# Frequency Domain Design


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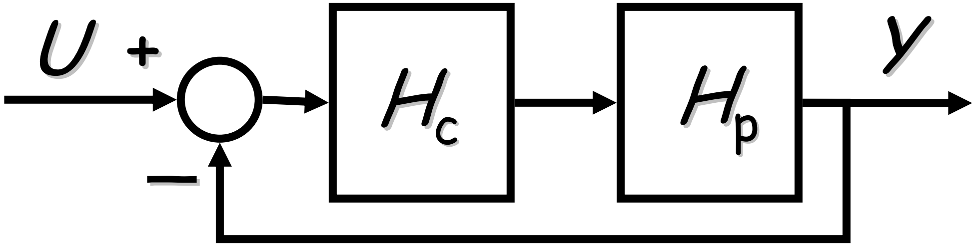
- Frequency diagrams of open and closed systems
- Sensitivity
- Design of lead and lag networks

# Open and closed systems






$$H_p = \frac{1}{s+1}$$

$$H_c = K$$

| | open loop | closed loop |
|---|---|---|
| | $H_L = \frac{K}{s+1}$ | $H = \frac{K}{s+(1+K)}$ |
| $\omega = 0$ | $K$ | $K/_{1+K}$ |
| $\omega \to \infty$ | $\frac{K}{j\omega}$ | $\frac{K}{j\omega}$ |

# Open and closed systems

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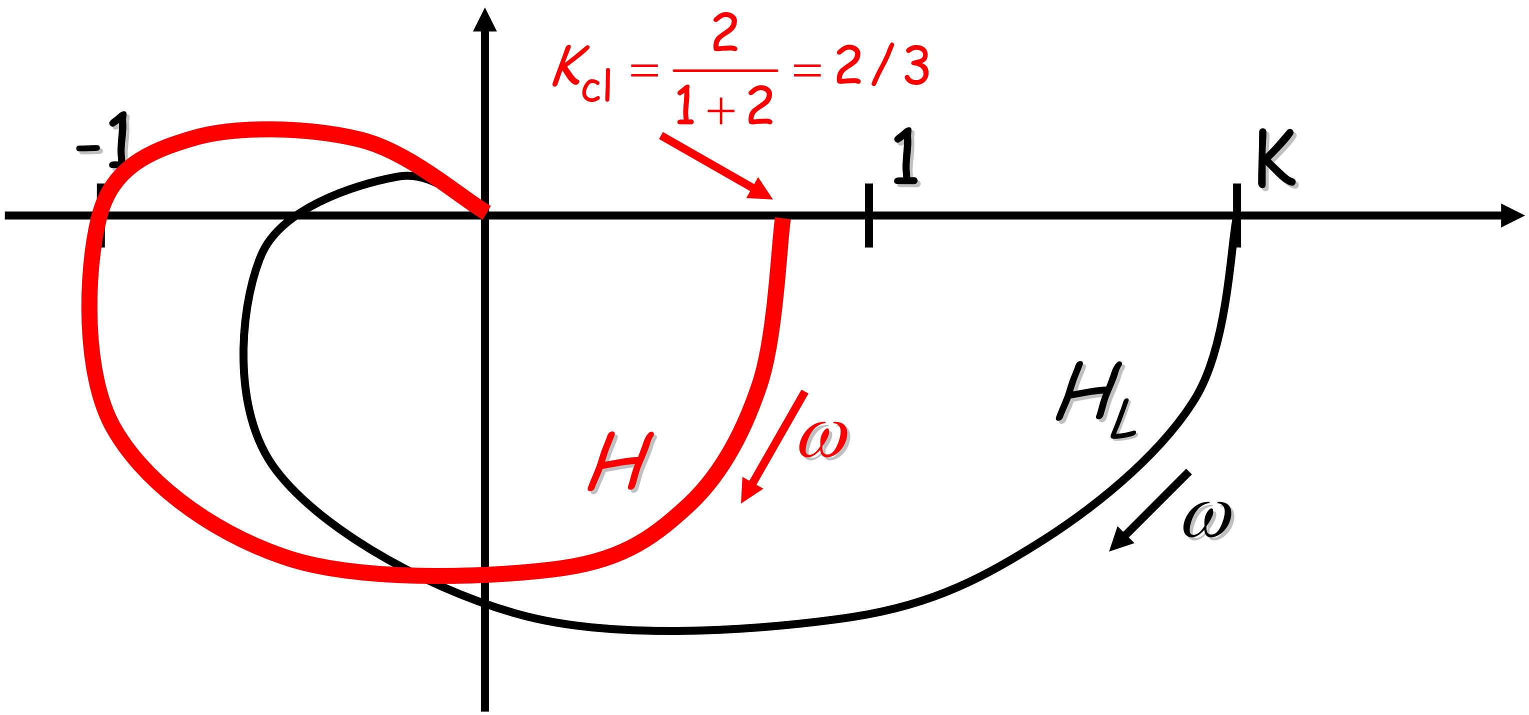
Feedback only effects the low-frequencies

If $K$ large, for $\omega \to 0$, $H_{\text{closed loop}} \to 1$

| | open loop | closed loop |
|---|---|---|
| | $H_L = \frac{K}{s+1}$ | $H = \frac{K}{s+(1+K)}$ |
| $\omega = 0$ | $K$ | $K/(1+K)$ |
| $\omega \to \infty$ | $\frac{K}{j\omega}$ | $\frac{K}{j\omega}$ |

Feedback only effects the low-frequencies

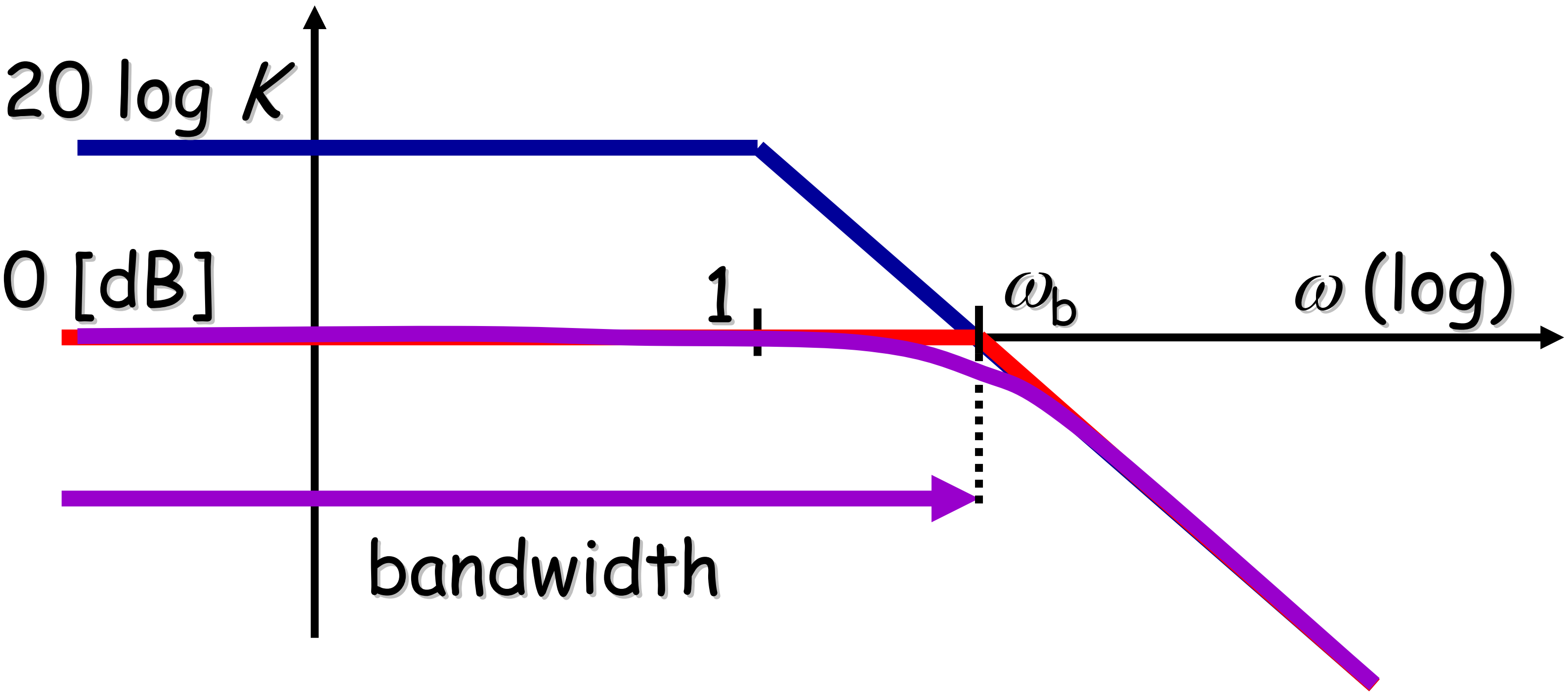
If $K$ large, for $\omega \to 0$, $H_{\text{closed loop}} \to 1$



# Open and closed systems

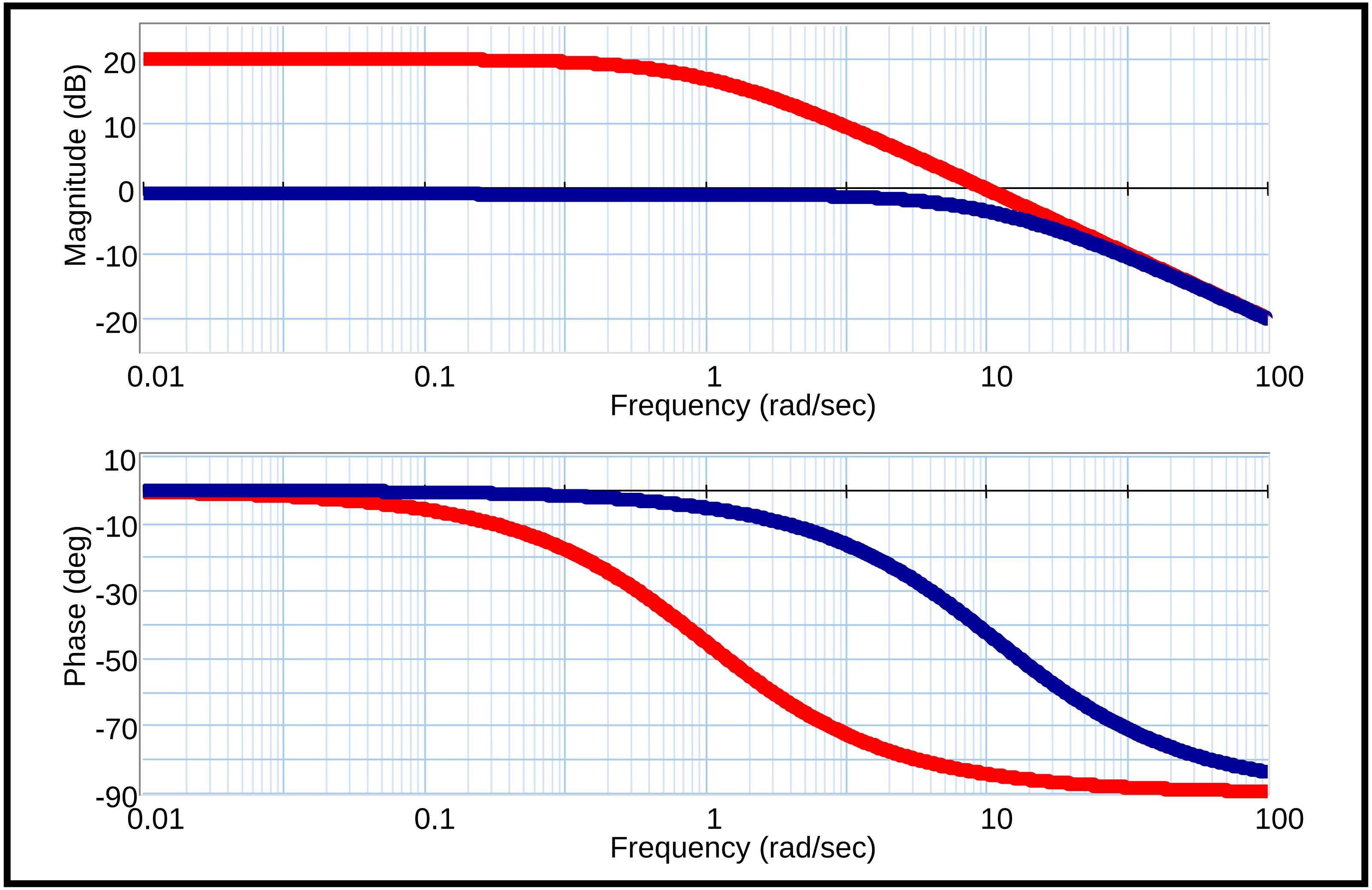
Feedback only effects the low-frequencies

If $K$ large, for $\omega \rightarrow 0$, $H_{cl} \rightarrow 1$



# 20-sim: 10/(s+1)




Magnitude (dB)
20
10
0
-10
-20
0.01
0.1
1
10
100
Frequency (rad/sec)
Phase (deg)
10
-10
-30
-50
-70
-90
0.01
0.1
1
10
100
Frequency (rad/sec)

