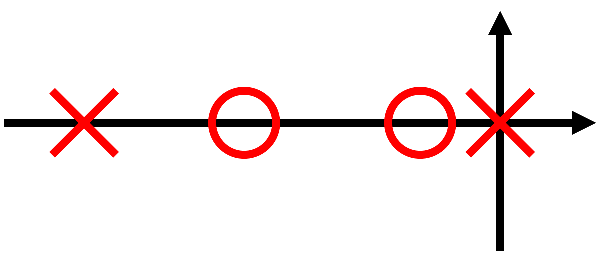
PI-response


1.5
1
0.5
0
SignalMonitor1
0
2
4
6
8
10
time
PI
P
$K_p = 0.45 \times \mathbf{10} = 4.5$
$\tau_i = 0.8 \times 1.9 \approx 1.5$

PID-controller (series)



$$H_c = K_p\left(1+\frac{1}{s\tau_i}\right)\left(\frac{s\tau_d+1}{\beta s\tau_d+1}\right) = \frac{K_p}{\tau_i}\left(\frac{s\tau_i+1}{s}\right)\left(\frac{s\tau_d+1}{\beta s\tau_d+1}\right)$$



lead/lag network
with pole in origin

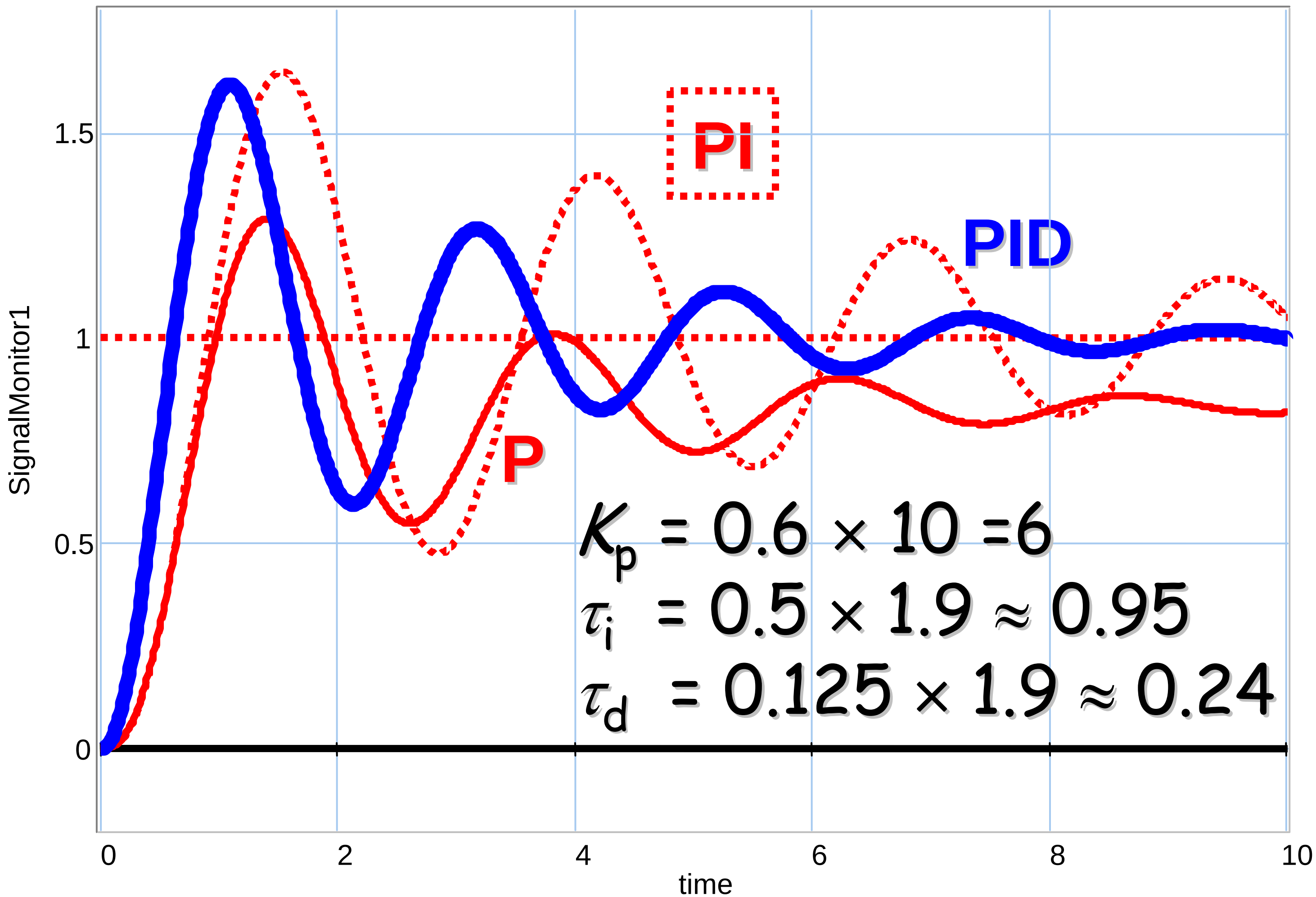
β = typically 0.1

β = taming factor

Ziegler-Nichols rules

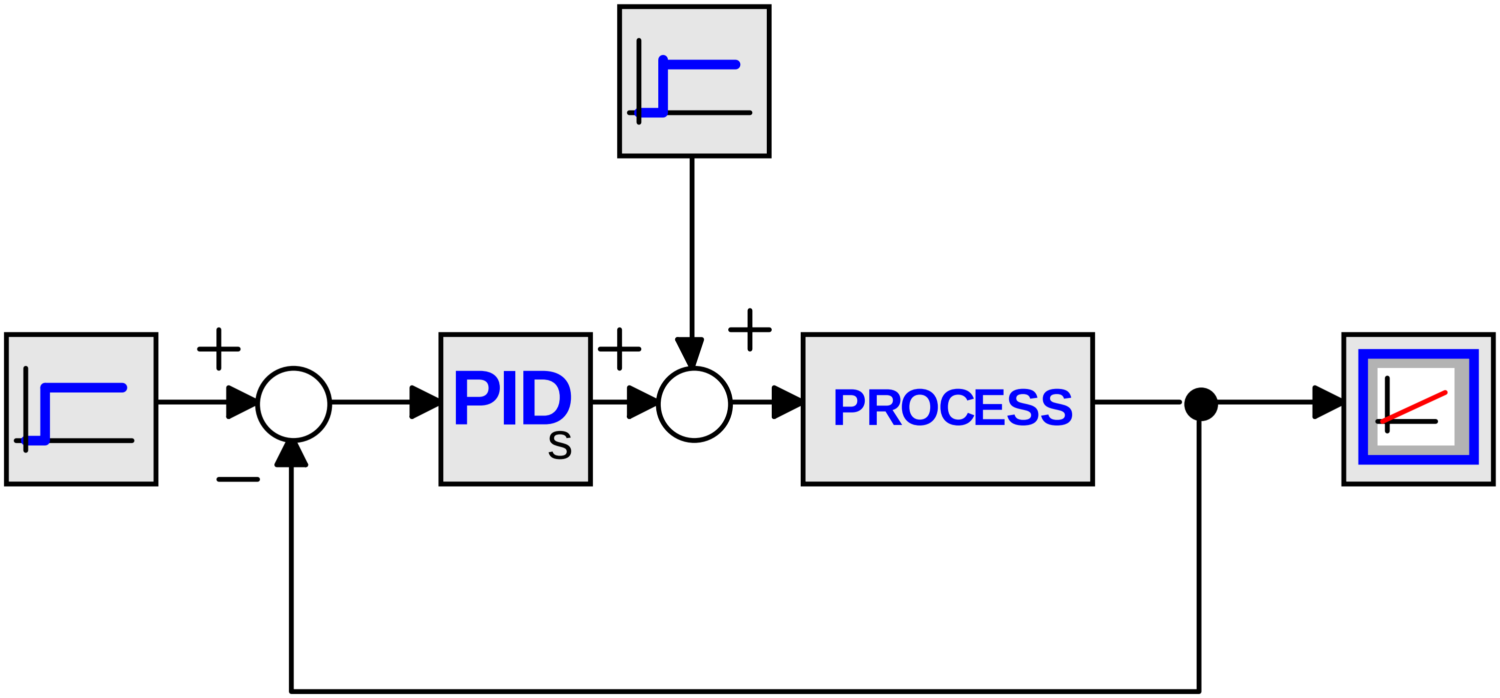
P-control	$K_p = 0.5 K_u$
PI-control series form	$K_p = 0.45 K_u$ $\tau_i = 0.8 T_u$
PID-control series form	$K_p = 0.6 K_u$ $\tau_i = 0.5 T_u$ $\tau_d = 0.125 T_u$