# Open and closed systems

$$H_{cl} = \frac{H_L}{1 + H_L}$$

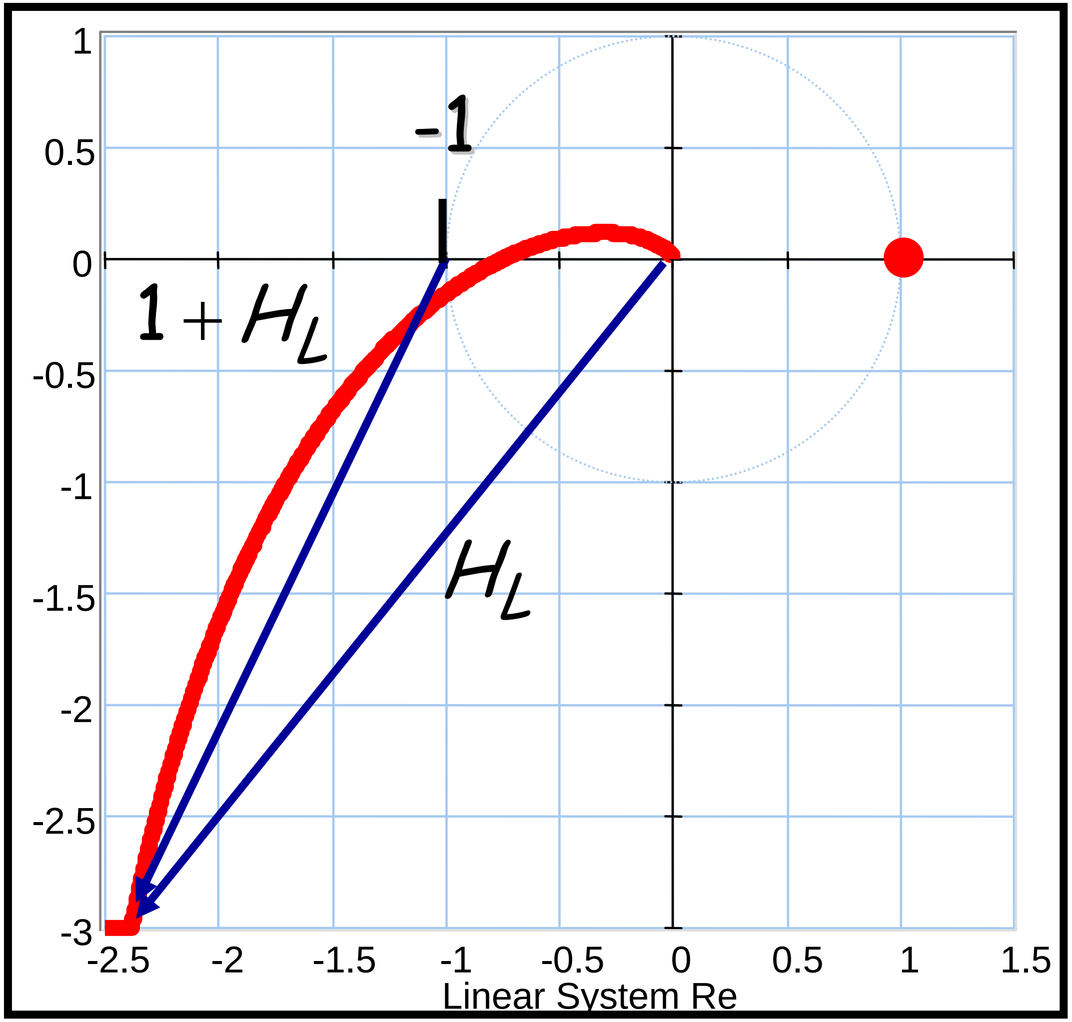
$$\omega = 0$$

$$|H_L| \approx |1 + H_L|$$

$$\arg(H_L) \approx \arg(1 + H_L)$$

$$\approx -\frac{\pi}{2}$$

$$H_{cl} \to 1$$



# Open and closed systems

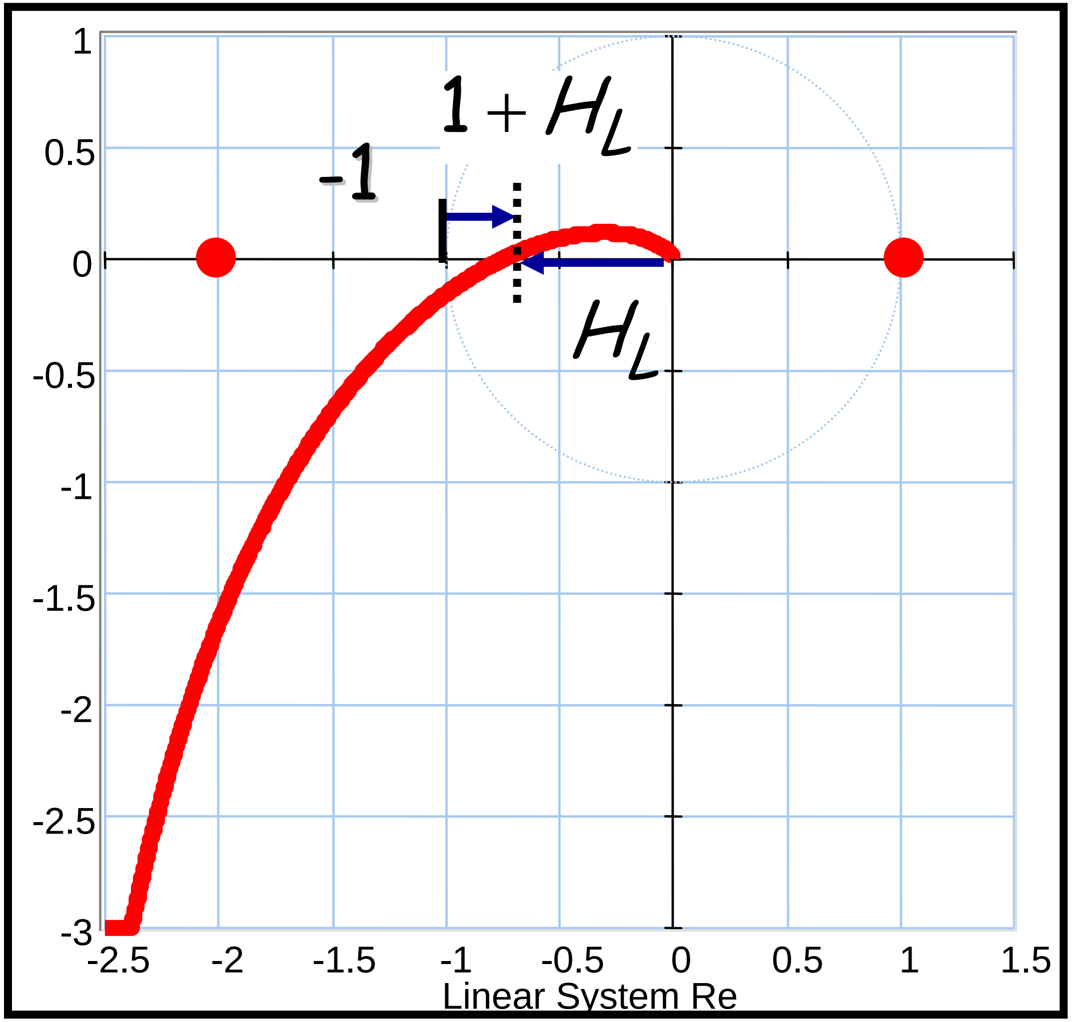
$$H_{cl} = \frac{H_L}{1 + H_L}$$

$$|H_L| \approx \frac{2}{3} \qquad |1 + H_L| \approx \frac{1}{3}$$

$$\arg(H_L) = -\pi$$

$$\arg(1 + H_L) = 0$$

$$H_{cl} \to 2e^{-j\pi}$$



# Open and closed systems

$$H_{cl} = \frac{H_L}{1 + H_L}$$

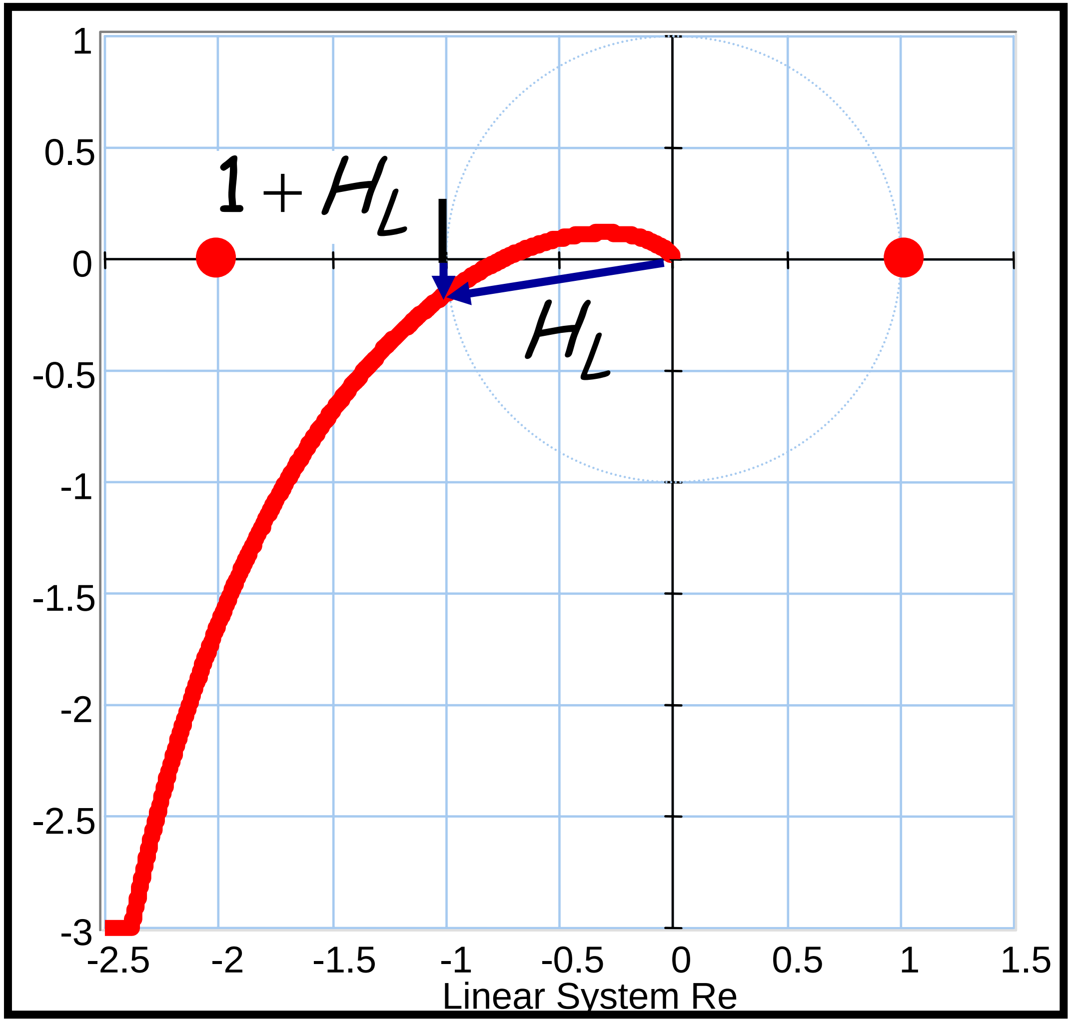
$$\left| H_L \right| \approx 1.25$$

$$\left| 1 + H_L \right| \approx 0.25$$

$$\arg\left( H_L \right) \approx -\pi$$

$$\arg(1 + H_L) \approx -\frac{\pi}{2}$$

$$H_{cl} \rightarrow 5e^{-j\frac{\pi}{2}}$$



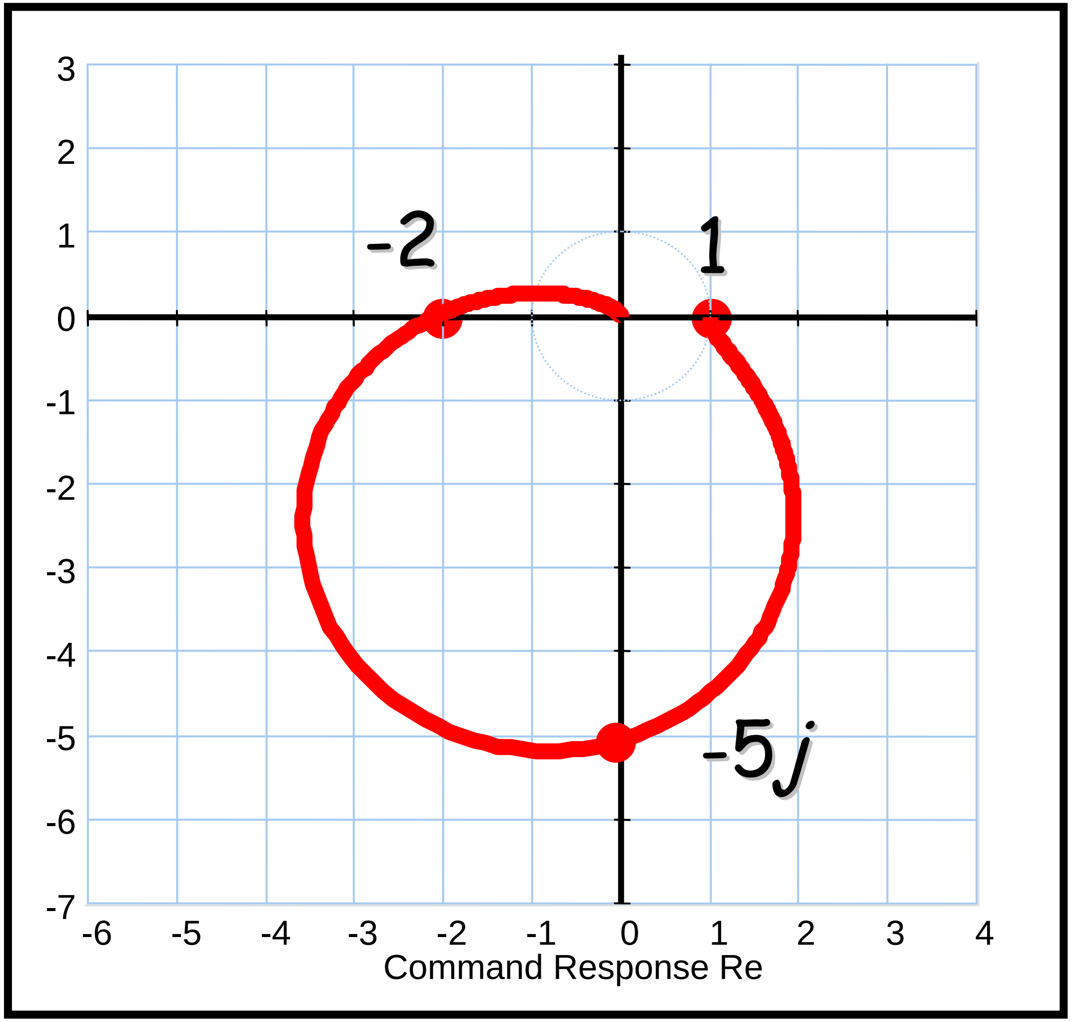
# Open and closed systems



$$H_{cl} = \frac{H_L}{1 + H_L}$$

$$H_{cl} \rightarrow 5e^{-j\frac{\pi}{2}}$$

Stable because
-1 outside plot of $H_L$
(open system)

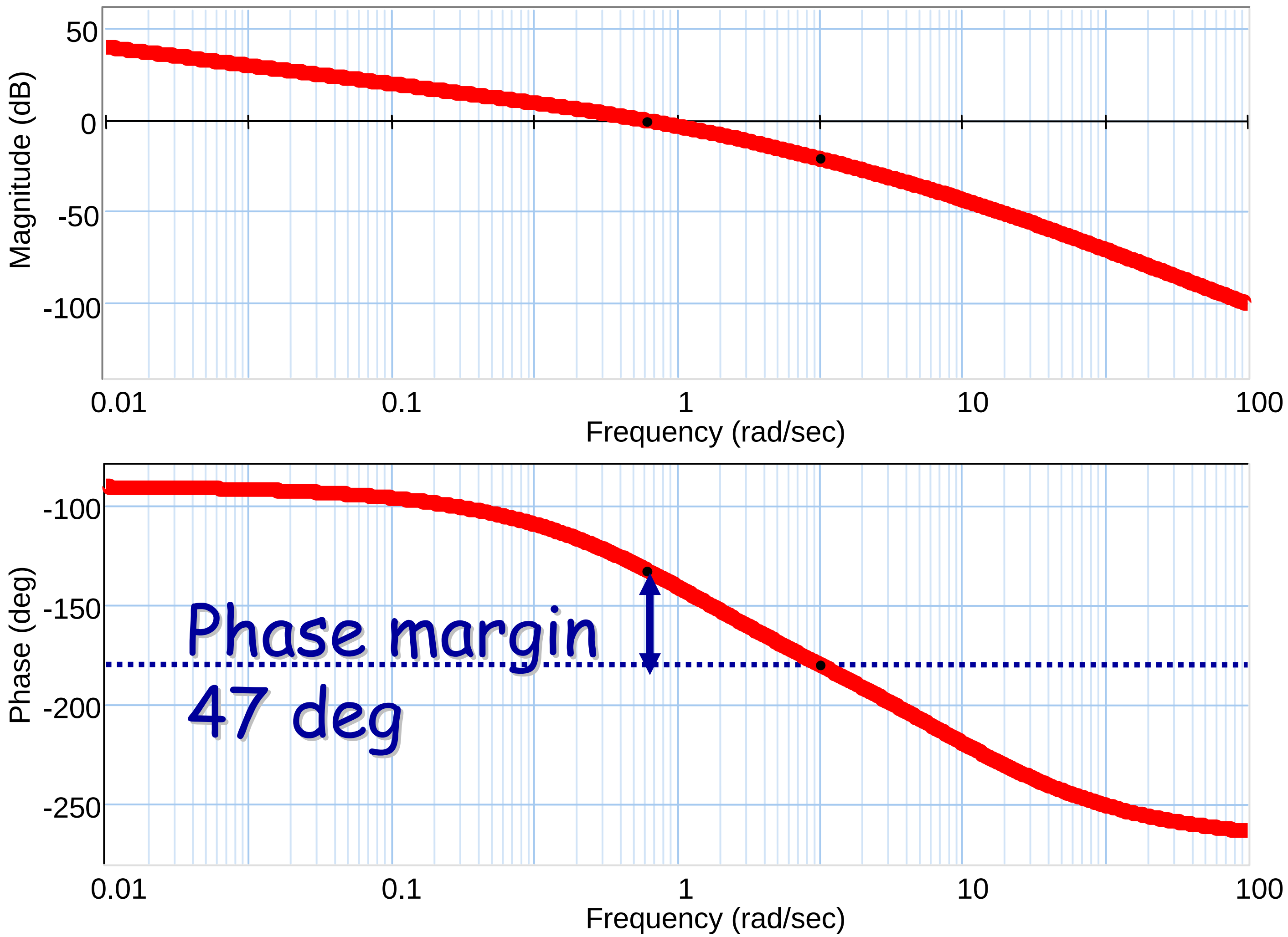


# Design

- Design a proportional controller such that the system has a phase margin of 70 degrees ($z \approx 0.7$) for the process:

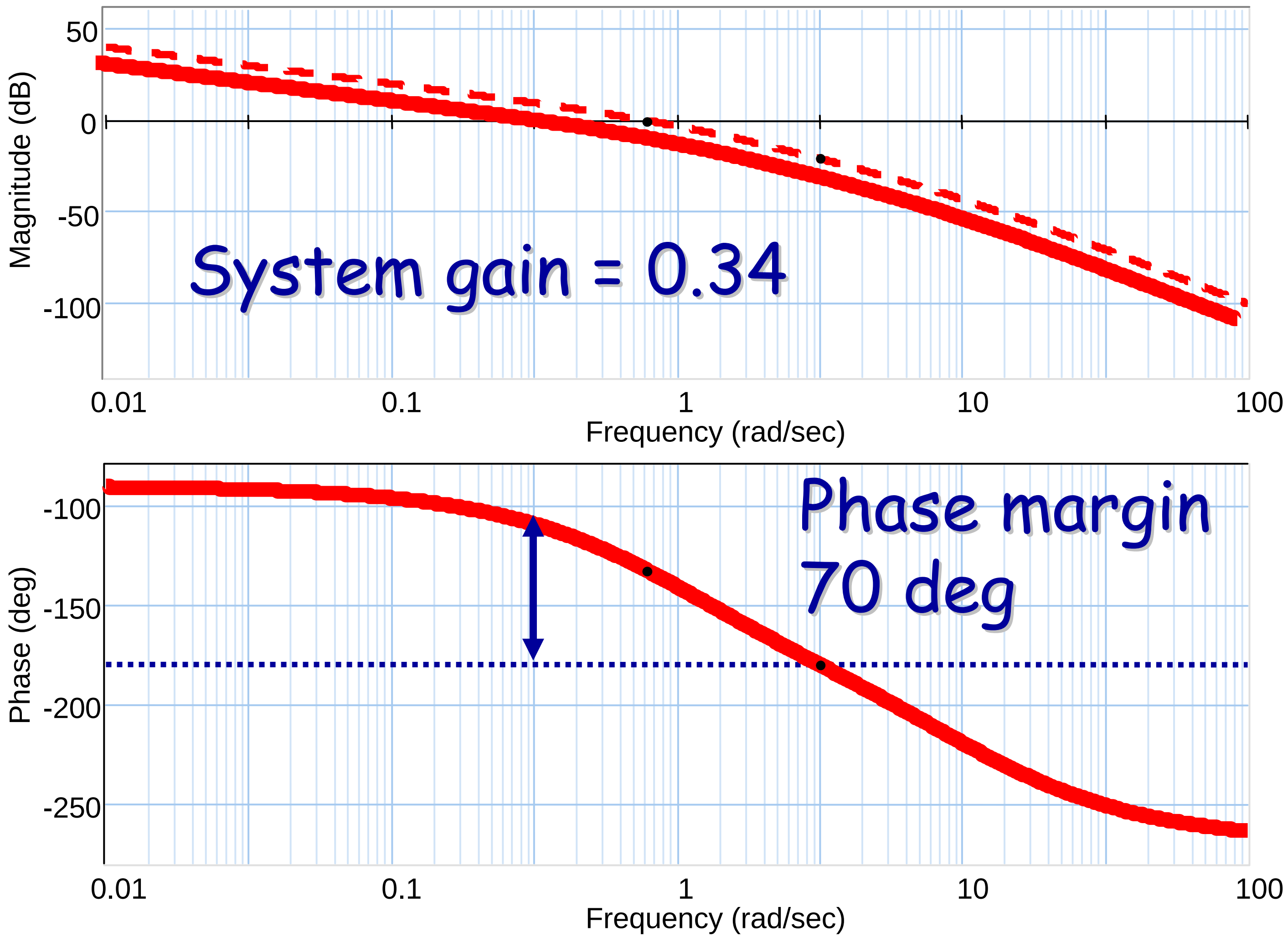
$$H(j\omega) = \frac{10}{j\omega(j\omega+1)(j\omega+10)}$$

# Design


50
0
-50
-100
Magnitude (dB)
0.01
0.1
1
10
100
Frequency (rad/sec)
-100
-150
-200
-250
Phase (deg)
Phase margin
47 deg
0.01
0.1
1
10
100
Frequency (rad/sec)

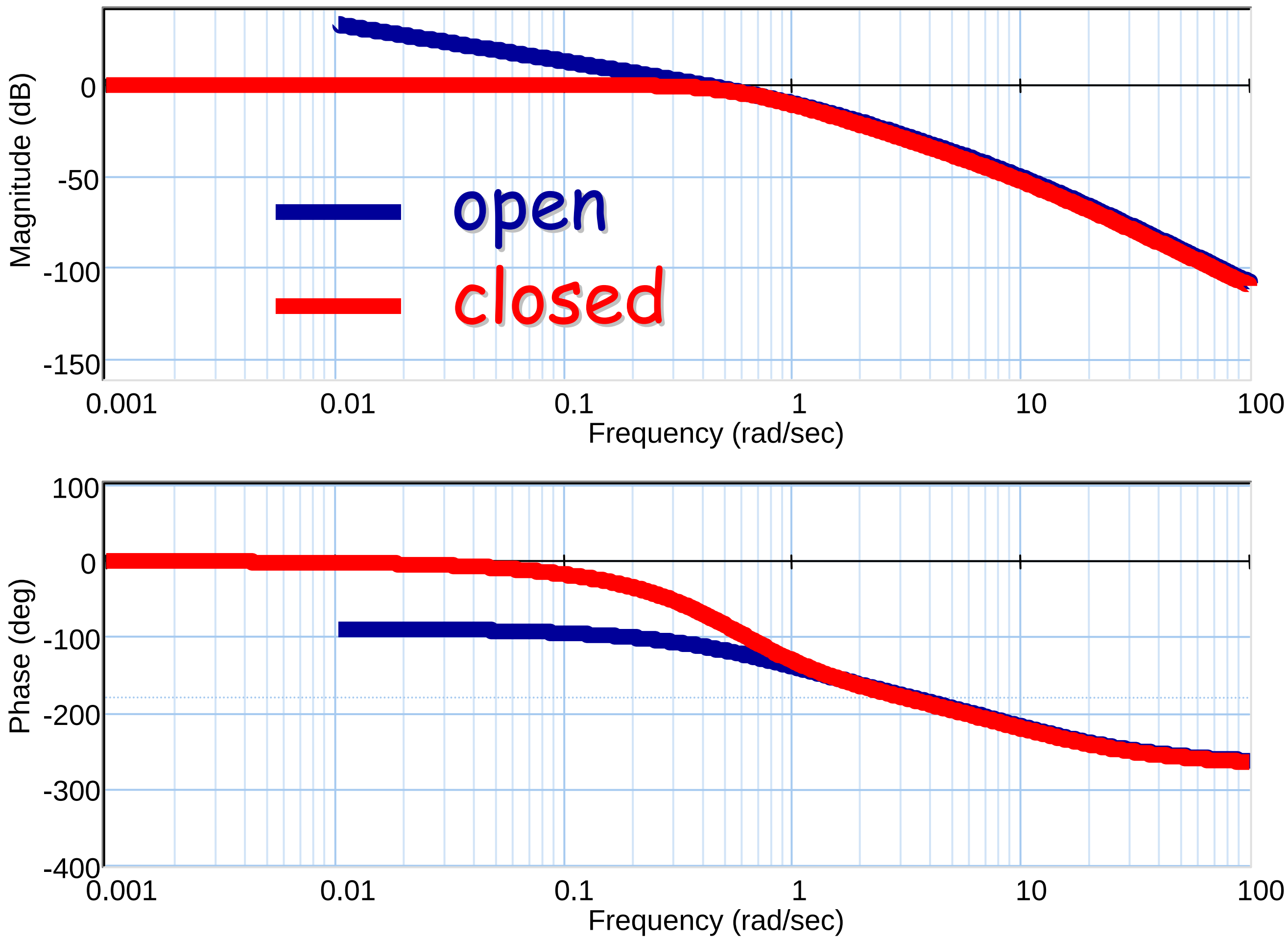
# Design




50
0
-50
-100
Magnitude (dB)
System gain = 0.34
0.01
0.1
1
10
100
Frequency (rad/sec)
-100
-150
-200
-250
Phase (deg)
Phase margin
70 deg
0.01
0.1
1
10
100
Frequency (rad/sec)

# Closed system (Bode)

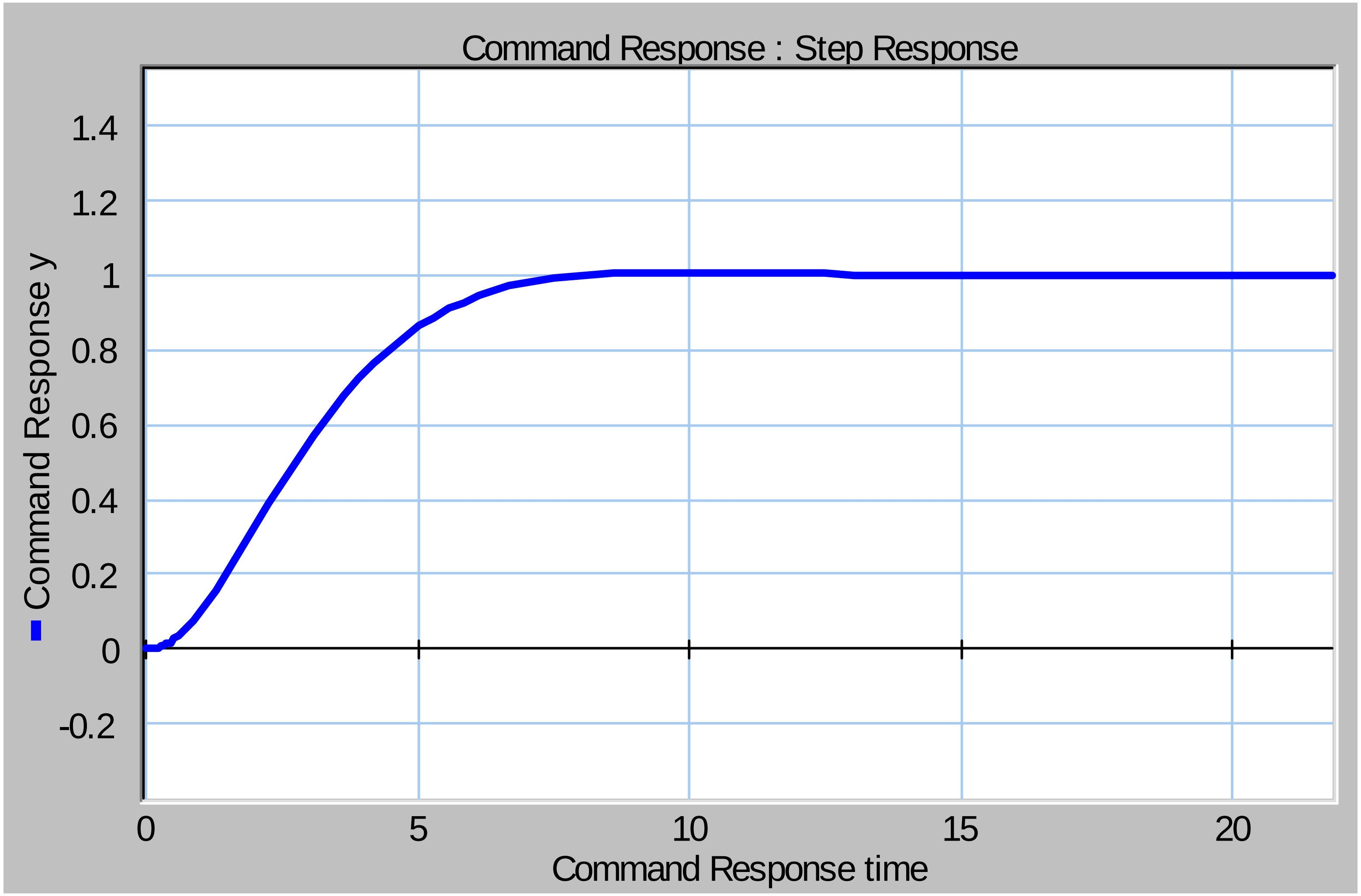




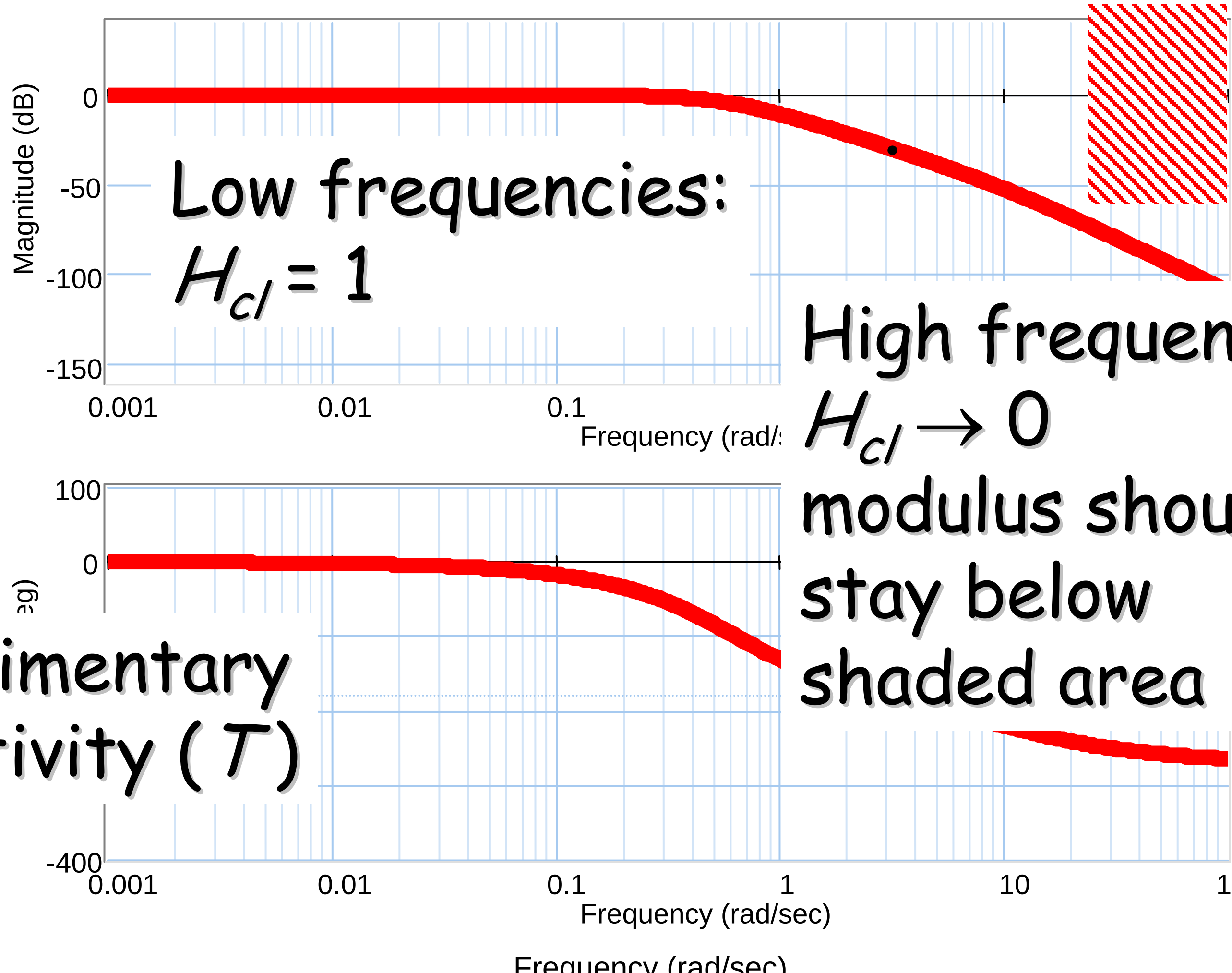

Frequency (rad/sec)

# Step response





# Closed system (Bode)




Low frequencies:
$H_{cl} = 1$

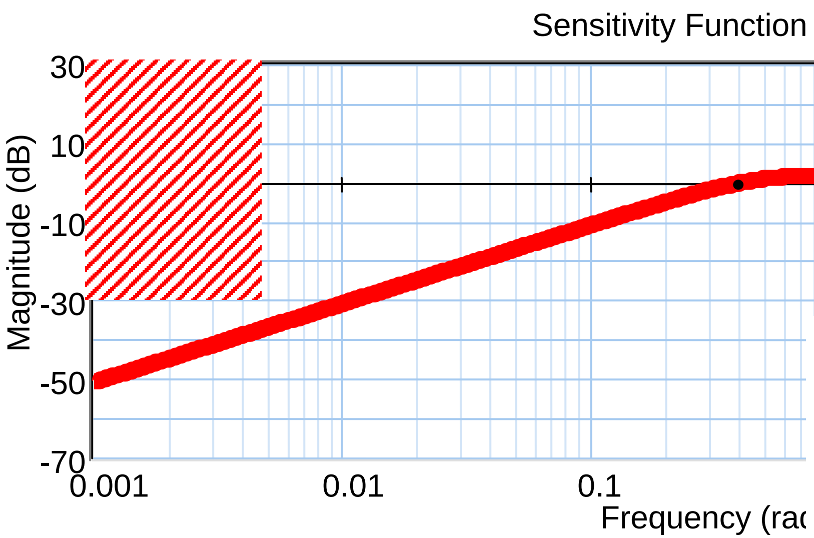
High frequencies:
$H_{cl} \rightarrow 0$
modulus should stay below shaded area

C/R

Complimentary Sensitivity ($T$)

# Sensitivity (S)





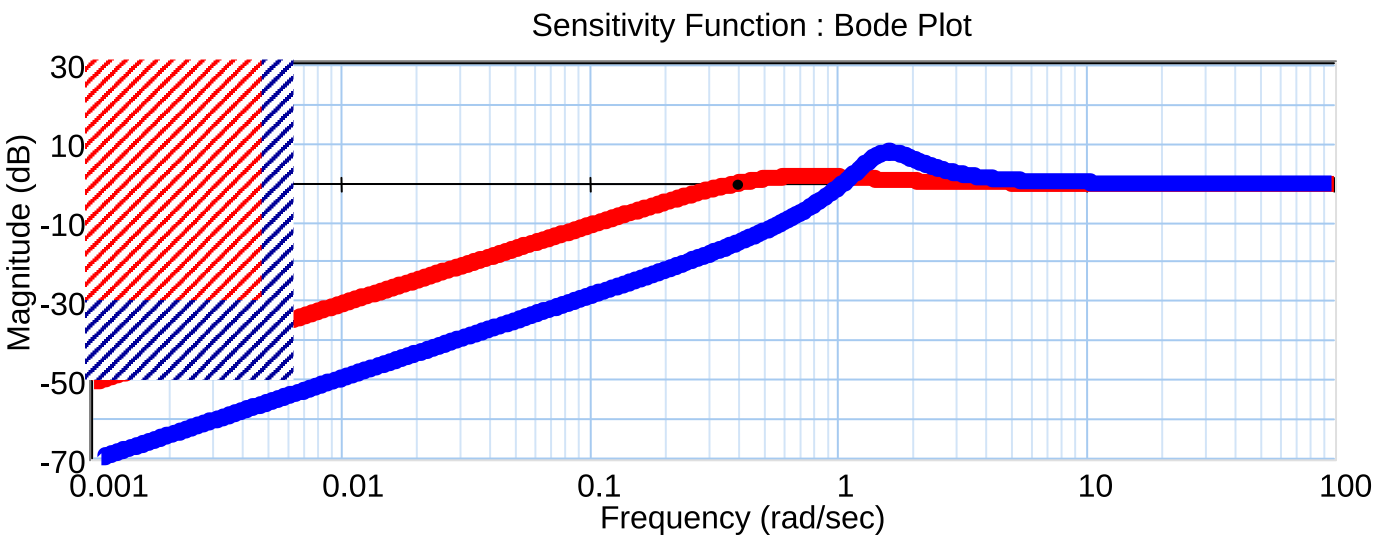

Low frequencies:
*Sensitivity* → 0
Good suppression of low-frequency disturbances

**20-sim**

demo

Frequency (rad/sec)

# Sensitivity (*S*)




Higher gain: Better suppression of low-frequency disturbances, but amplification of higher frequencies

Frequency (rad/sec)

# Bode Sensitivity integral (theorem of Westcott)

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$S(j\omega)$ = Sensitivity (Afwijkingsverhouding)

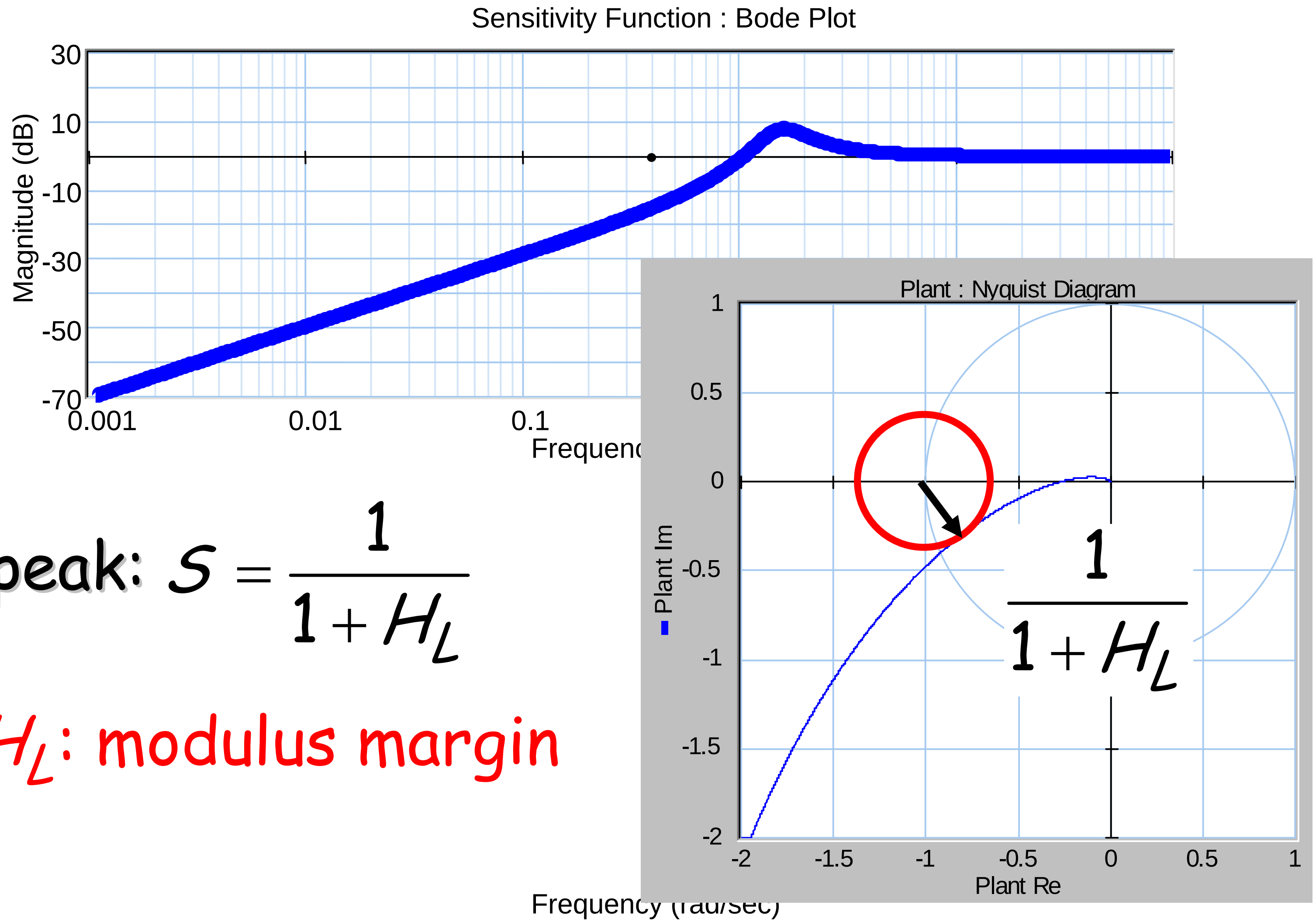
$$S = \frac{1}{1 + H_L}$$

If $H_L$ has at least two more poles than zero's:

$$\int_0^\infty \log\left|S(j\omega)\right| d\omega = 0$$

All improvements in one area, have to be paid for by a deterioration in another area (compare waterbed)

# Sensitivity (S)

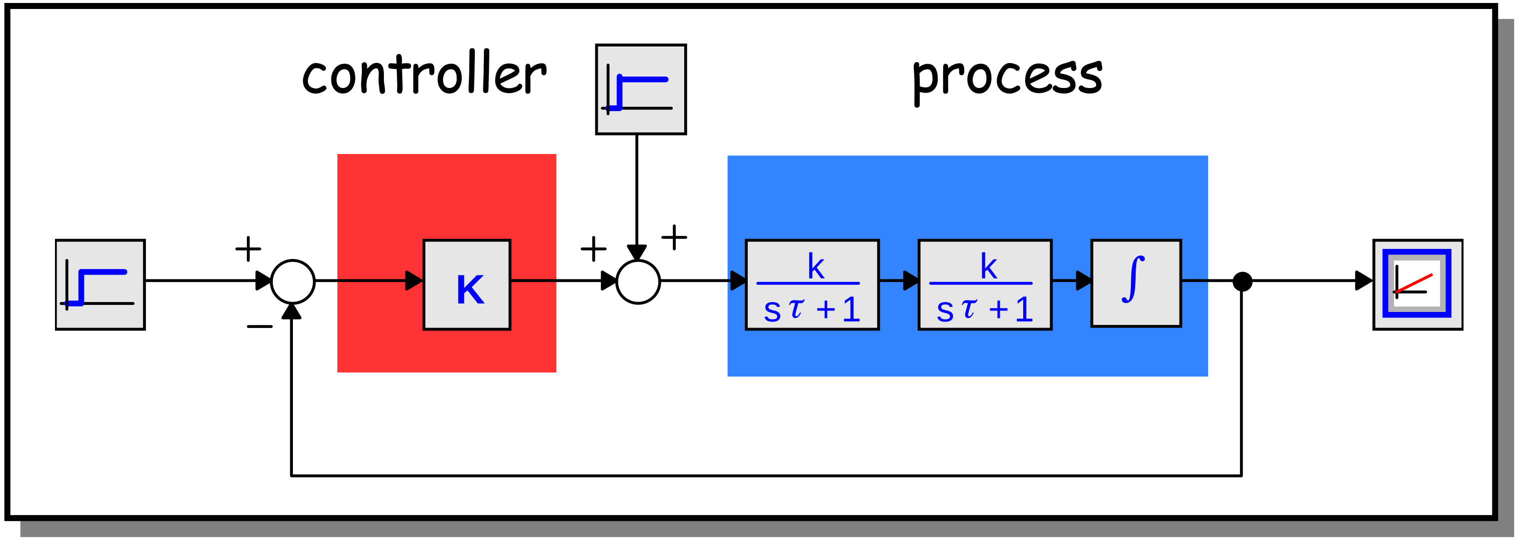



peak: $S = \frac{1}{1+H_L}$

$1+H_L$: modulus margin

# Disturbance suppression

Consider the following system



# Bode

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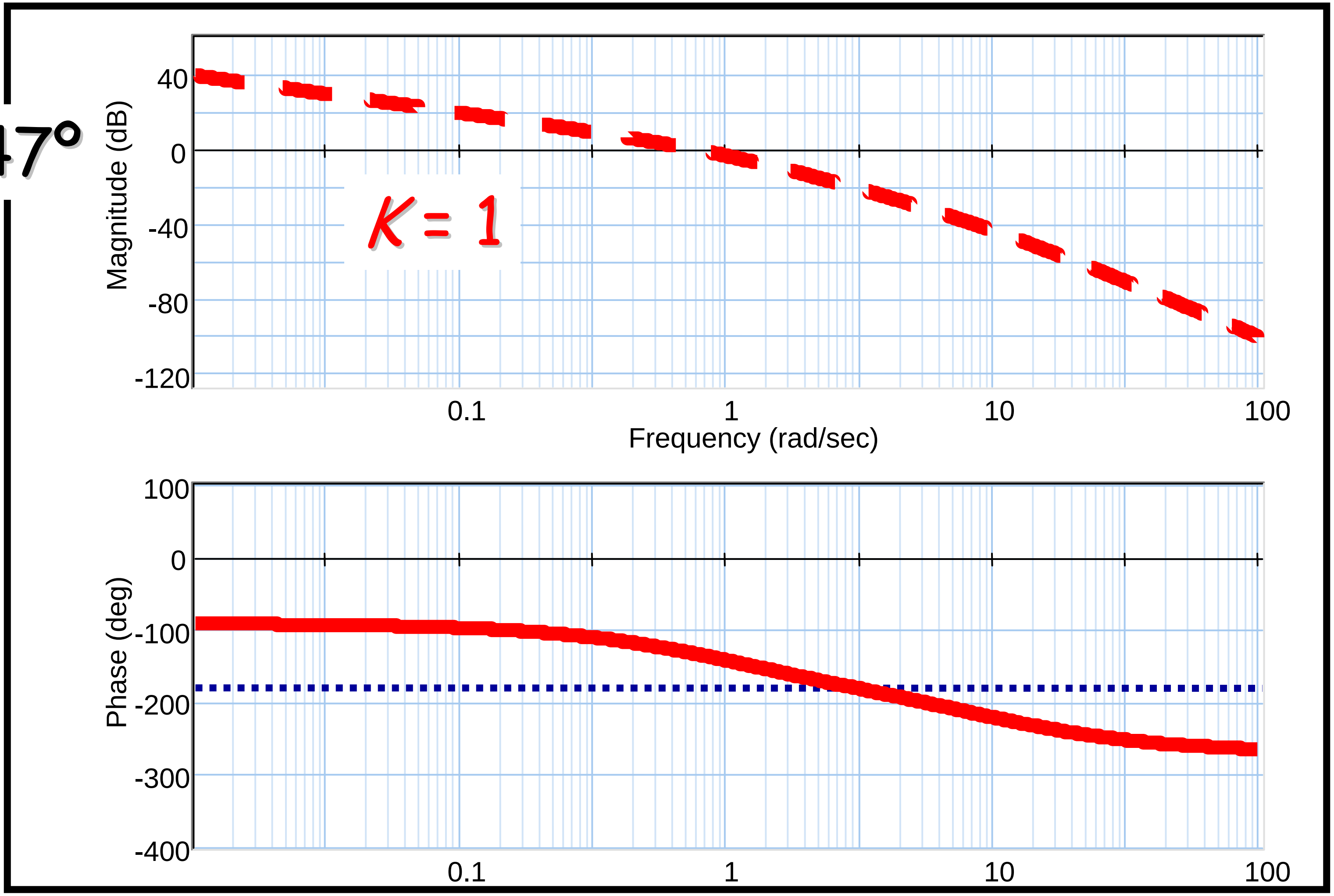
PM =47°