PID-response

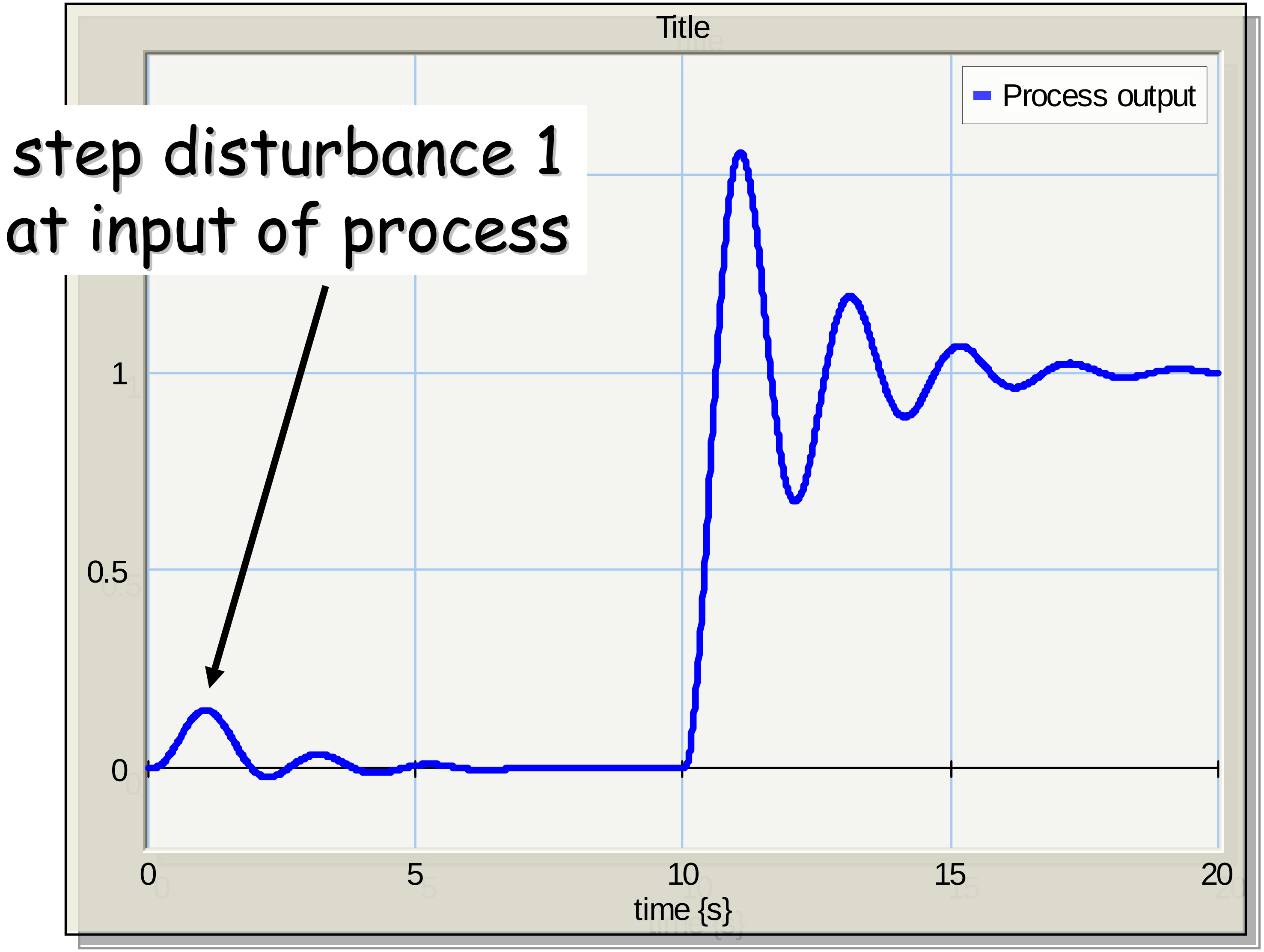


$$K_p = 0.6 \times 10 = 6$$
$$\tau_i = 0.5 \times 1.9 \approx 0.95$$
$$\tau_d = 0.125 \times 1.9 \approx 0.24$$

PID-response



Disturbance rejection

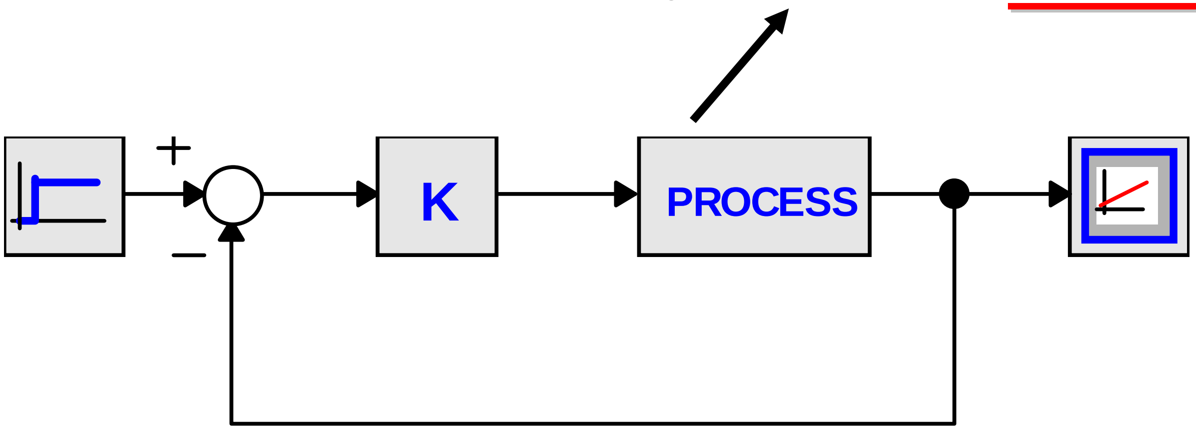


Tuning (τ and T_d)

$$H_{\text{process}}(s) = \frac{K\,e^{-0.1s}}{(s+1)}$$

small dead time
large time constant

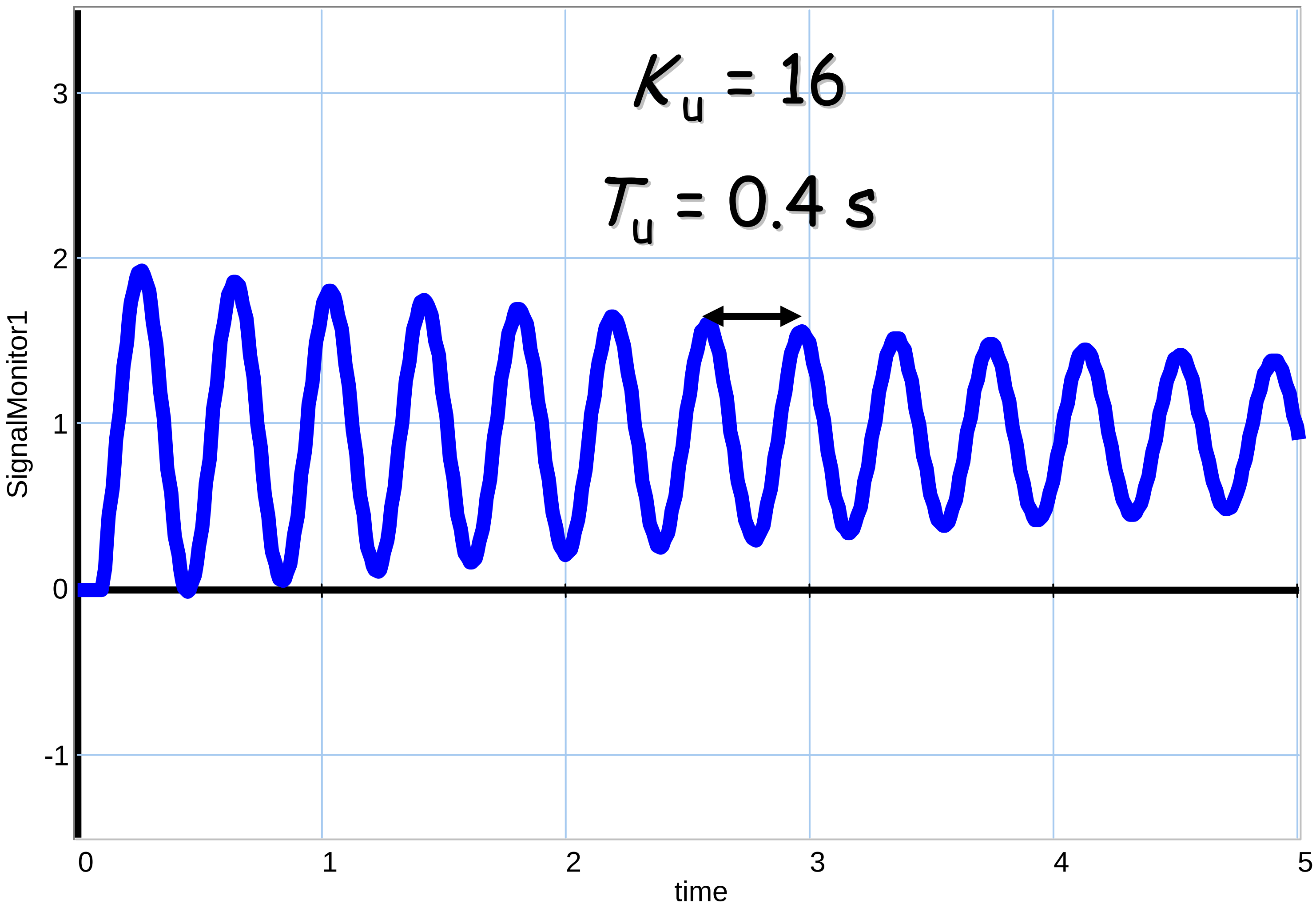
assumed to be unknown



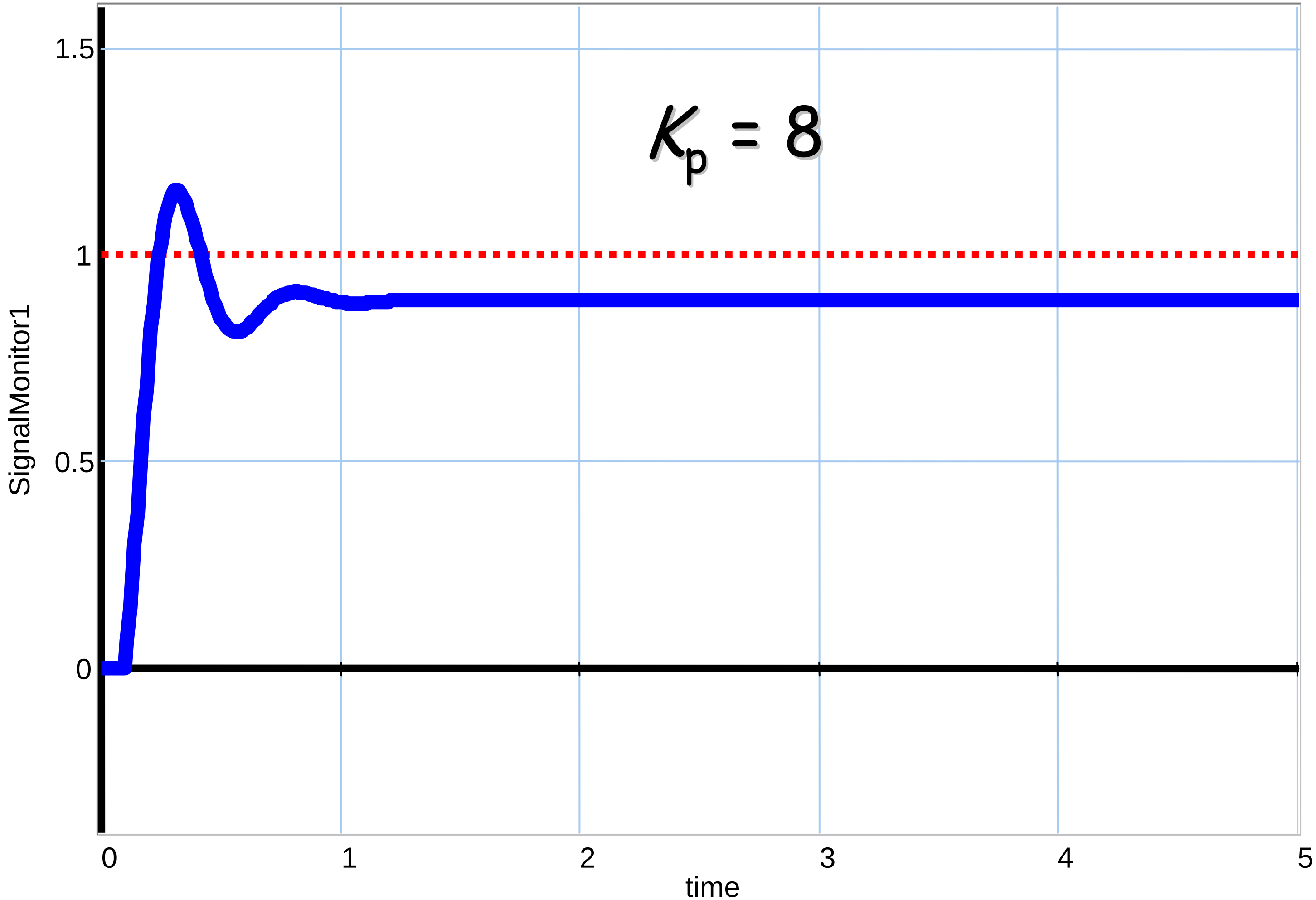
20-sim: PID_tuning_P_Td

Bring feedback system at the border of instability

Border of instability