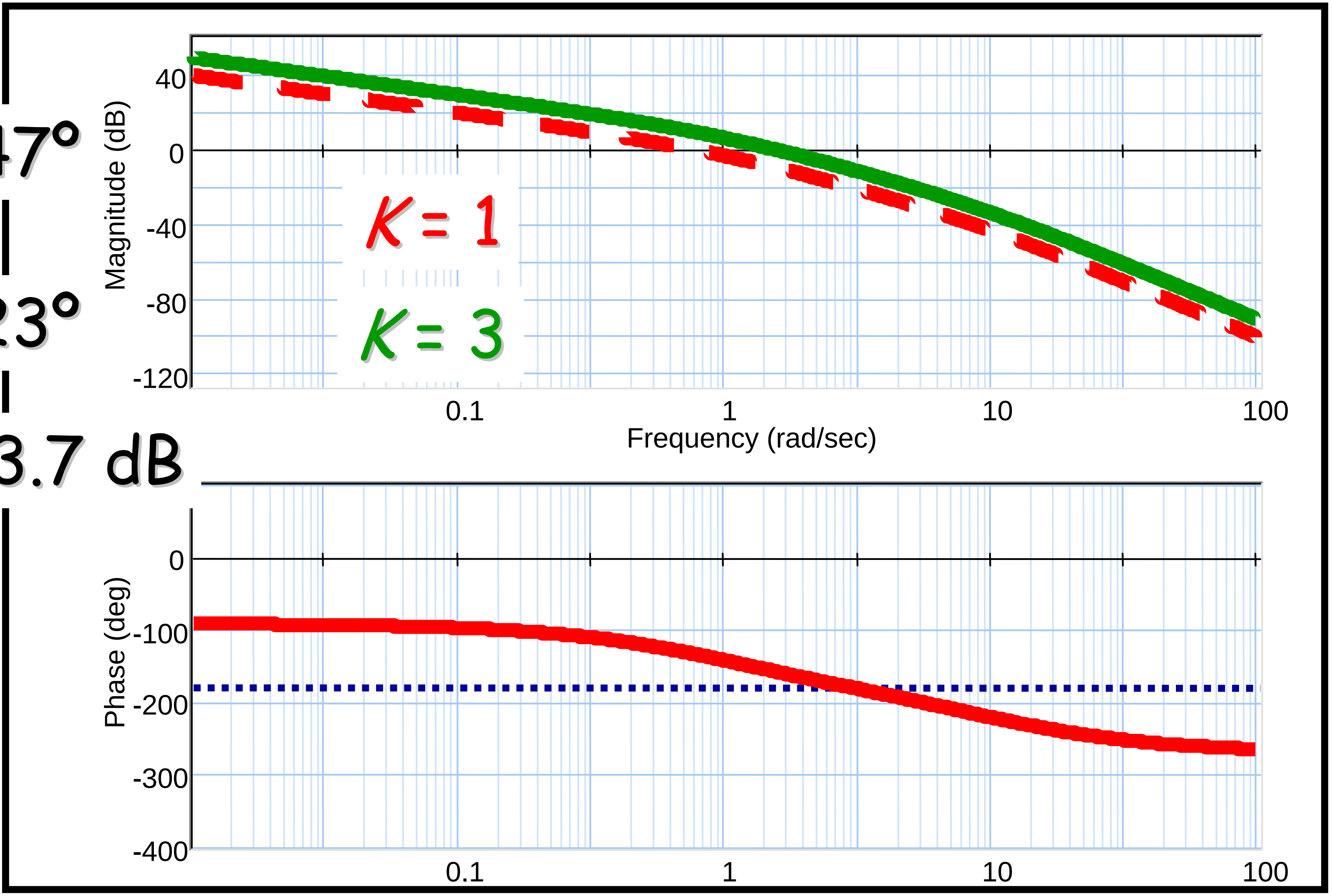
# Bode



PM =47°

PM =23°

GM = 3.7 dB



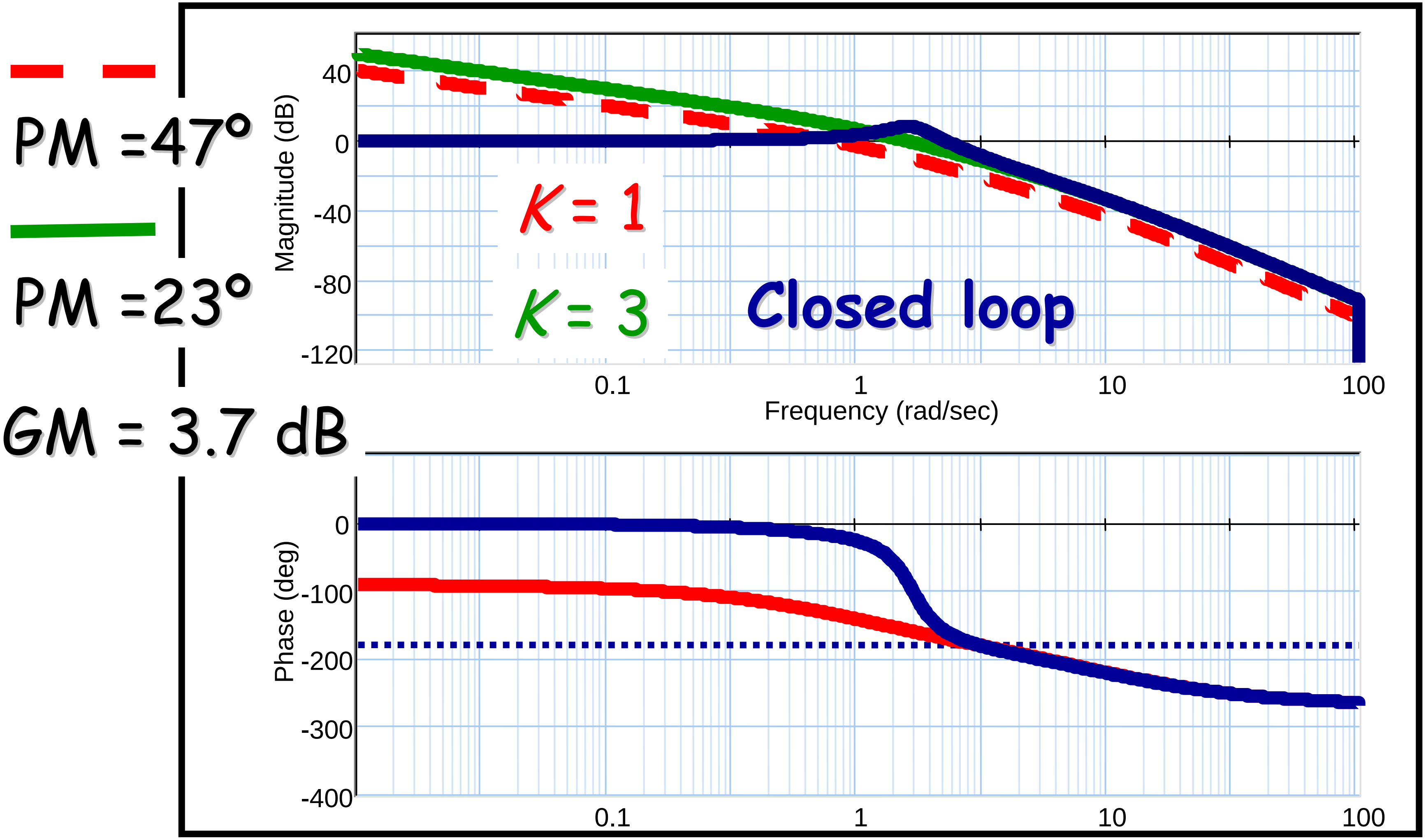
# Bode



PM =47°

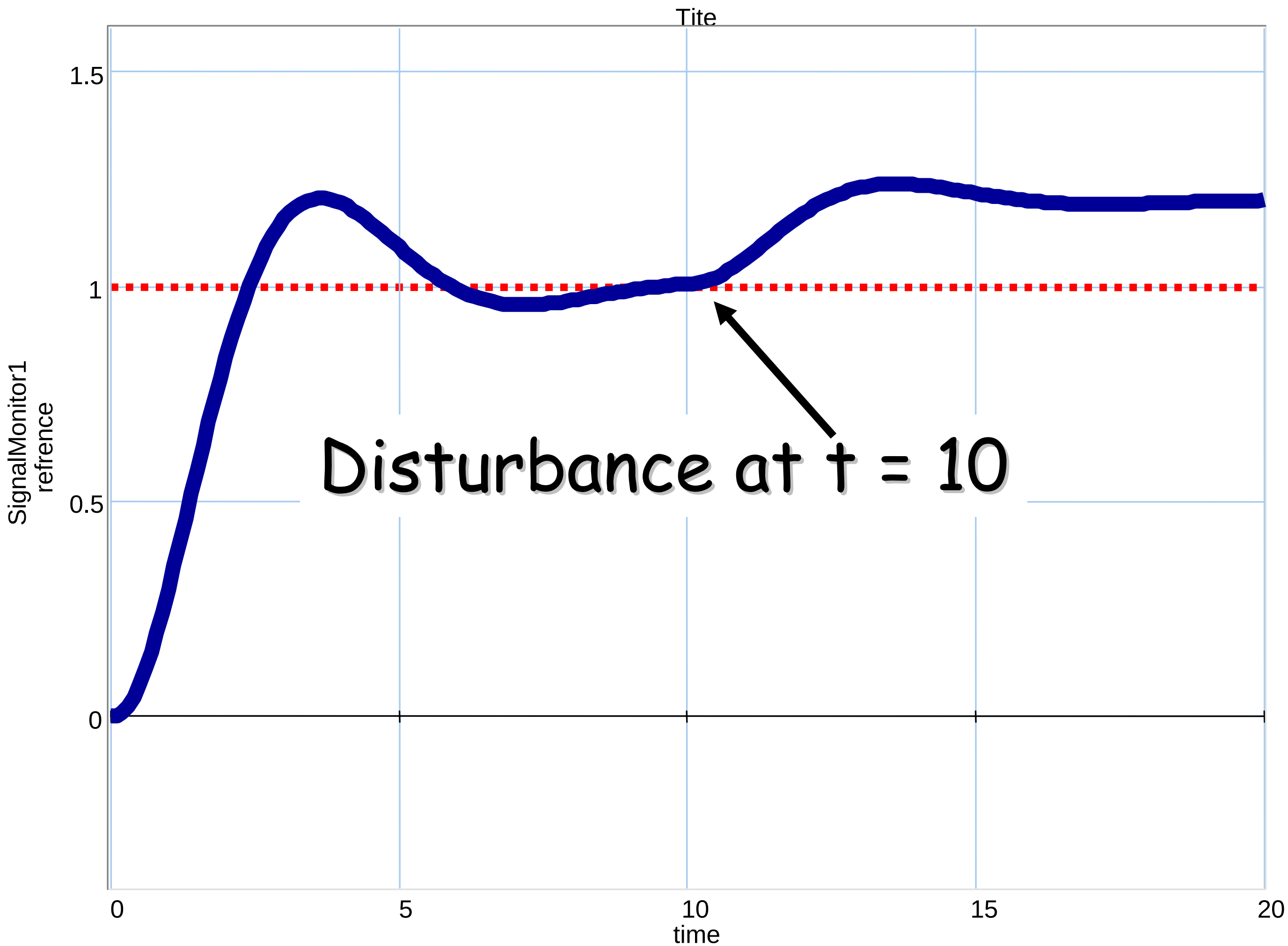
PM =23°

GM = 3.7 dB



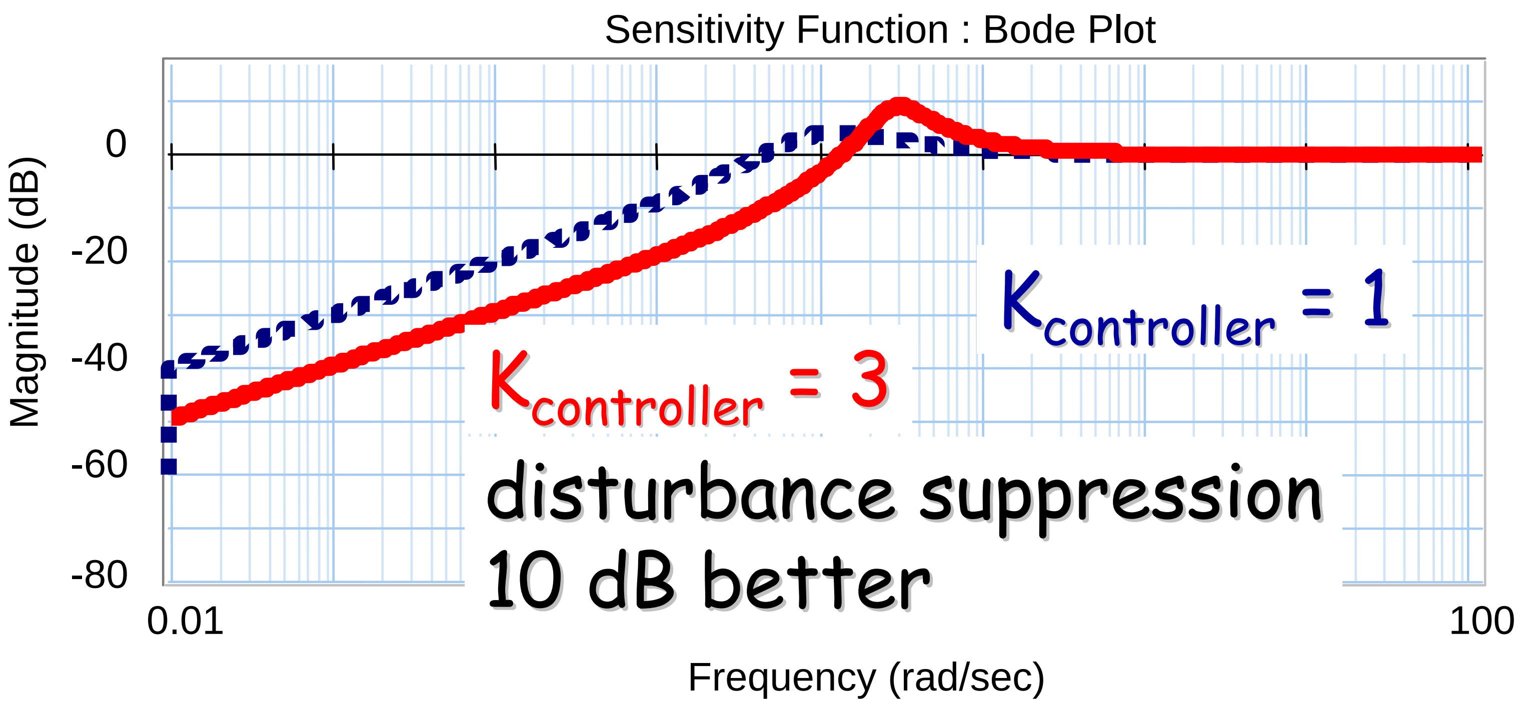
# Response K = 1




Tite
1.5
1
0.5
0
SignalMonitor1
refrence
0
5
10
15
20
time
Disturbance at t = 10

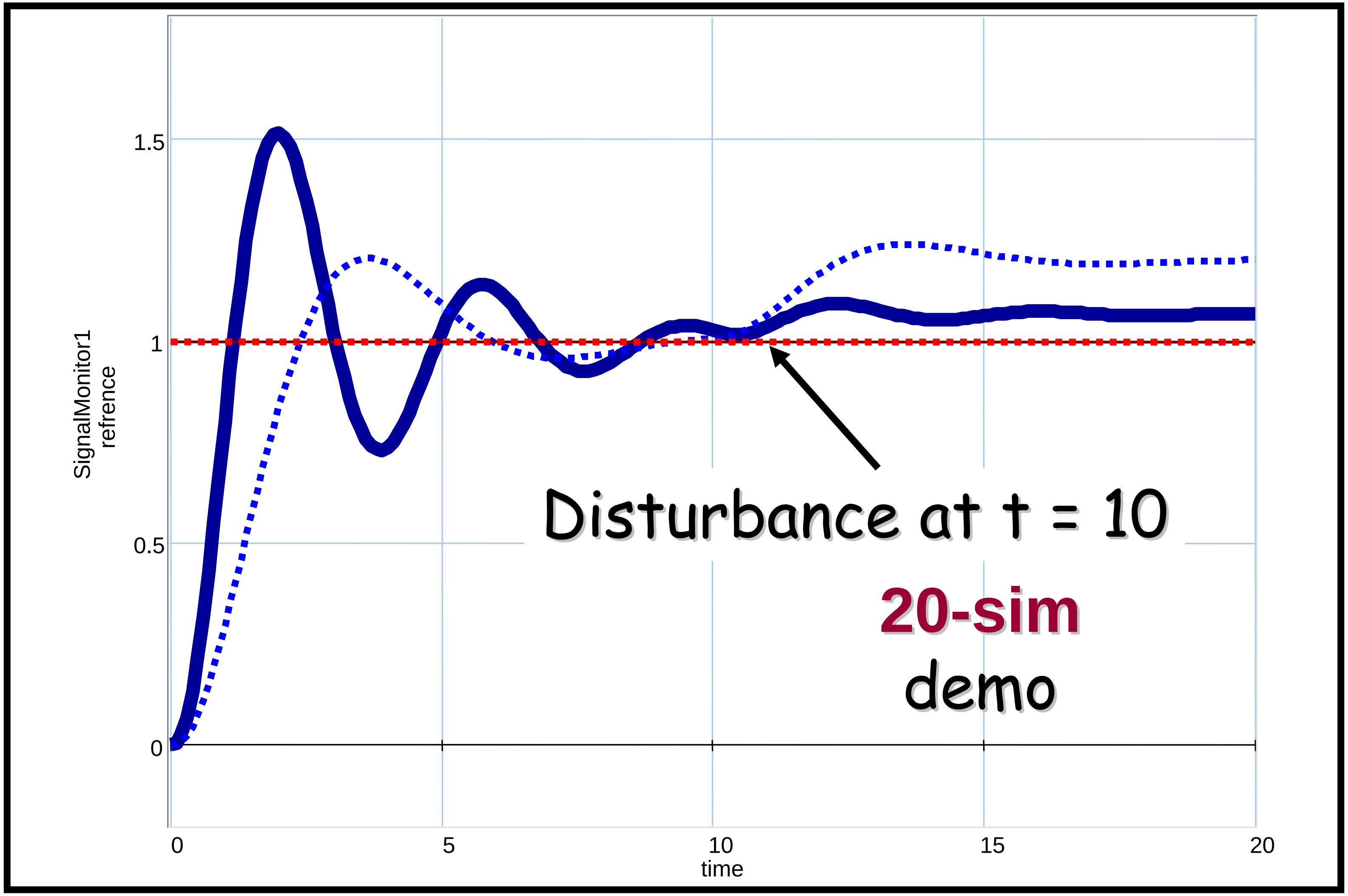
# Sensitivity



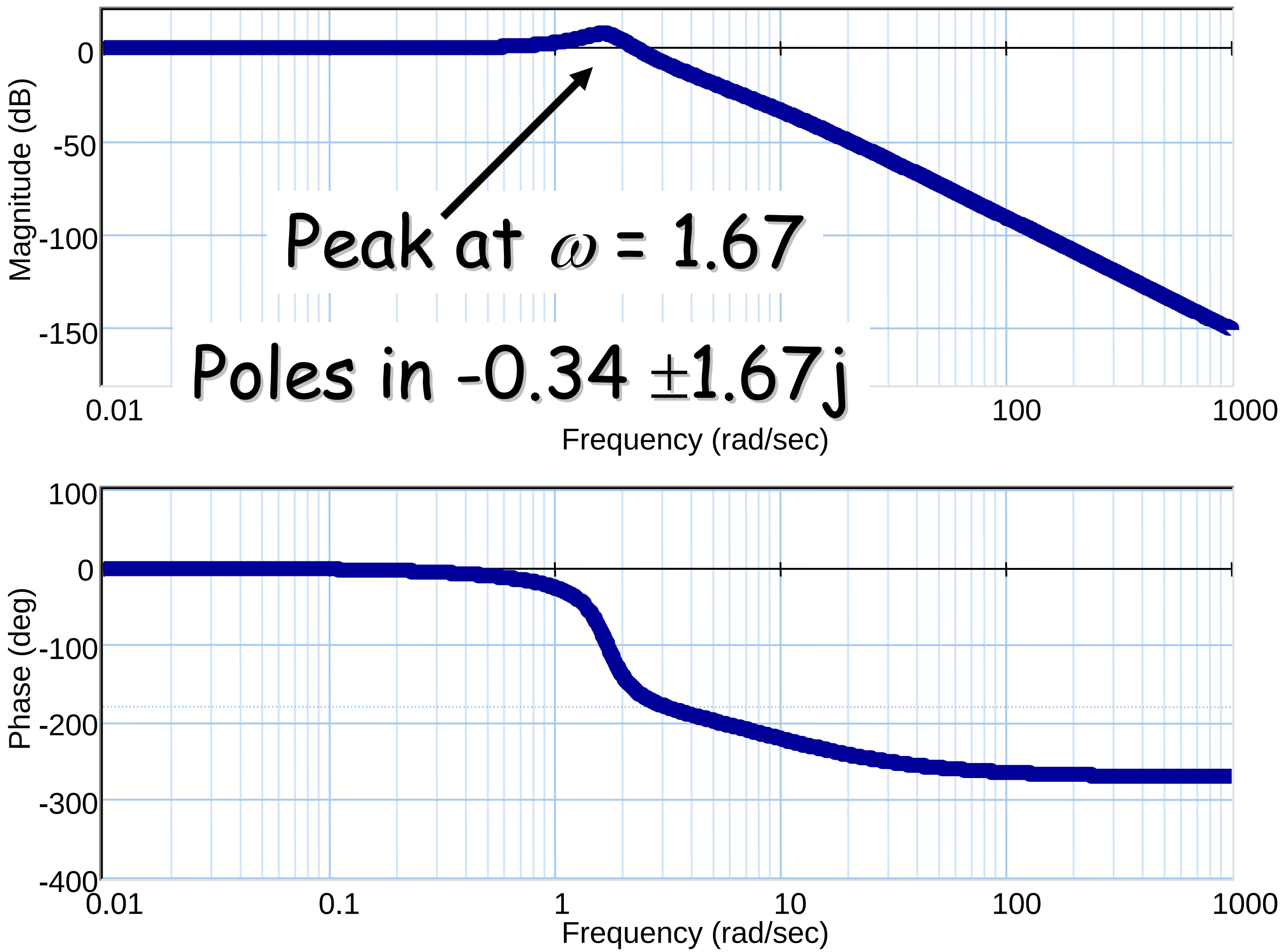


# Response K = 3




1.5
1
0.5
0
SignalMonitor1
refrence
0
5
10
15
20
time
Disturbance at t = 10
20-sim
demo

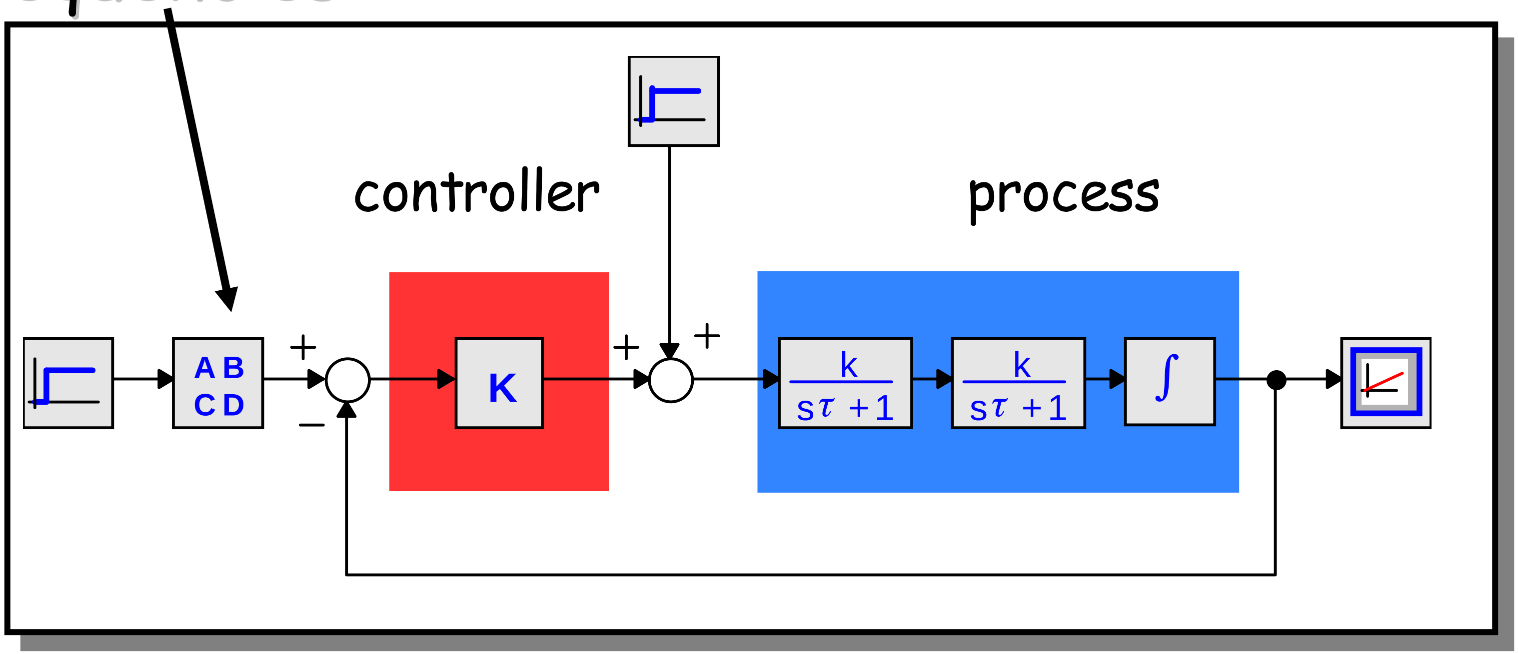
# Bode


Magnitude (dB)
0
-50
-100
-150
0.01
100
1000