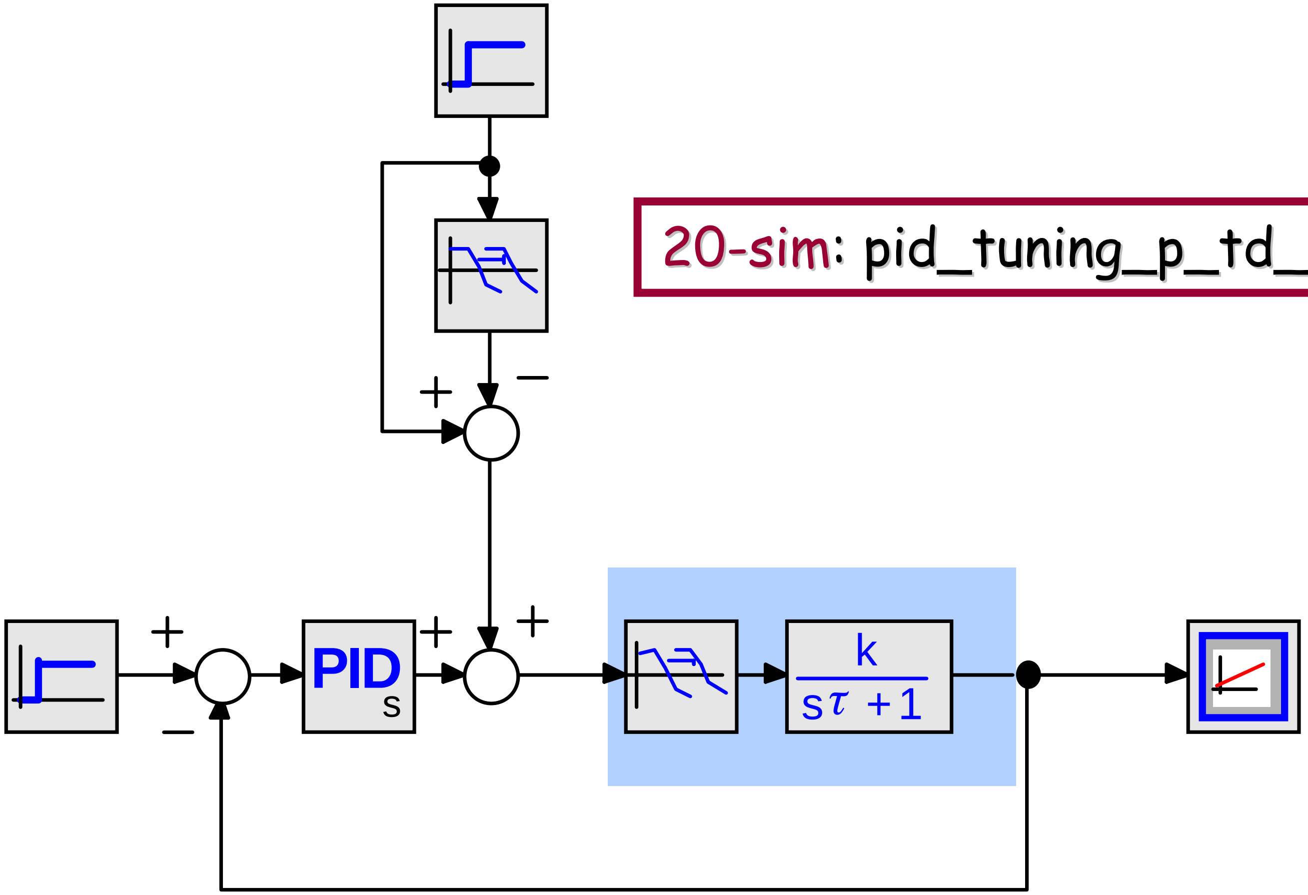
P-controller



time

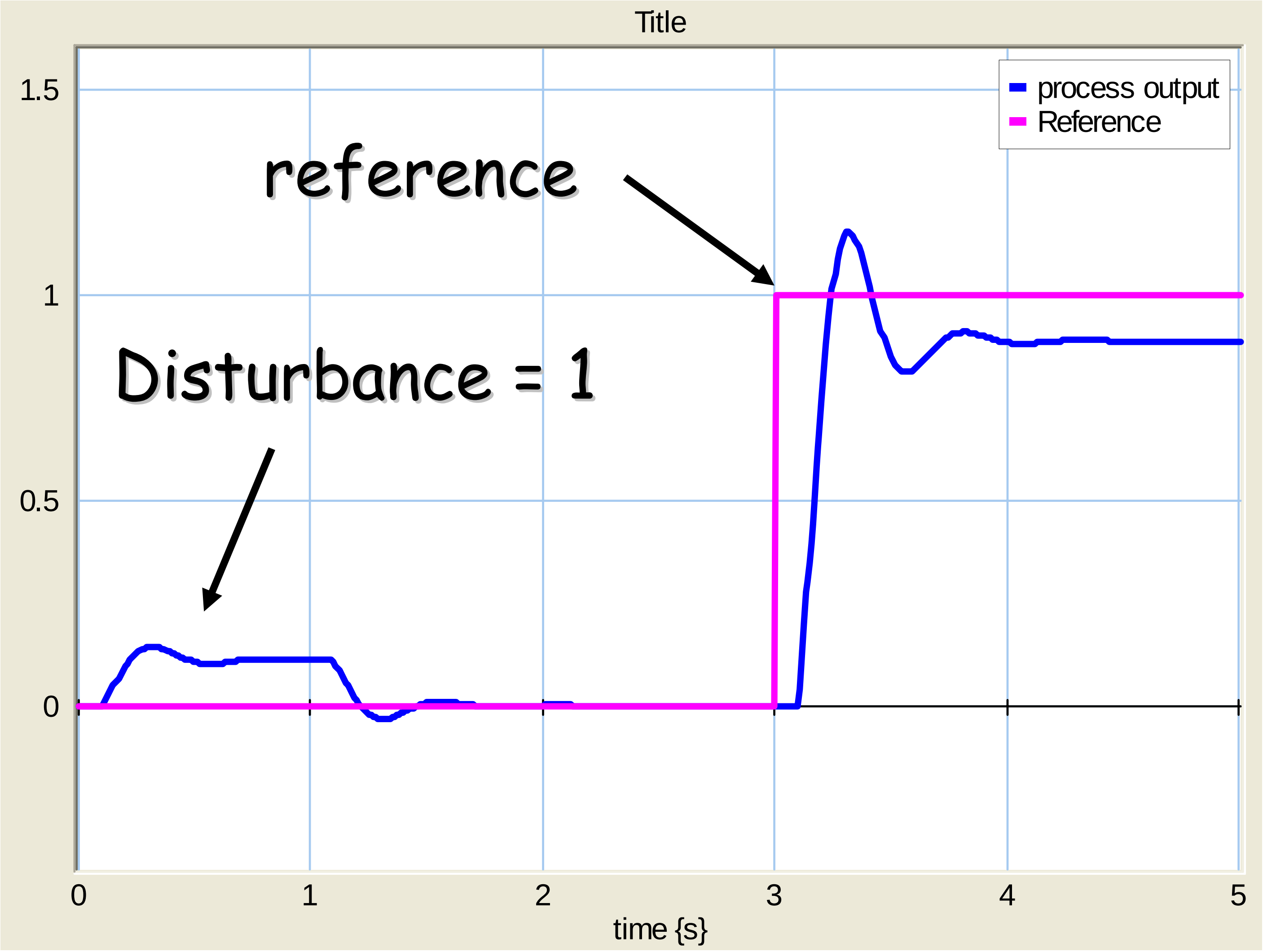
P-controller

20-sim: pid_tuning_p_td_with disturbance



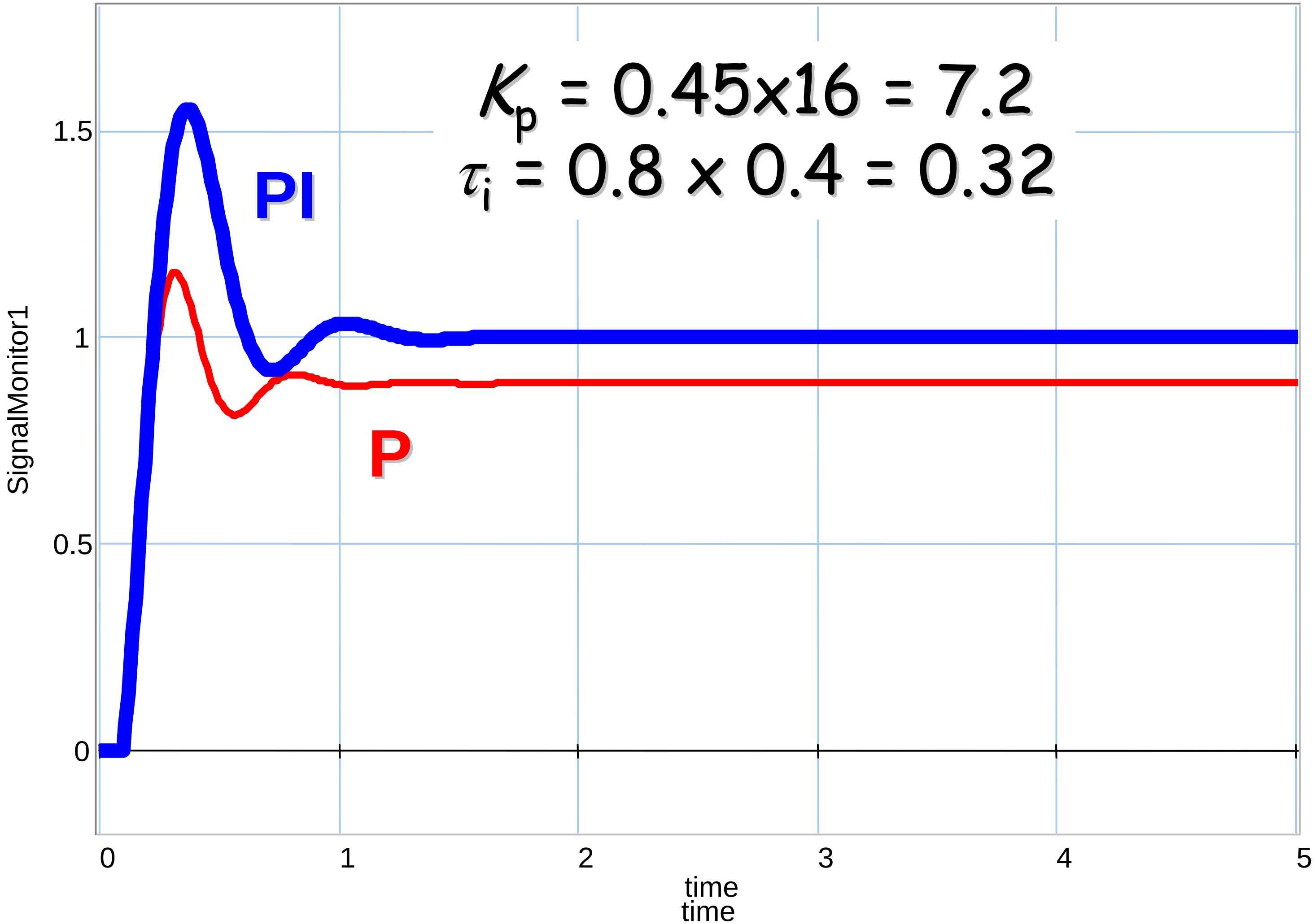
time

P-controller

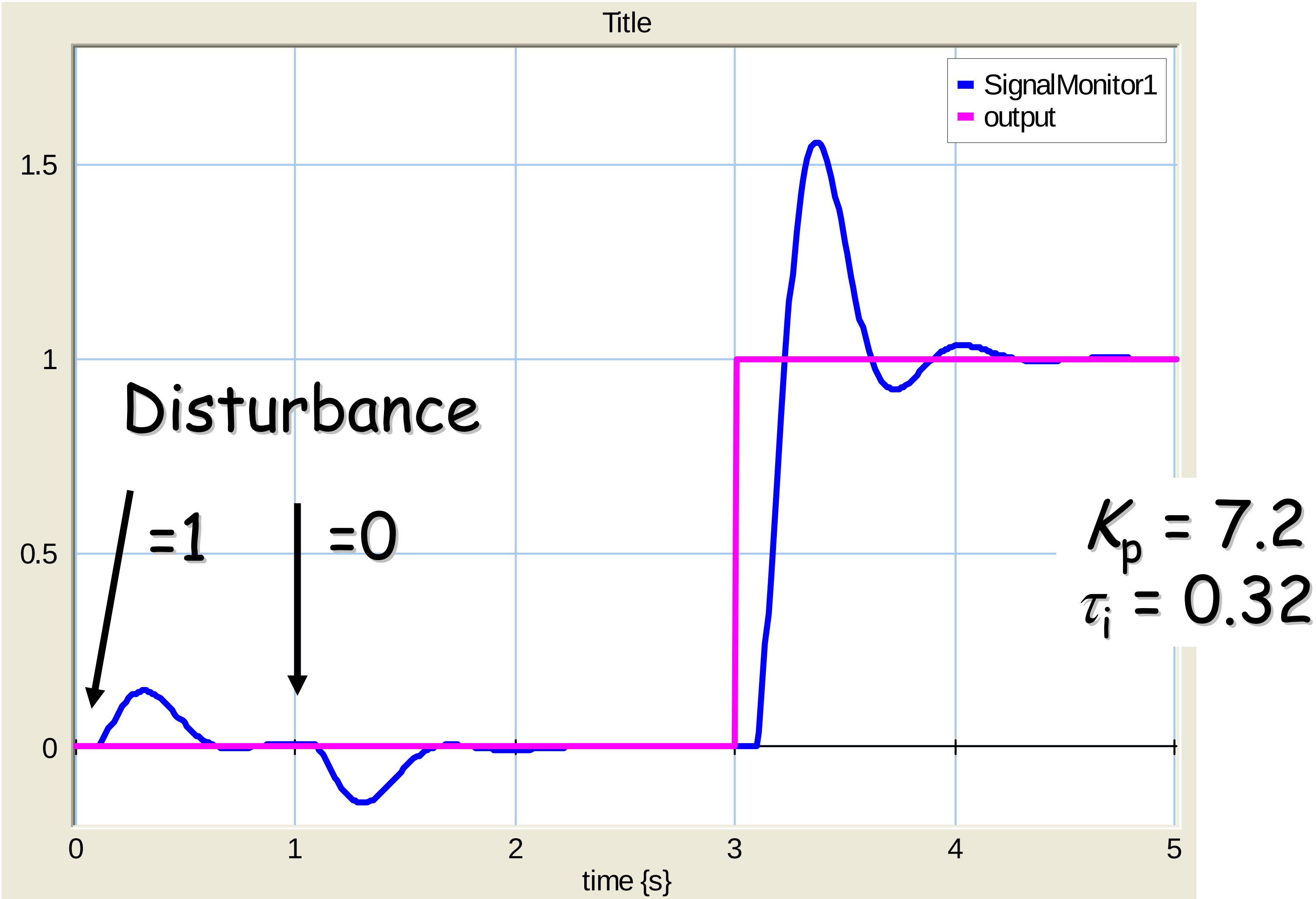


time

PI-controller

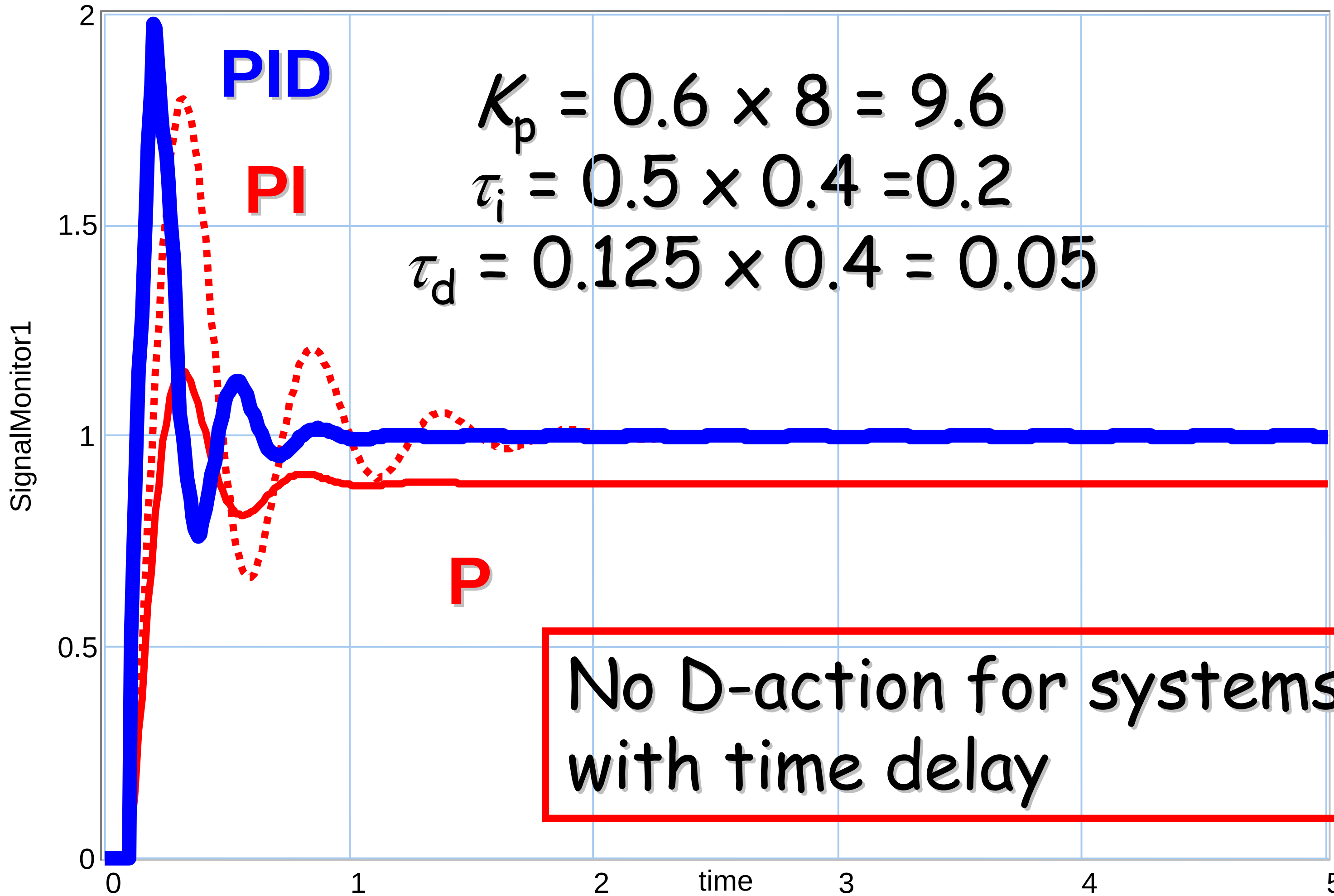


PI-controller



time

PID-controller



$K_p = 0.6 \times 8 = 9.6$

$\tau_i = 0.5 \times 0.4 = 0.2$

$\tau_d = 0.125 \times 0.4 = 0.05$

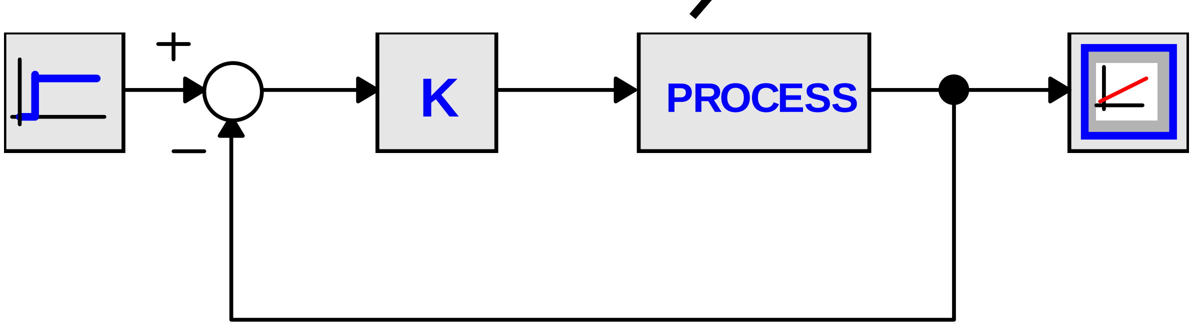
No D-action for systems with time delay

Tuning (τ and T_d)

$$H_{process}(s) = \frac{K\, e^{-1s}}{(s+0.1)}$$