Frequency (rad/sec)
Peak at ω = 1.67
Poles in -0.34 ±1.67j
Phase (deg)
100
0
-100
-200
-300
-400
0.01
0.1
1
10
100
1000
Frequency (rad/sec)

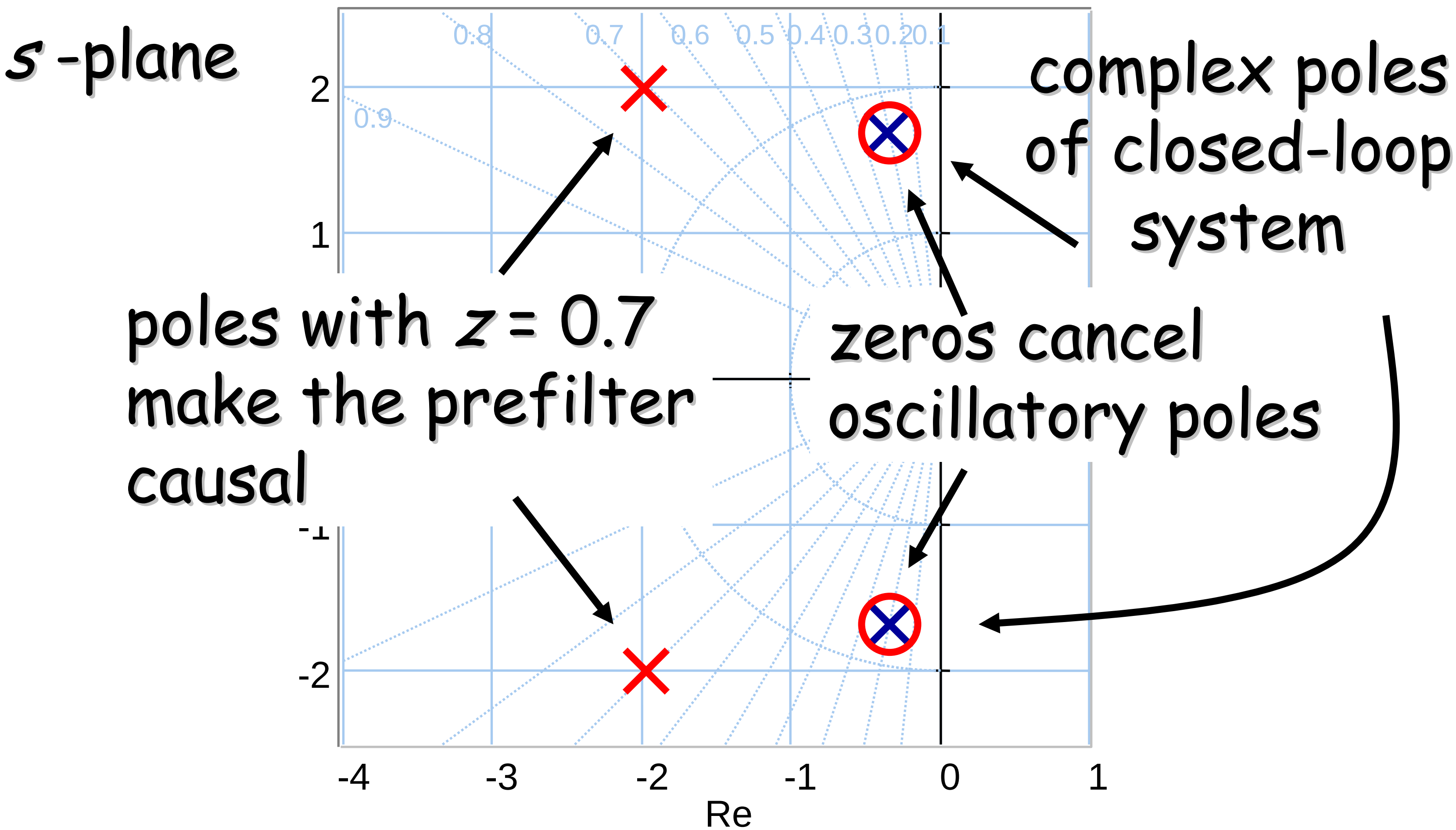
# Prefilter



attenuate the resonance frequencies



# Prefilter


s -plane
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.9
2
1
-1
-2
-4
-3
-2
-1
0
1
Re
complex poles of closed-loop system
poles with z = 0.7 make the prefilter causal
zeros cancel oscillatory poles

# Closed-loop Bode

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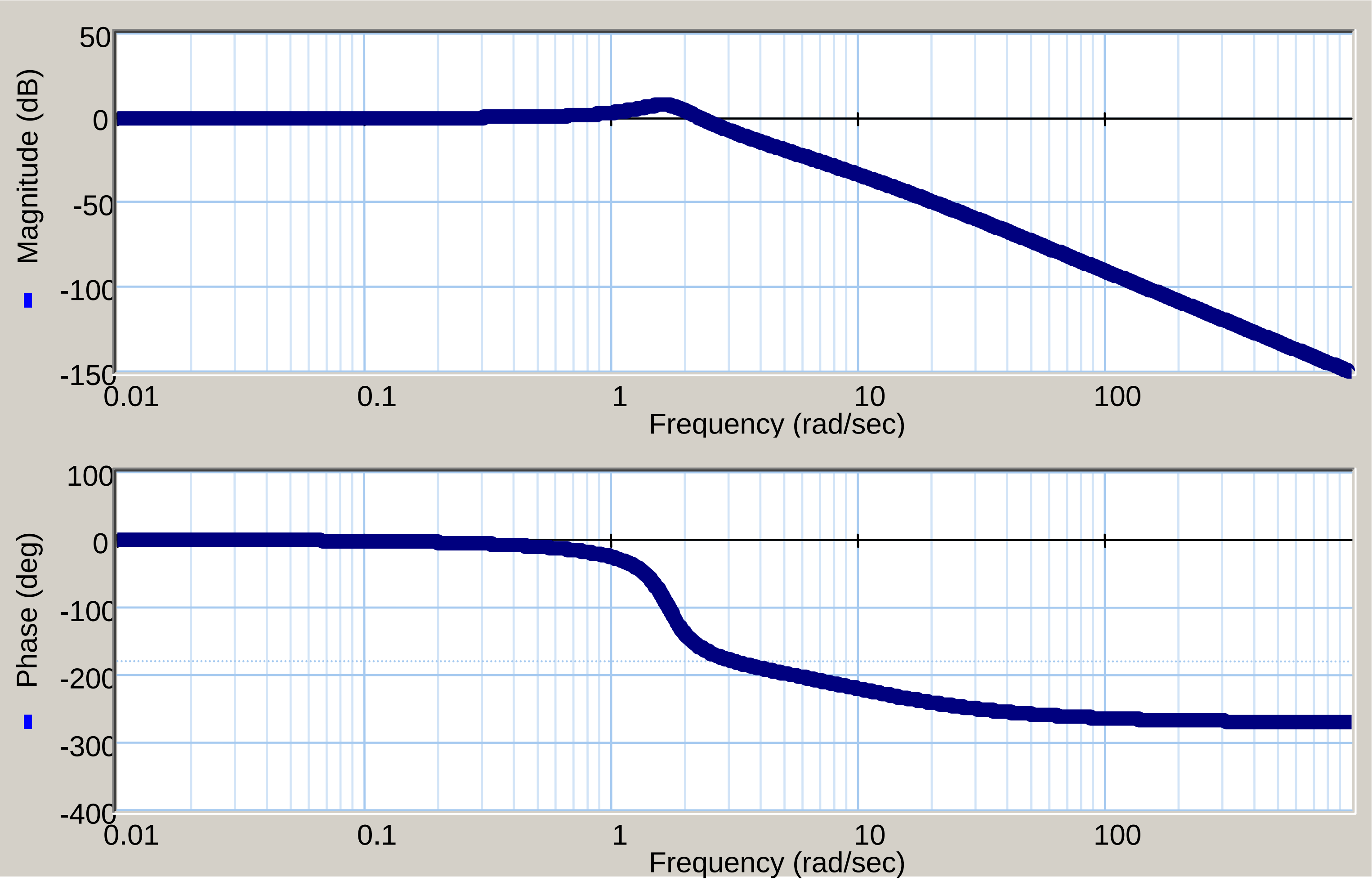
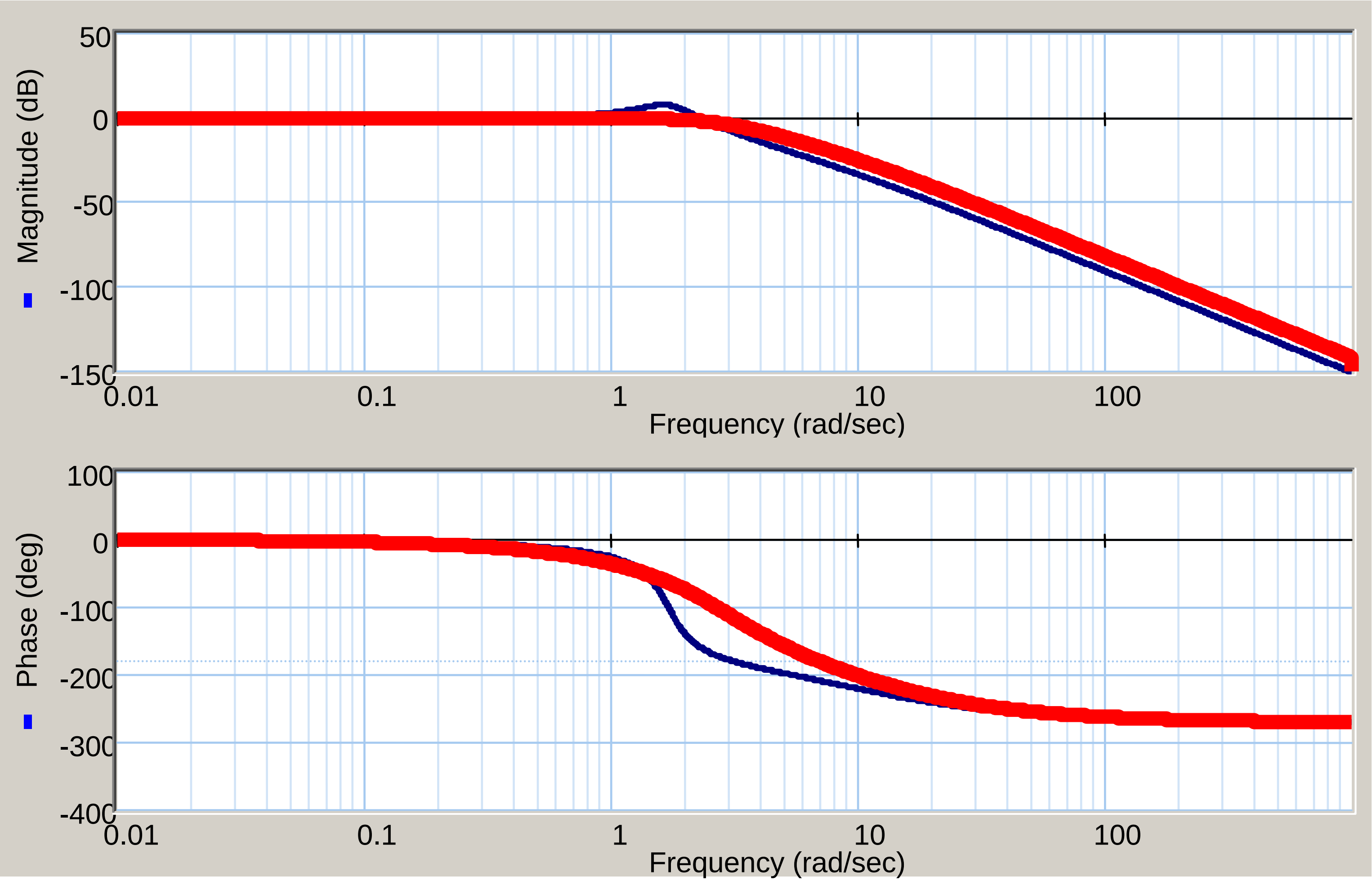
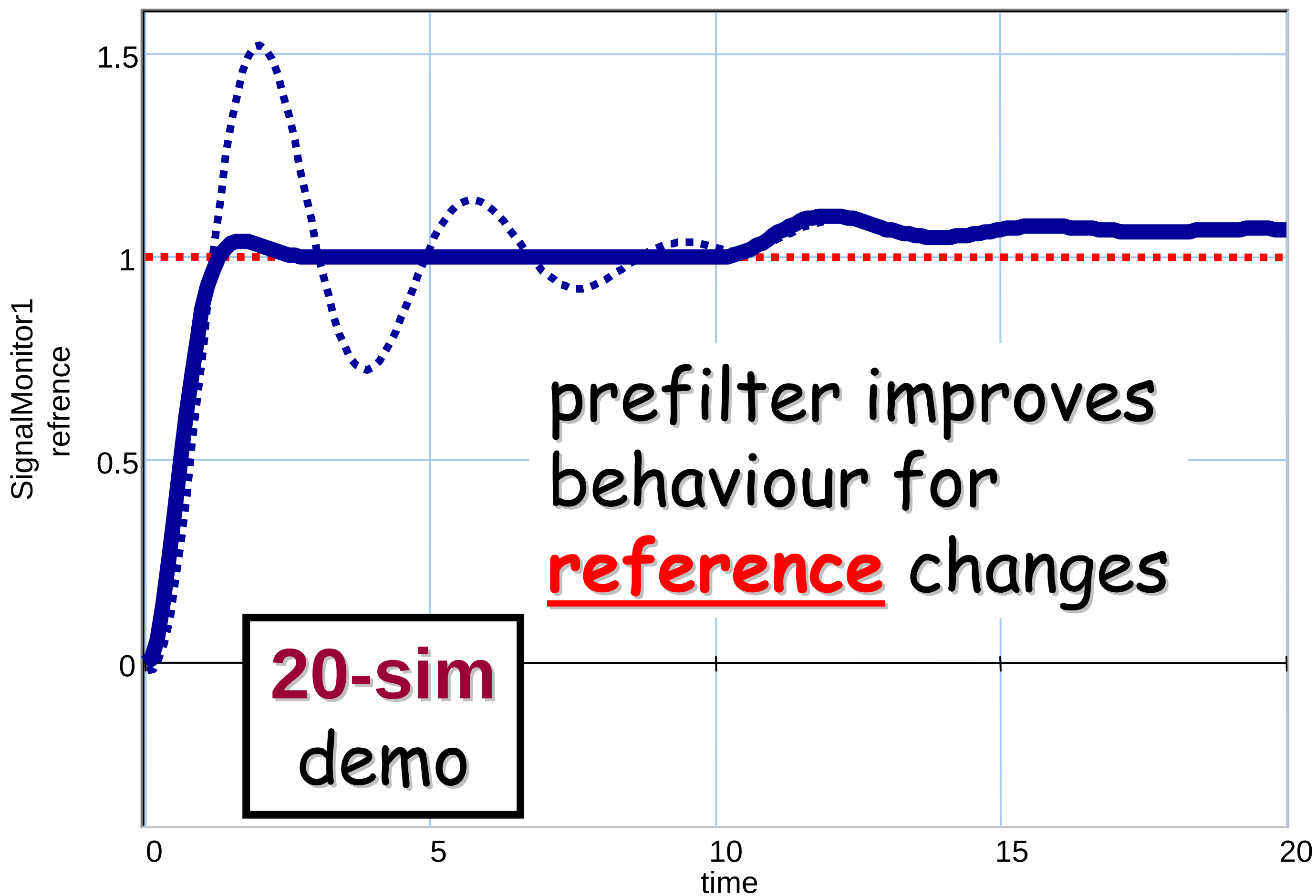


# Closed-loop Bode + prefilter





# Response


1.5
1
0.5
0
SignalMonitor1
refrence
0
5
10
15
20
time
prefilter improves behaviour for reference changes
20-sim
demo

# Review

- Disturbance suppression does not require high damping ratios
- Response on reference changes can be improved by means of a prefilter
- But…
  - gain and phase margins were small
  - robustness for parameter variations is small

# Solution



- Try to improve the robustness by designing more advanced compensators that simultaneously guarantee
  - good transients
  - high disturbance suppression

# Uncompensated system

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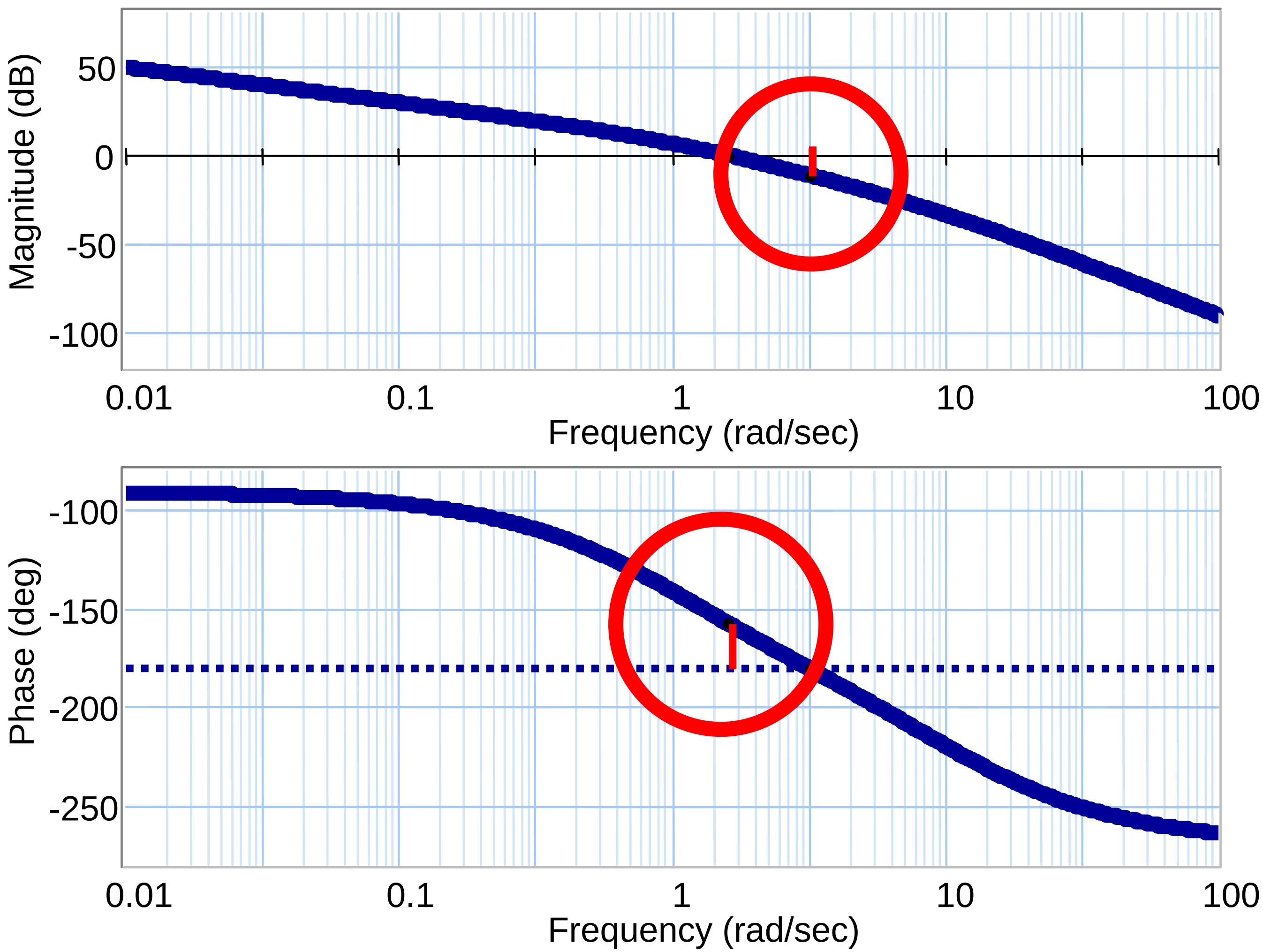
1:
decrease
HF-gain

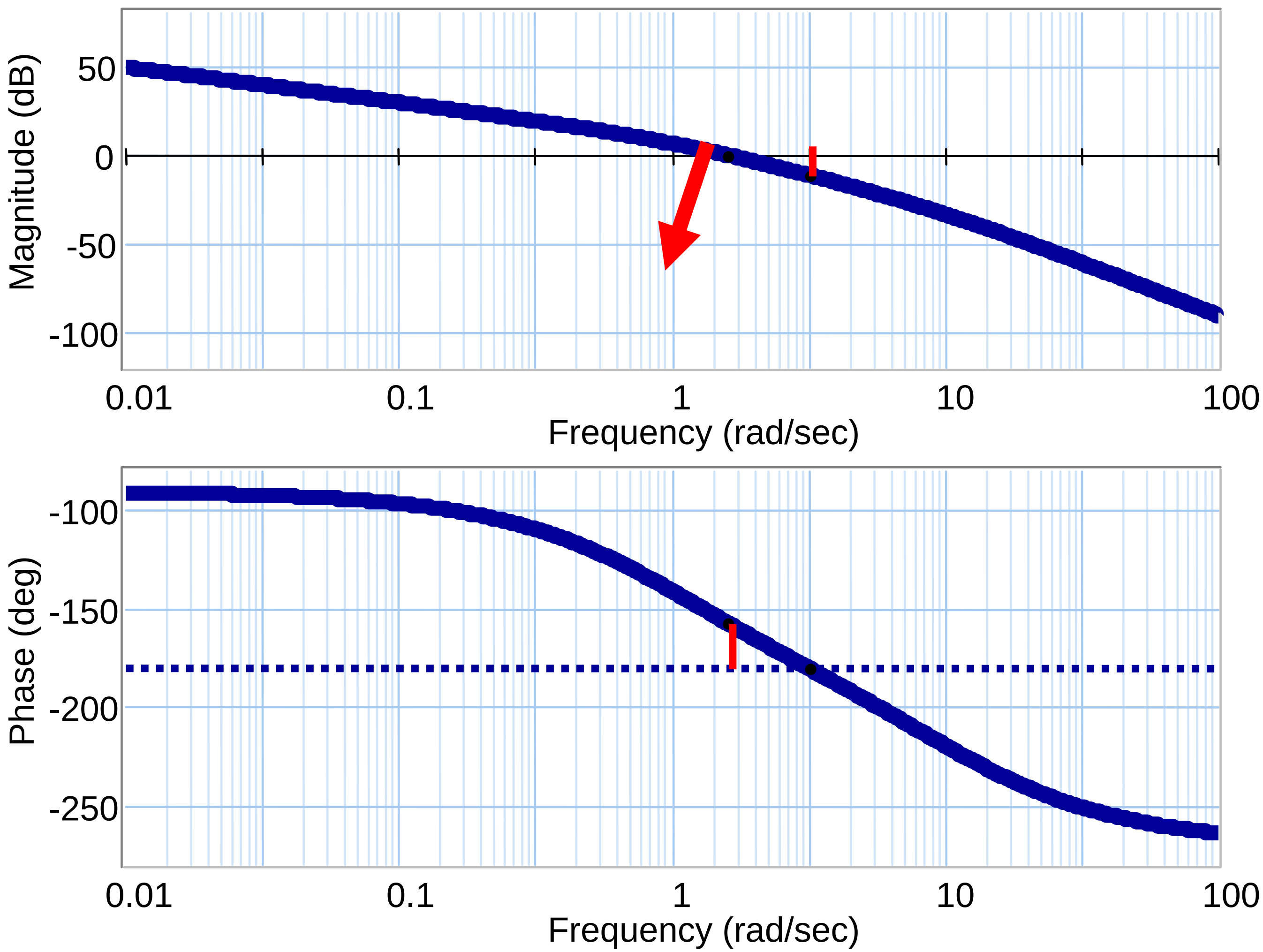
2:
decrease
HF-phase
shift



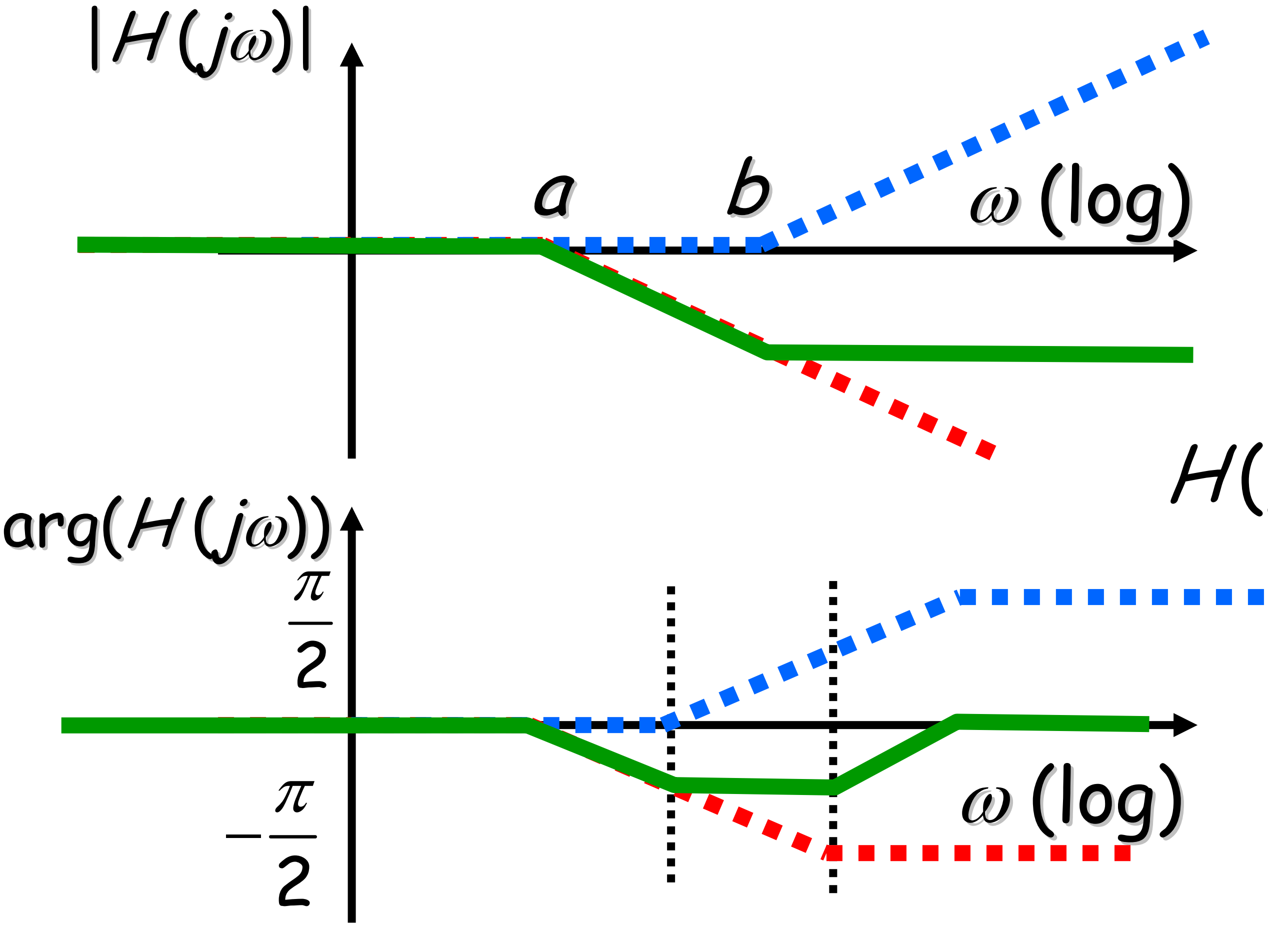
# Uncompensated system



1:
decrease
HF-gain



# Decrease HF gain




Lag network

$$H(j\omega) = \frac{a}{b}\frac{j\omega + b}{j\omega + a}$$

$$a < b$$

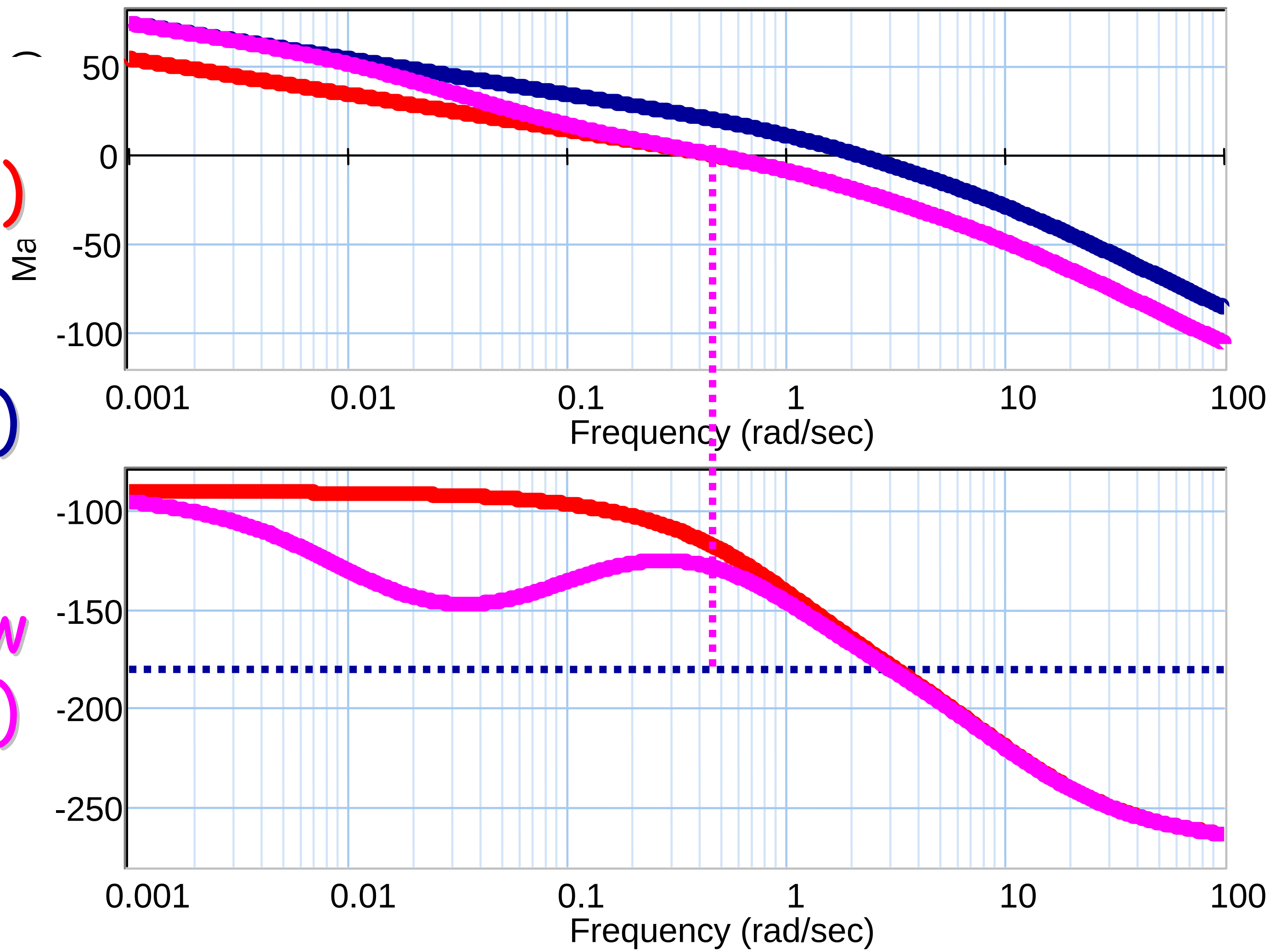
# Lag network



K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

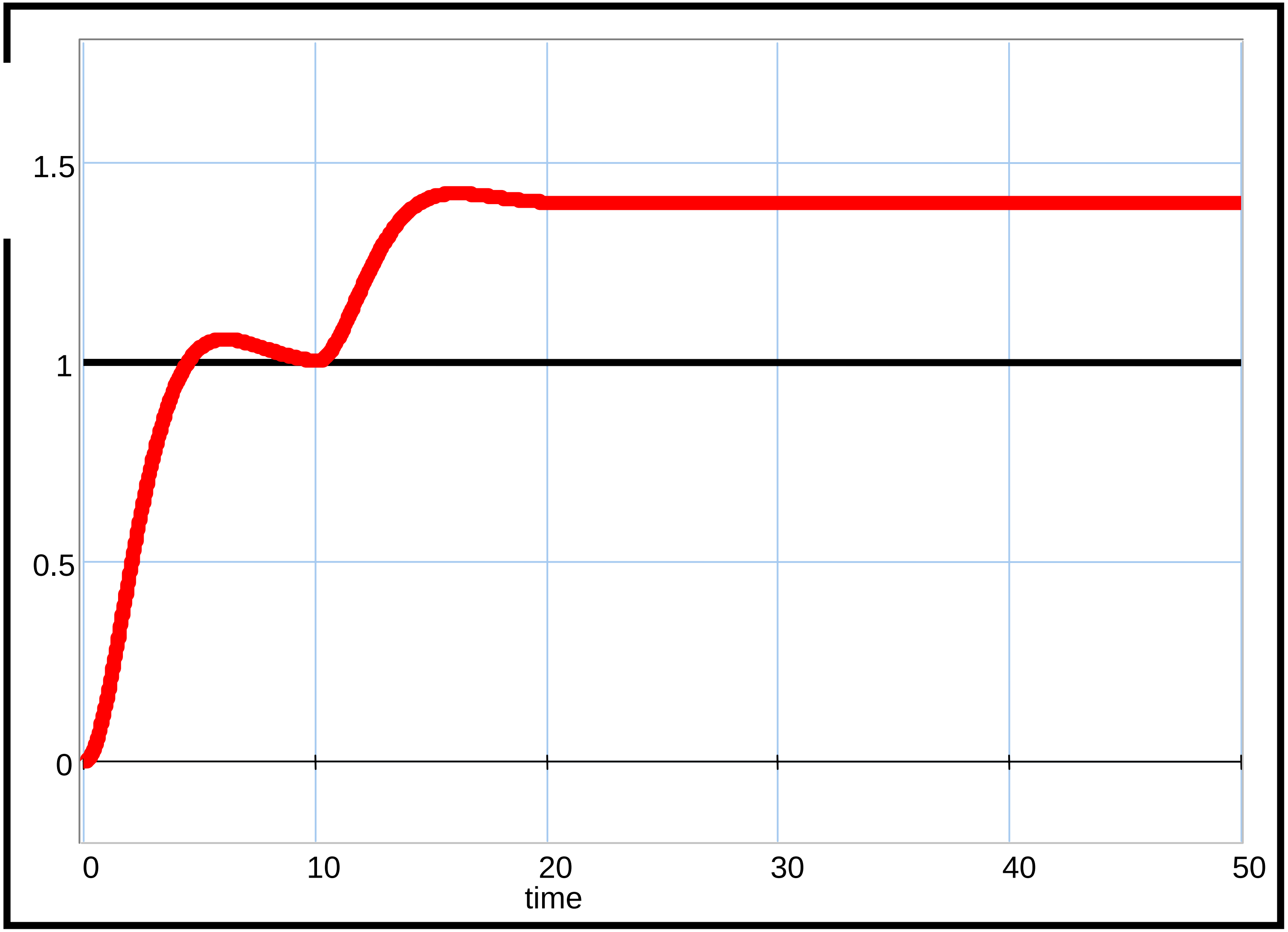
K = 5
+ Lag netw
(PM = 52°)