large dead time
small time constant

assumed to be unknown

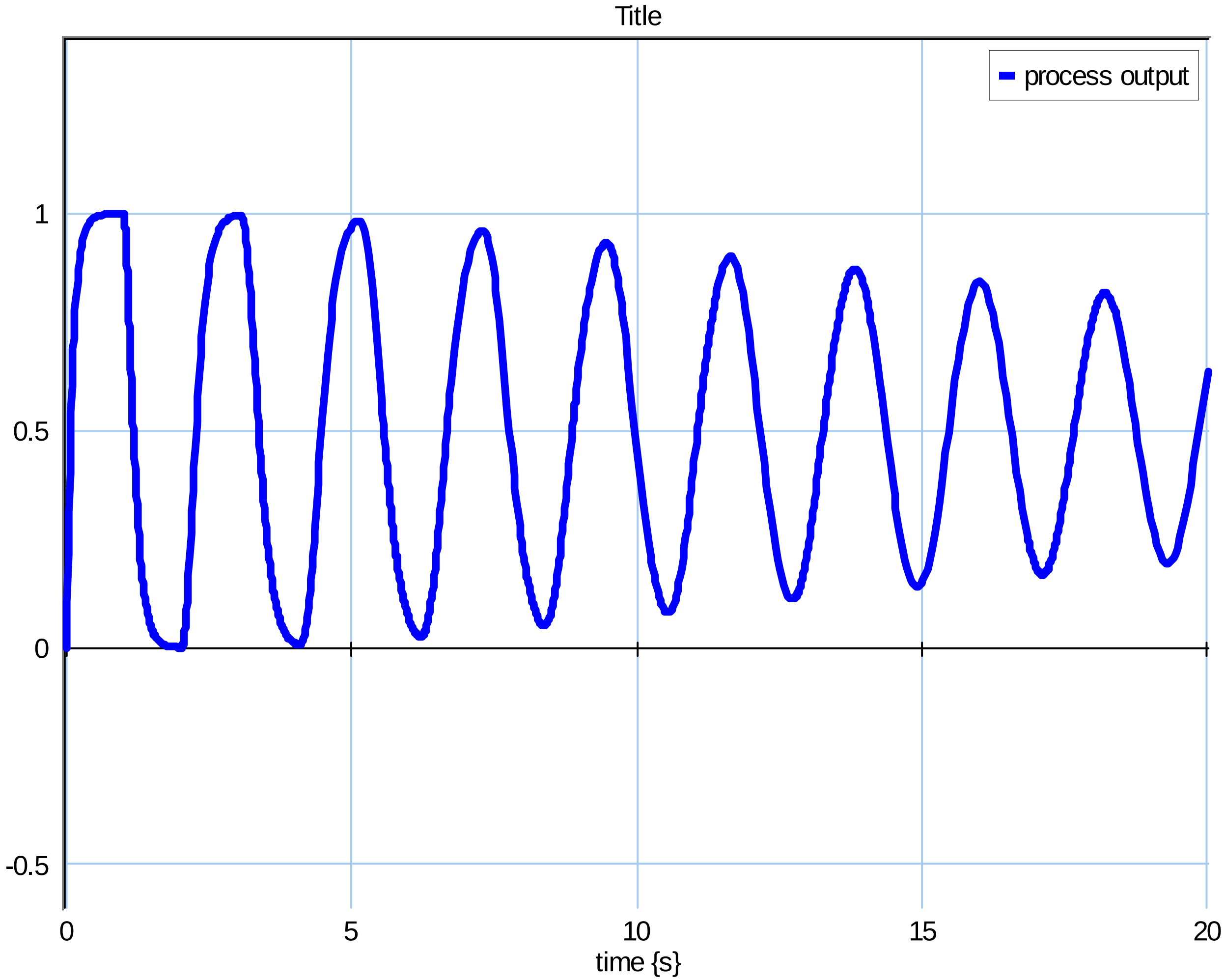


20-sim: PID_tuning_P_Td

Bring feedback system at the border of instability

Border of instability

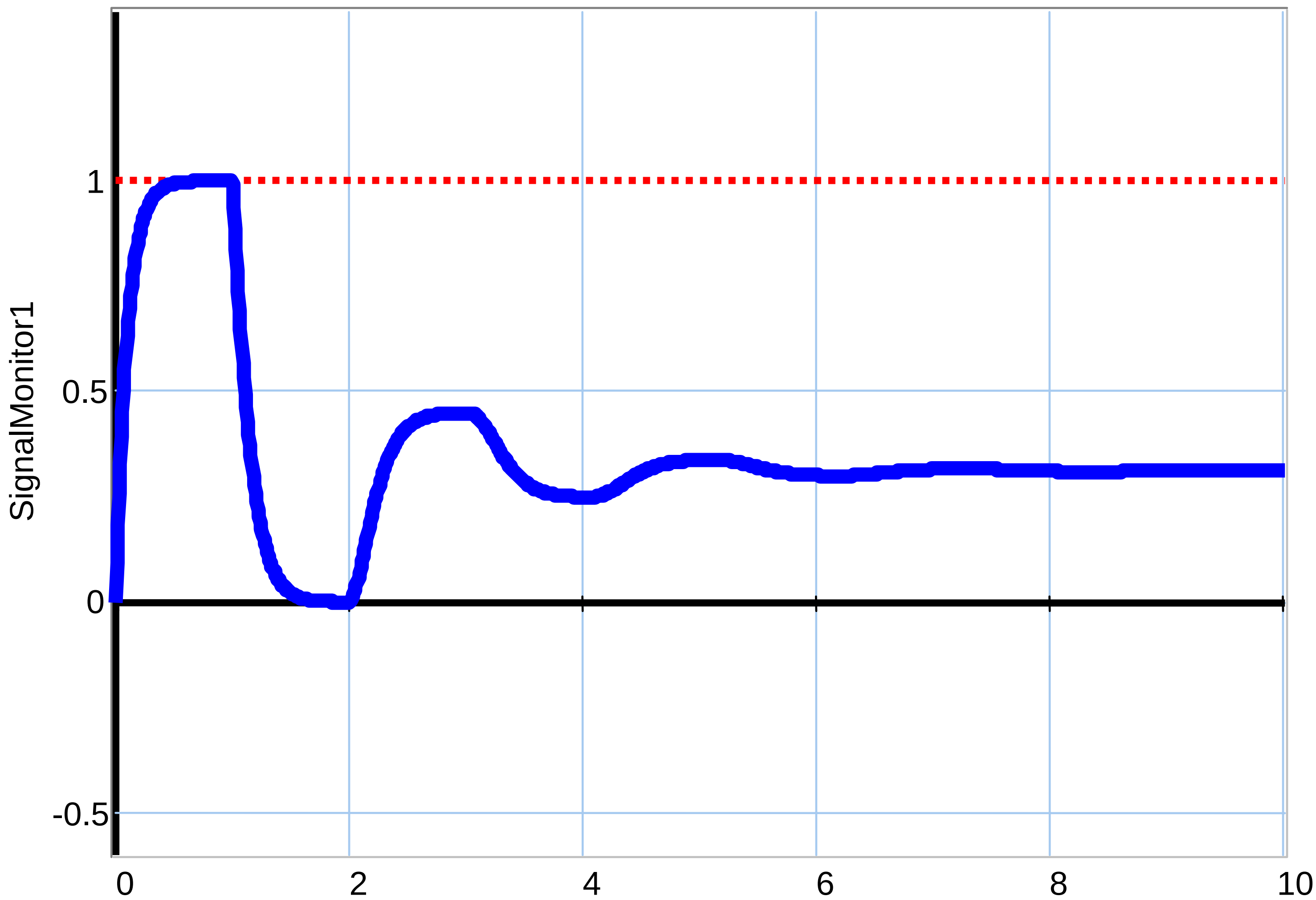
Control Engineering
University of Twente