# Responses



K = 0.5
(PM = 63°)

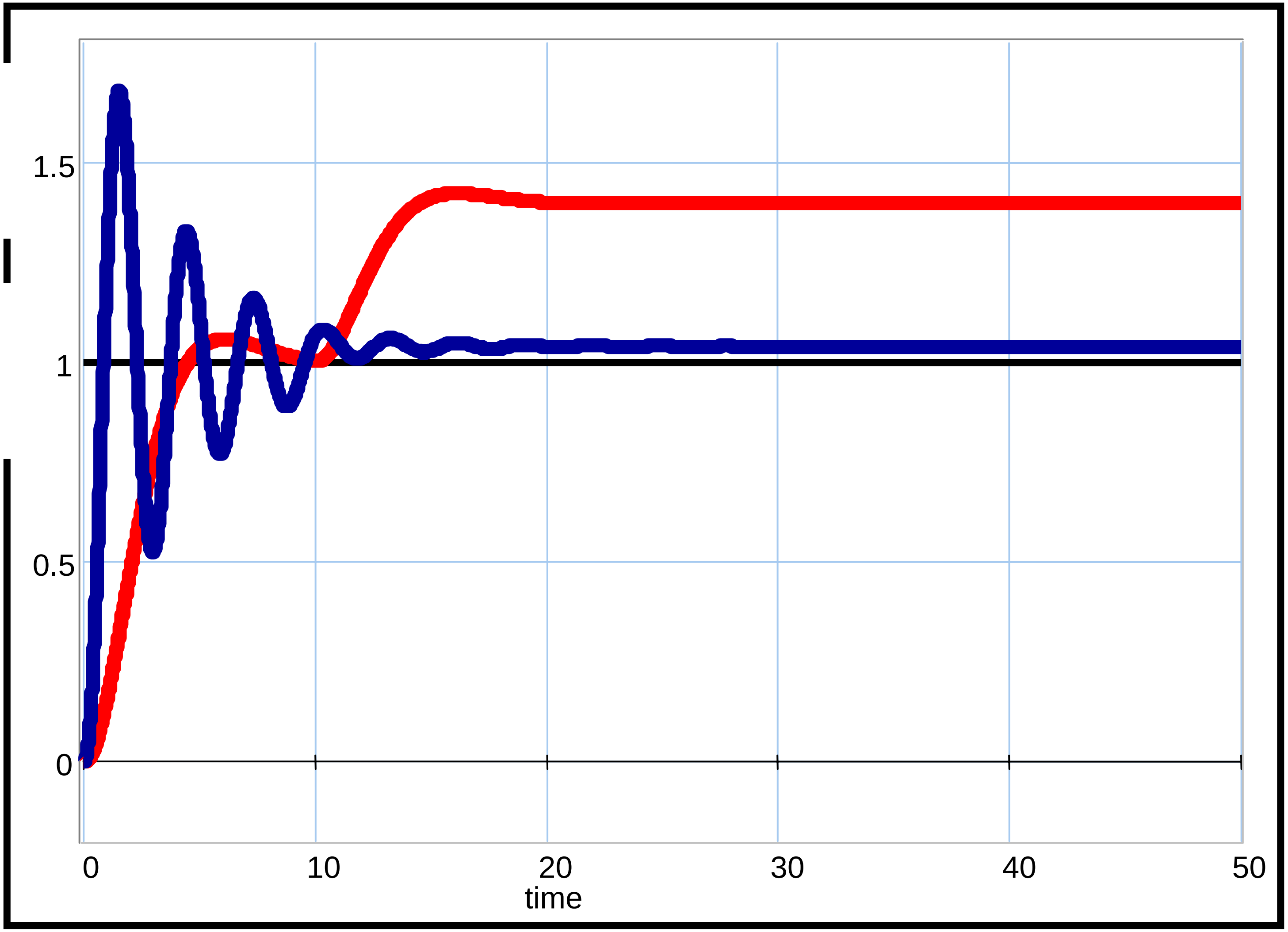


# Responses



K = 0.5
(PM = 63°)

K = 5
(PM = 13°)



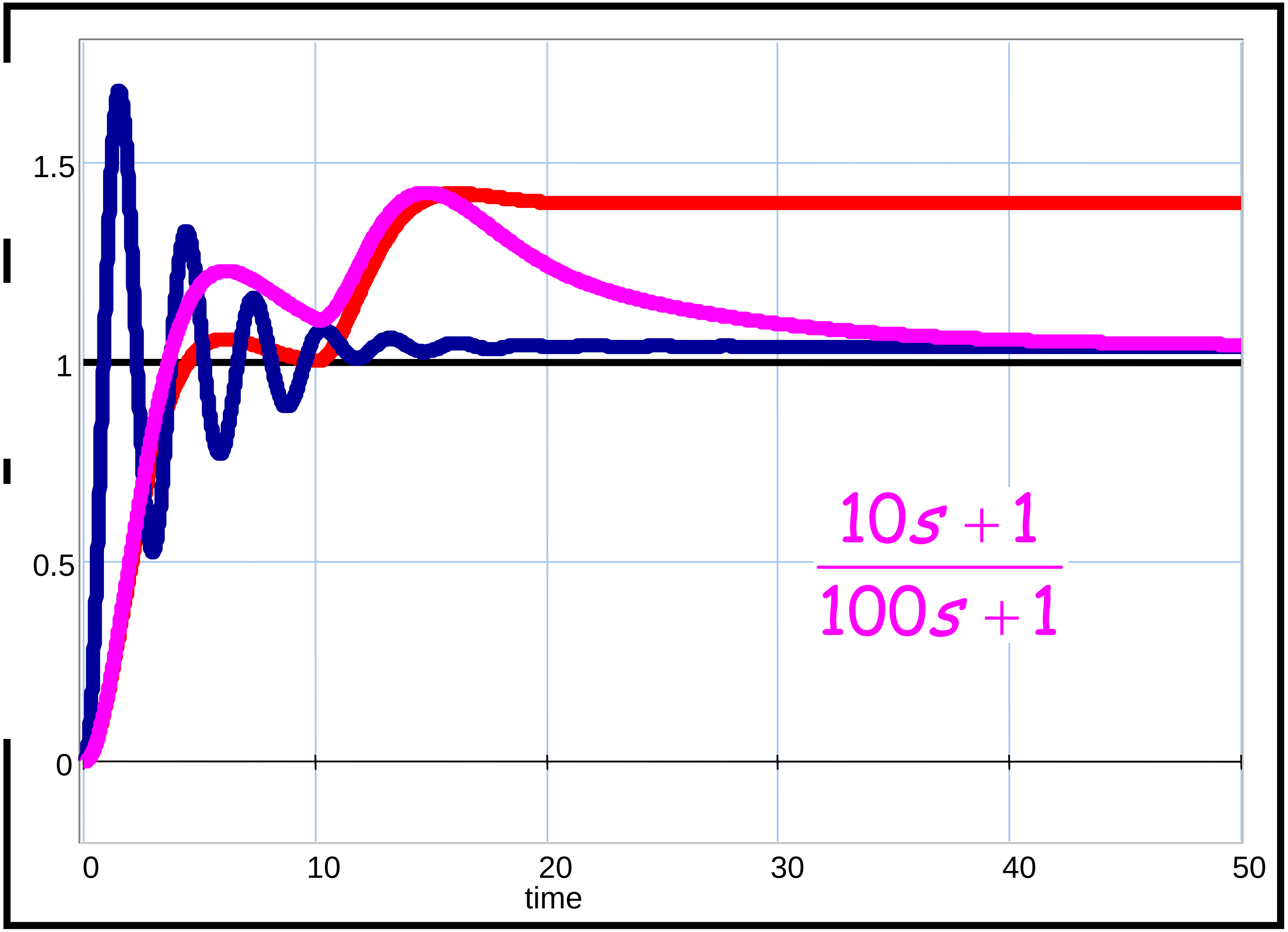
# Responses



K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

K = 5
+ Lag netw
(PM = 52°)

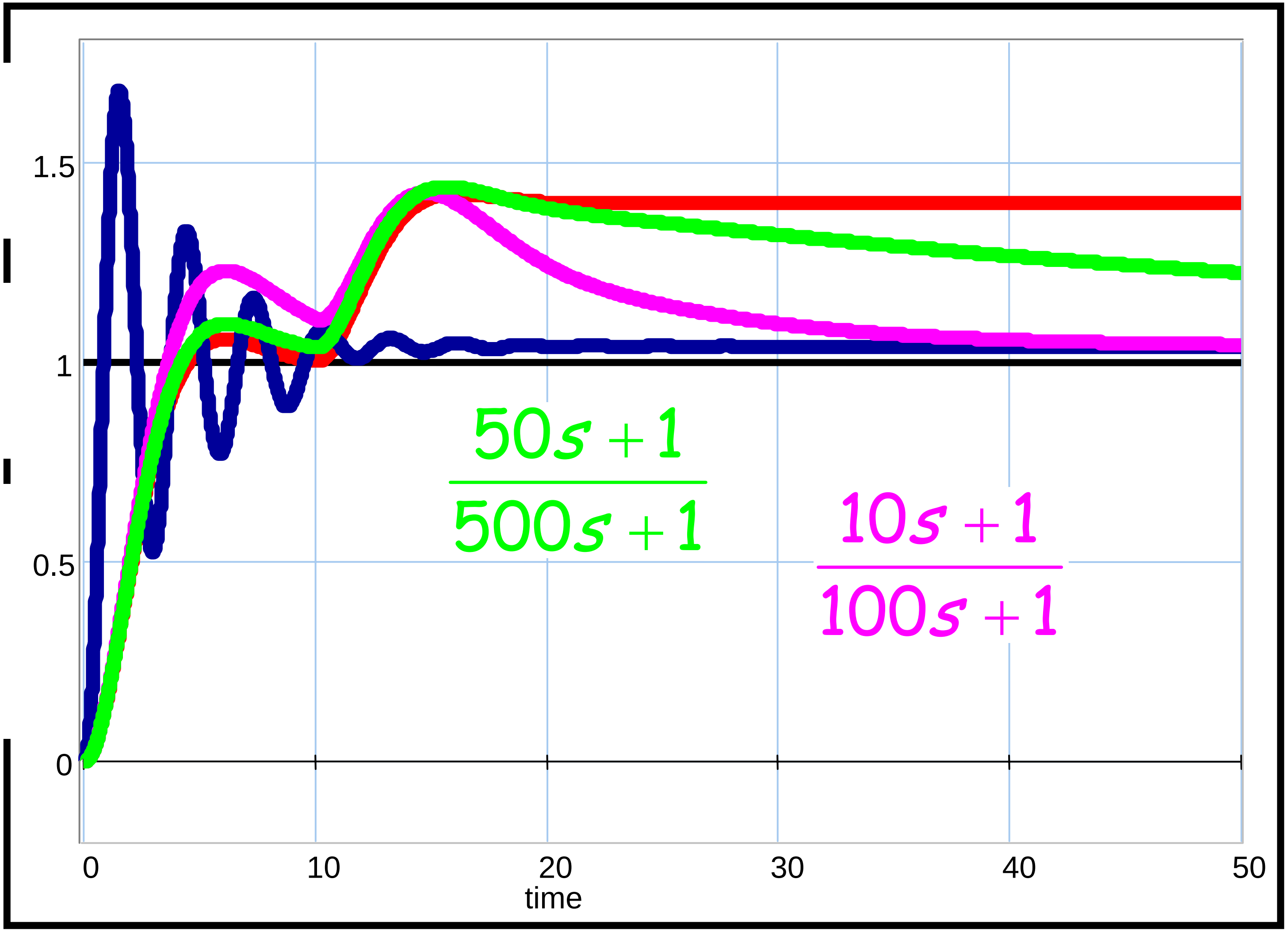


# Responses

K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

K = 5
+ Lag netw
(PM = 52°)



# Decrease HF phase shift

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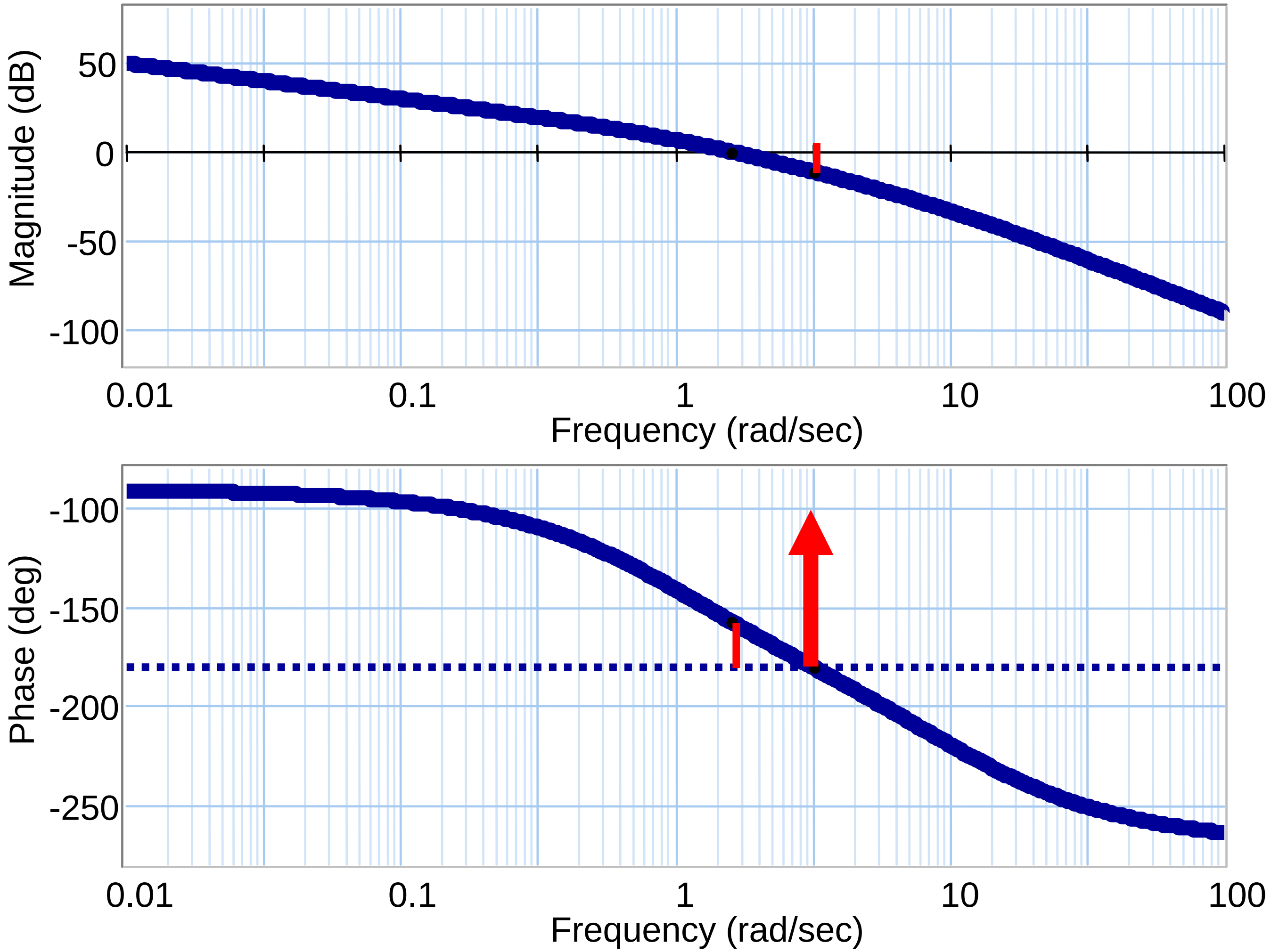
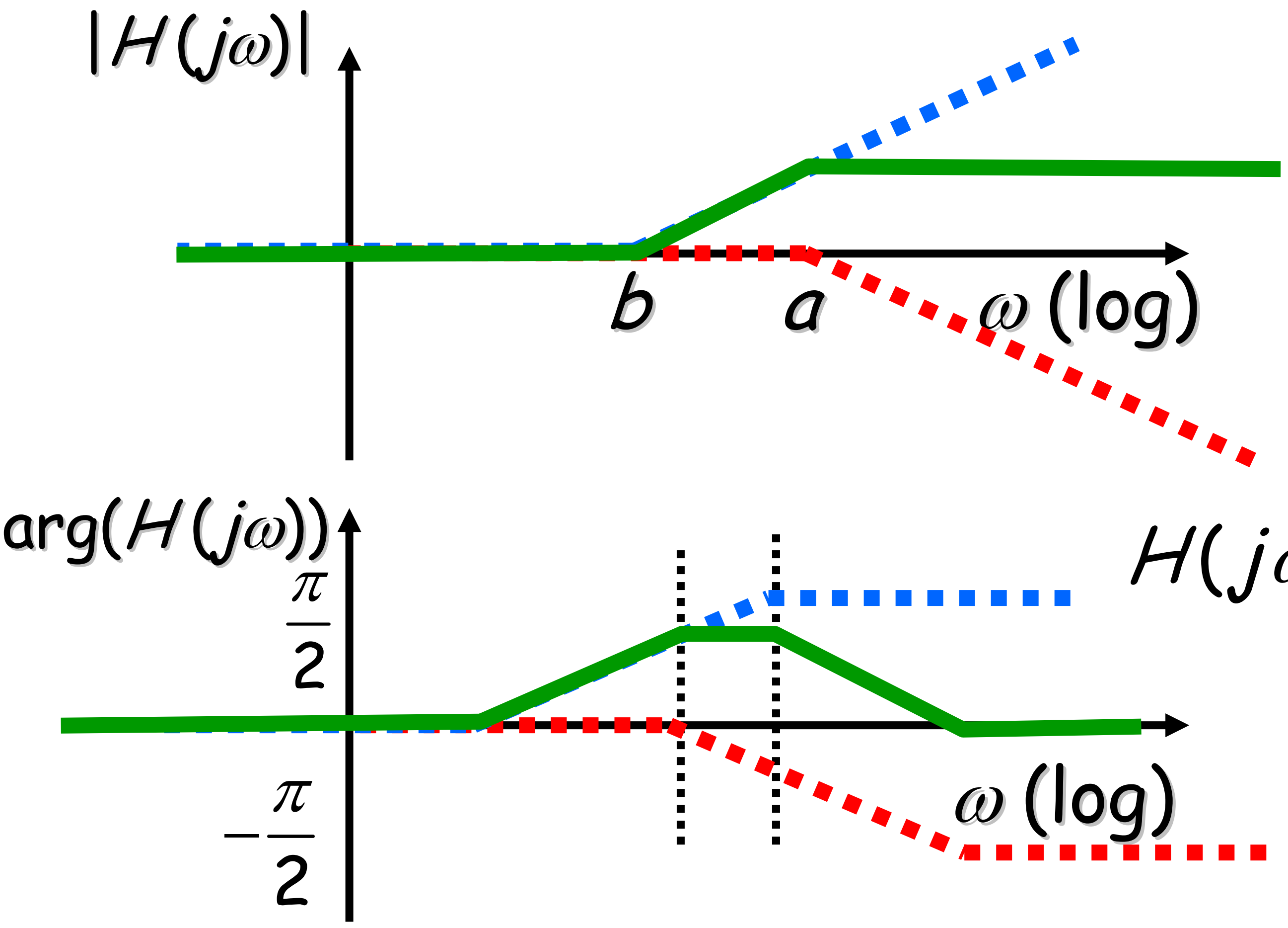
2:
decrease
HF-phase
shift