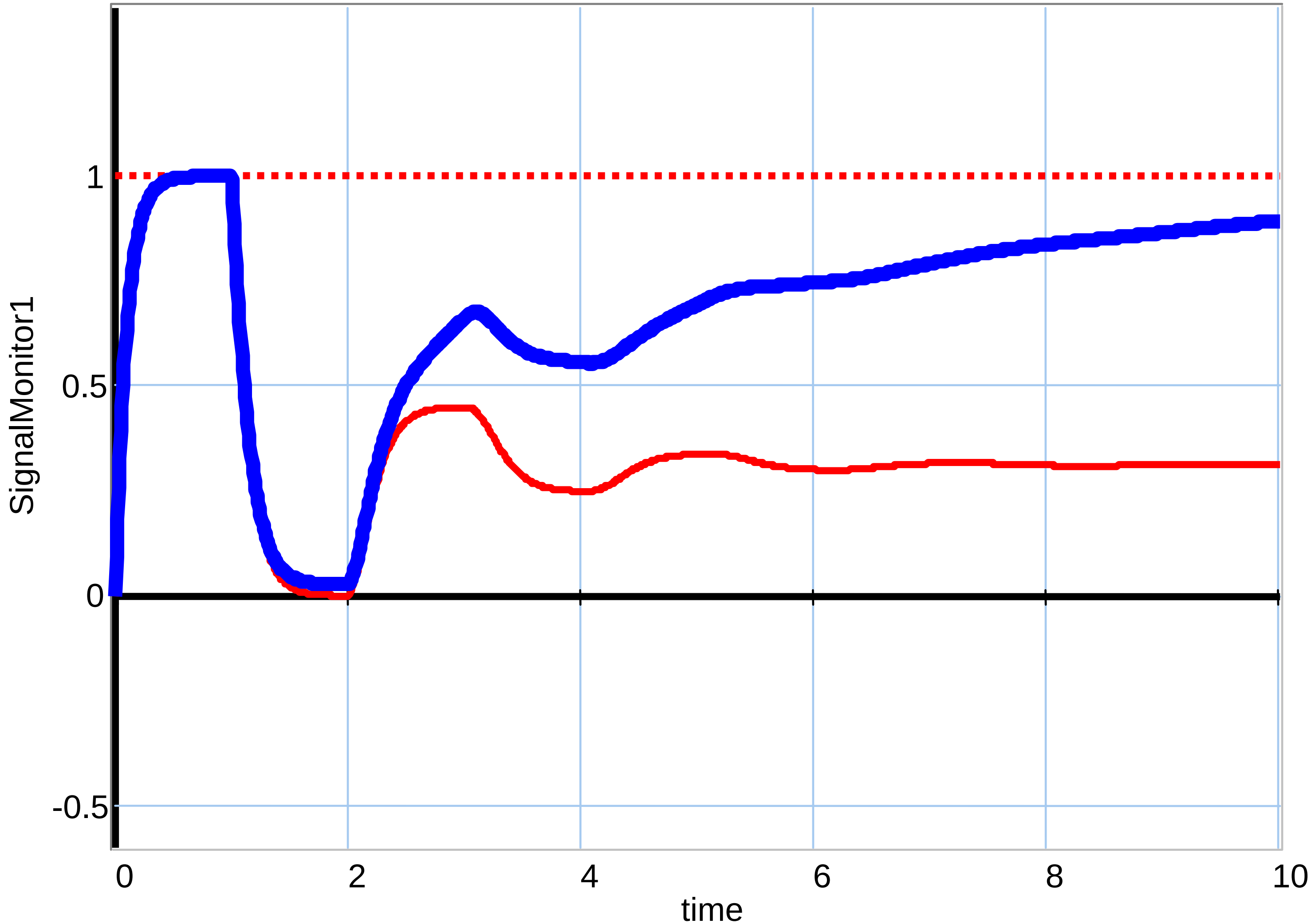
$K_u = 1$

$T_u = 2.1\ s$

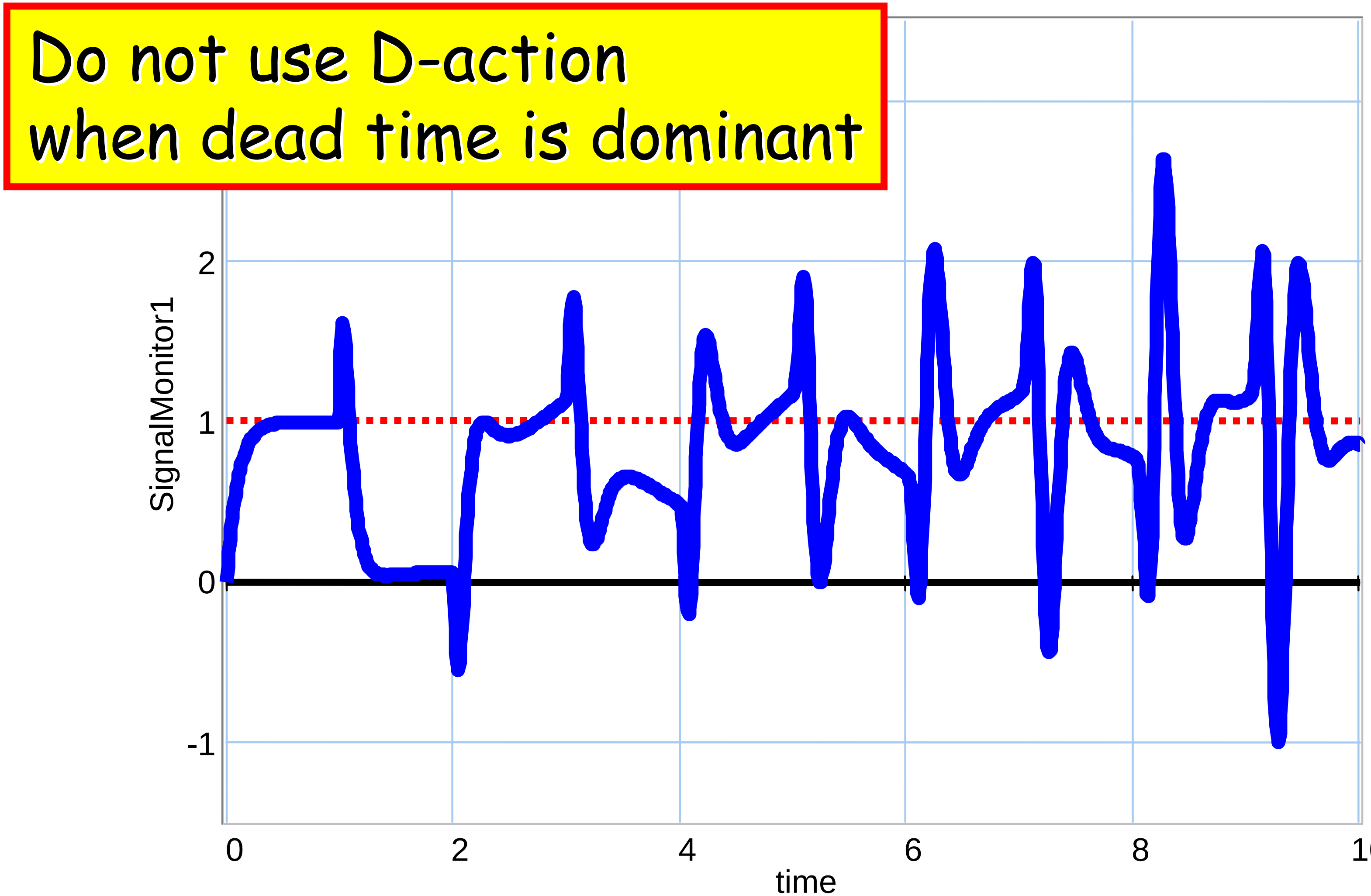
P-control



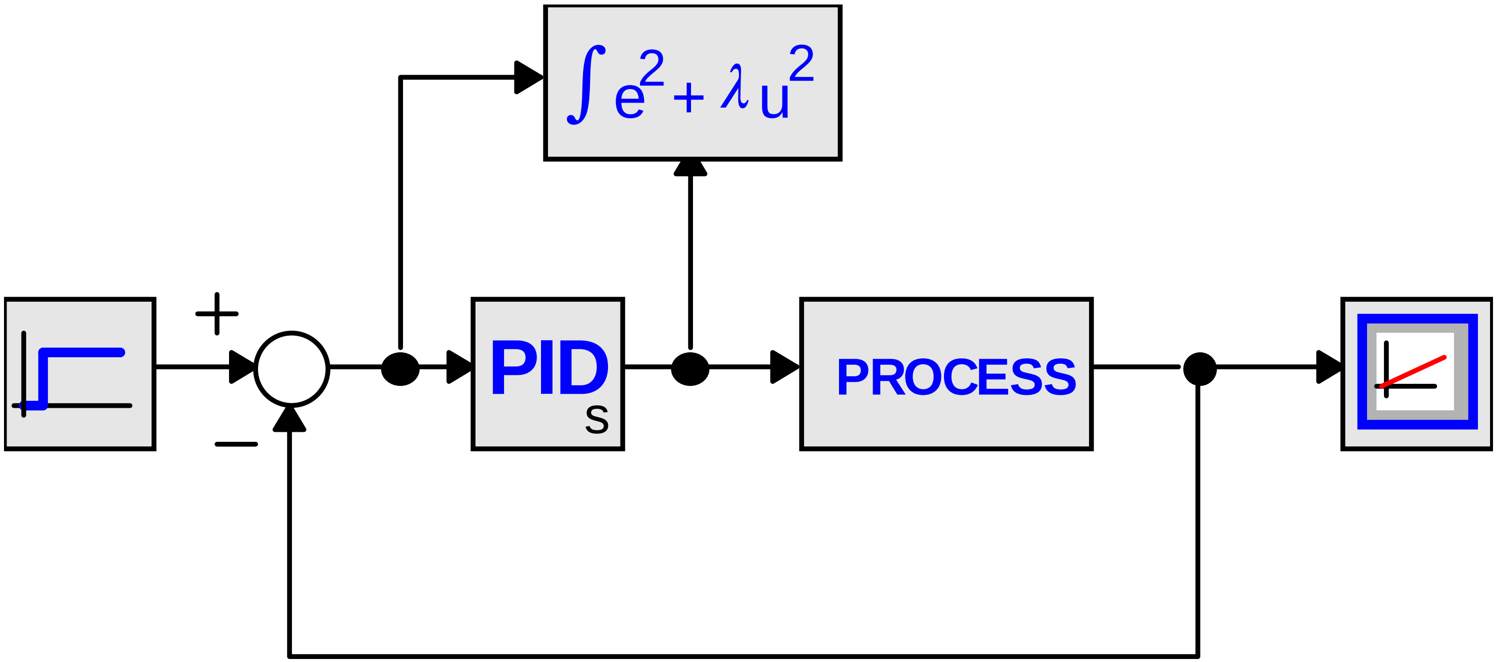
PI-control



PID-control


Do not use D-action
when dead time is dominant
SignalMonitor1
2
1
0
-1
0
2
4
6
8
10
time

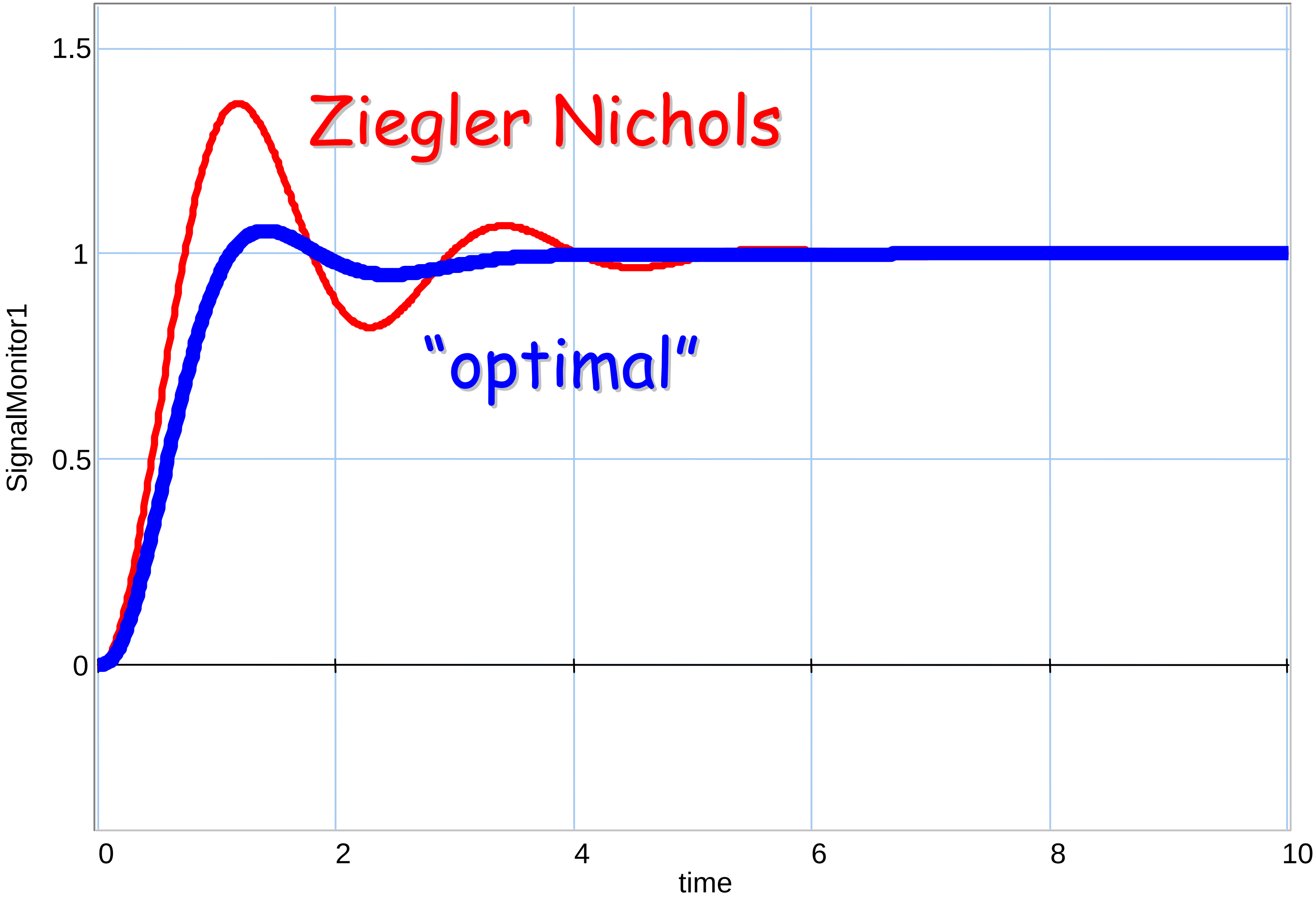
Optimal tuning



process must be known,
identified first,
or optimised on line

20-sim

Optimal tuning



Conclusions

- P(I)(D)-control suited for:
  - systems with an S-shaped response
  - systems with time delay
- Ziegler Nichols rules give 'reasonable' responses
  - (more suited for disturbance rejection than for tracking)
- Better tuning possible