# Combinations

**Lead network**




$$H(j\omega) = \frac{a}{b}\frac{j\omega + b}{j\omega + a}$$

$$b < a$$

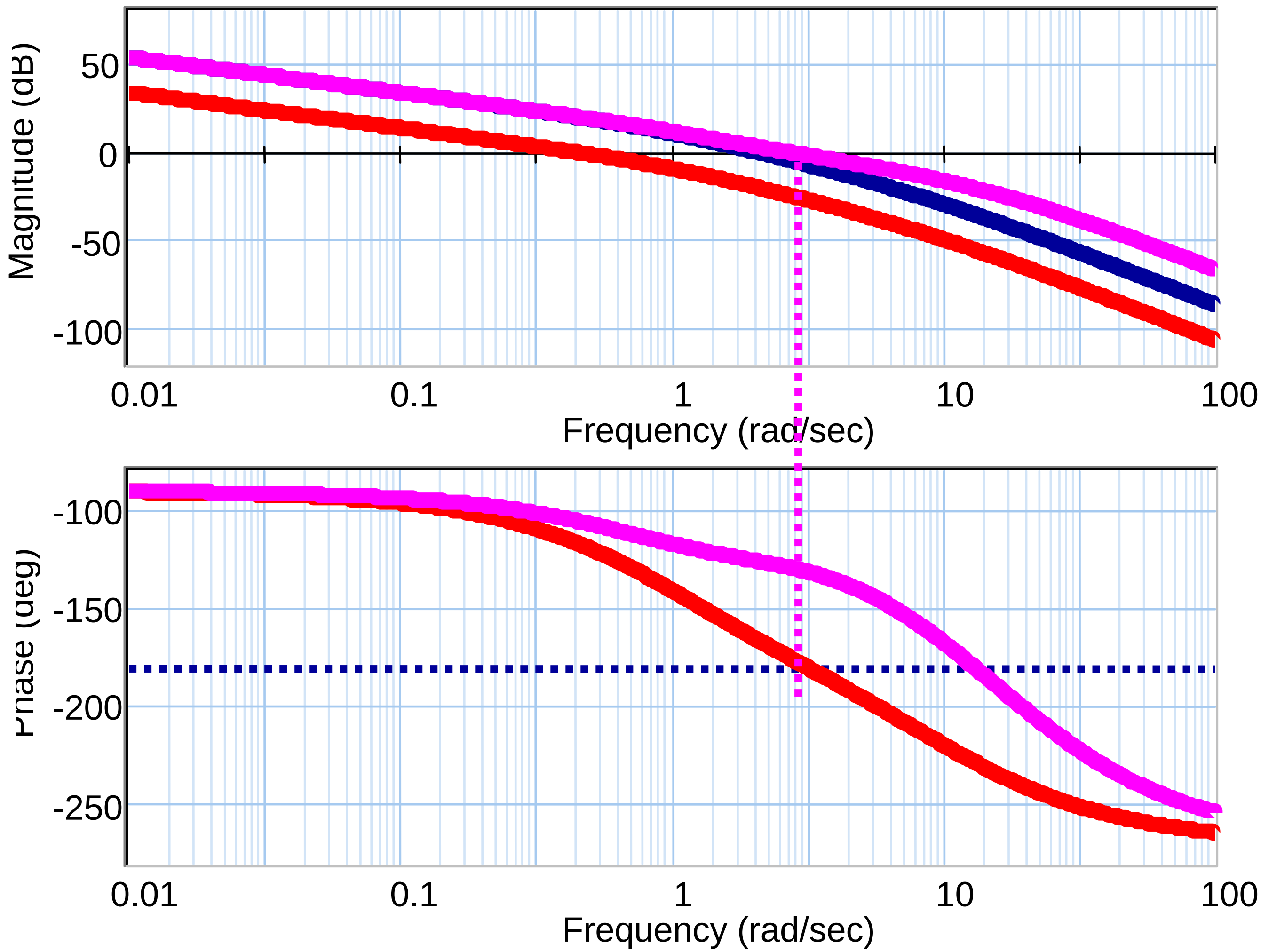
# Lead network (phase lead)



K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

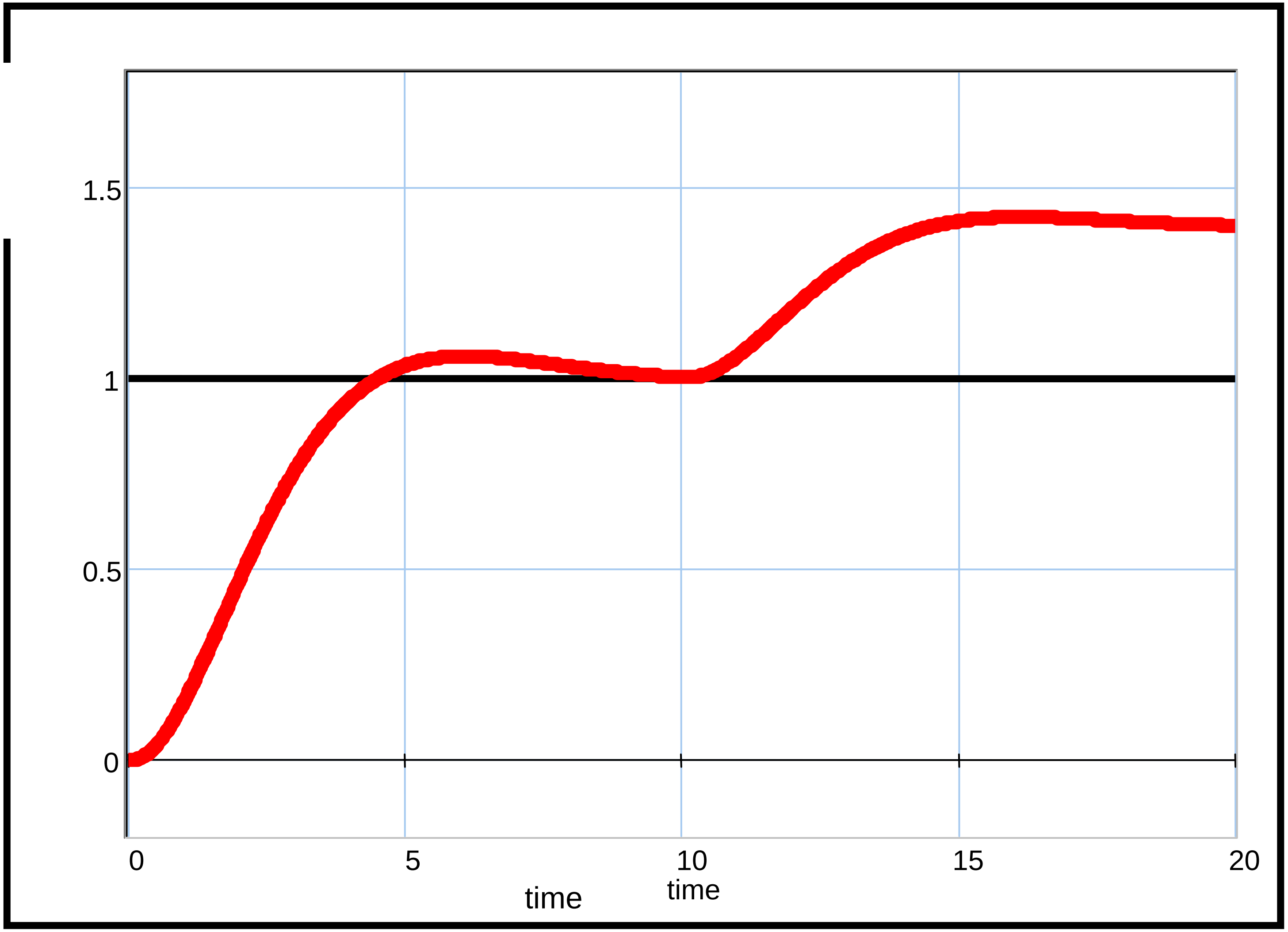
K = 5
+ Lead netw
(PM = 50°)



# Responses



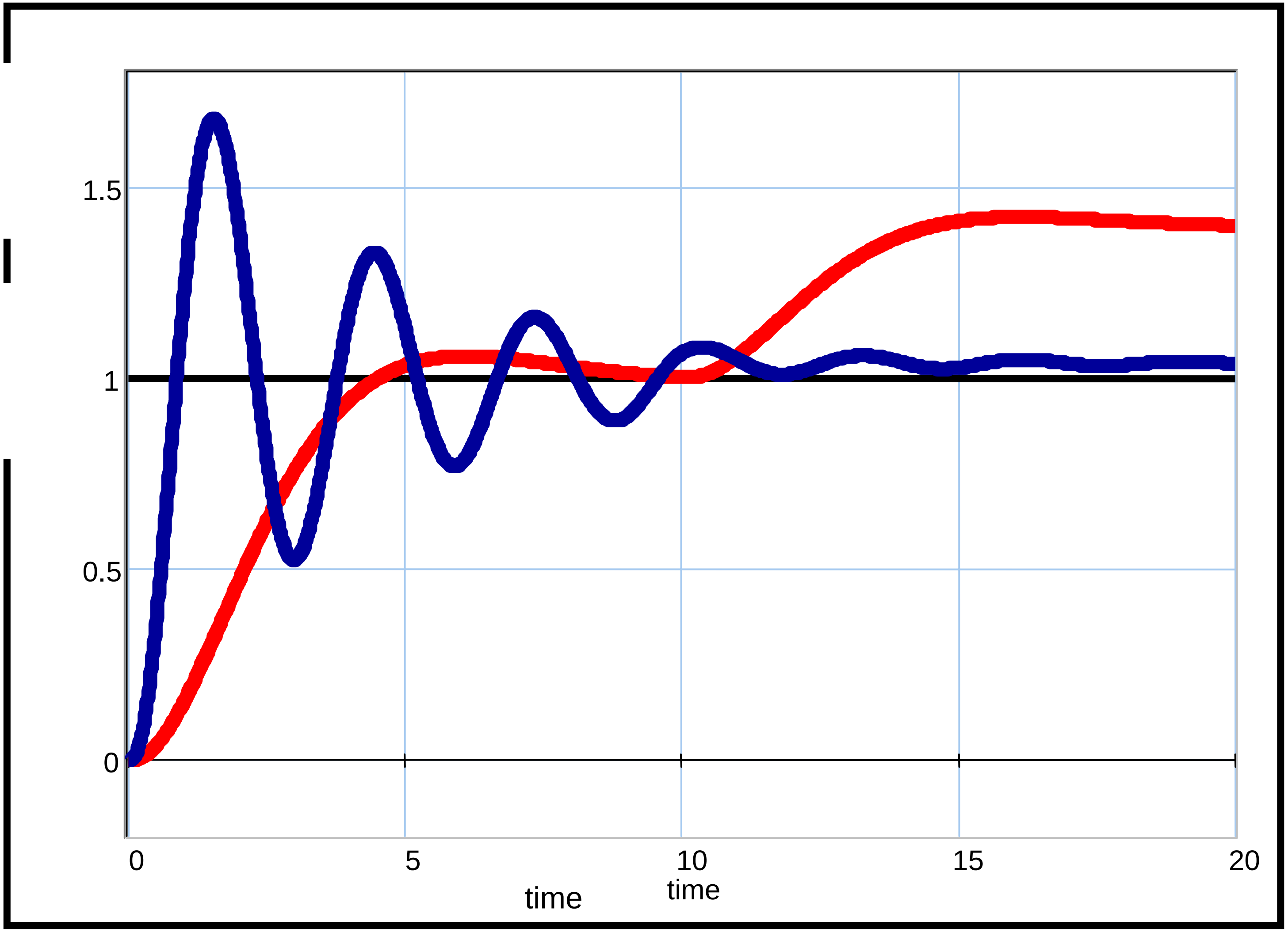
K = 0.5
(PM = 63°)



# Responses

K = 0.5
(PM = 63°)

K = 5
(PM = 13°)



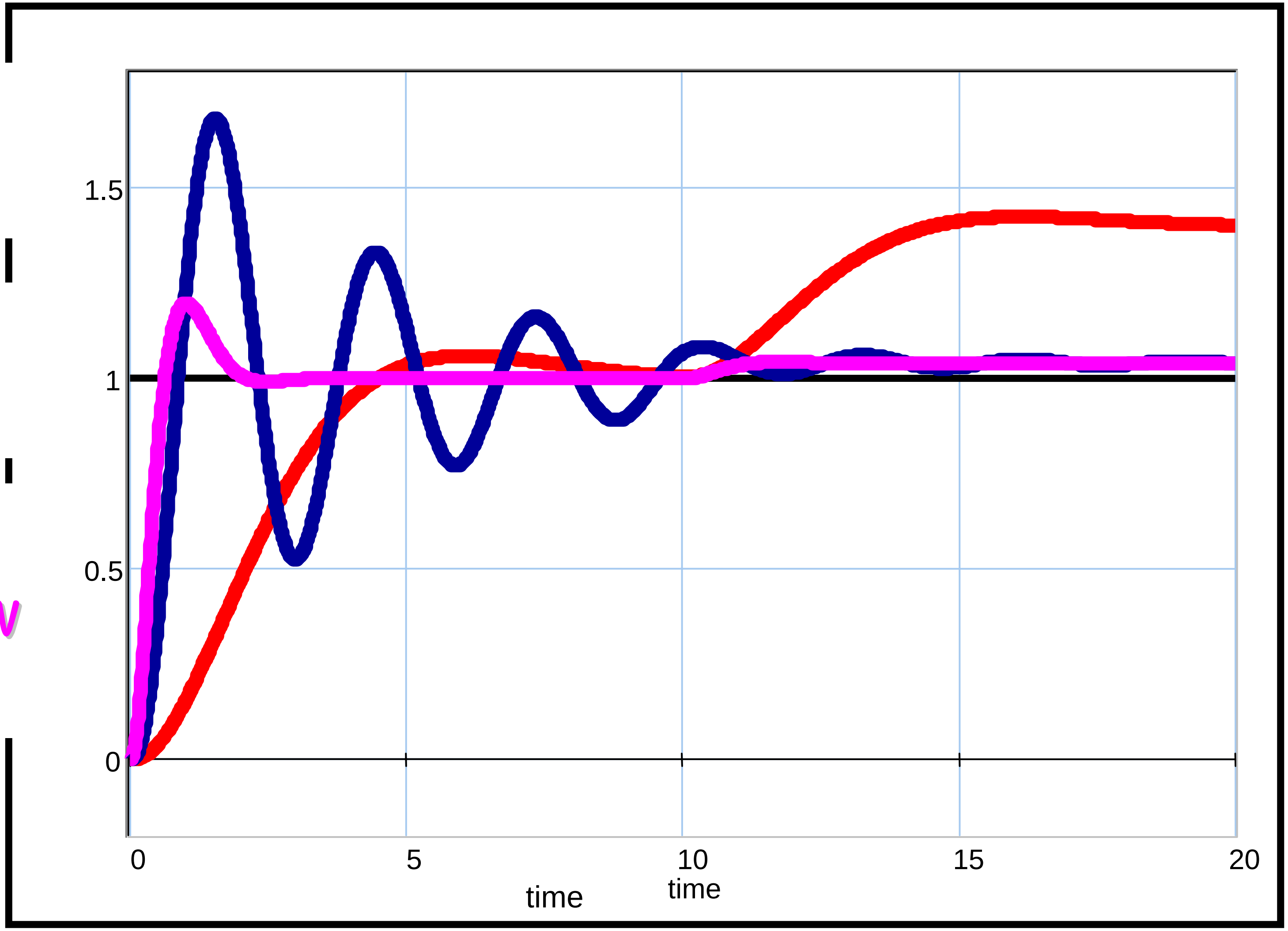
# Responses



K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

K = 5
+ Lead netw
(PM = 50°)



# Conclusions

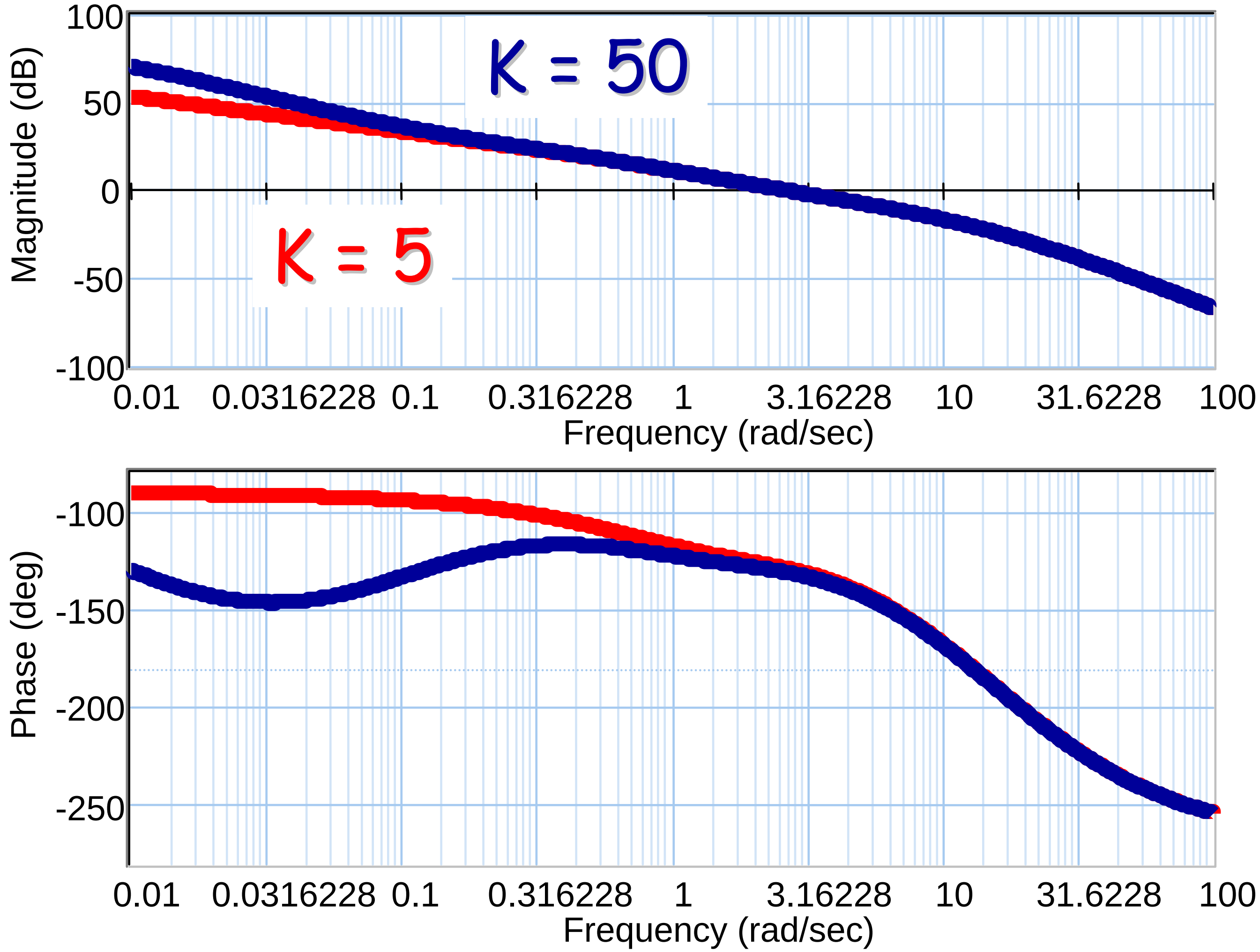


- Lag network:
  - dynamics approximately the same
  - (same bandwidth as low-gain system)
  - accuracy improved by increasing the low-frequency gain
- Lead network
  - Faster dynamics (increased bandwidth)
  - accuracy improved
  - Requires more powerful actuator

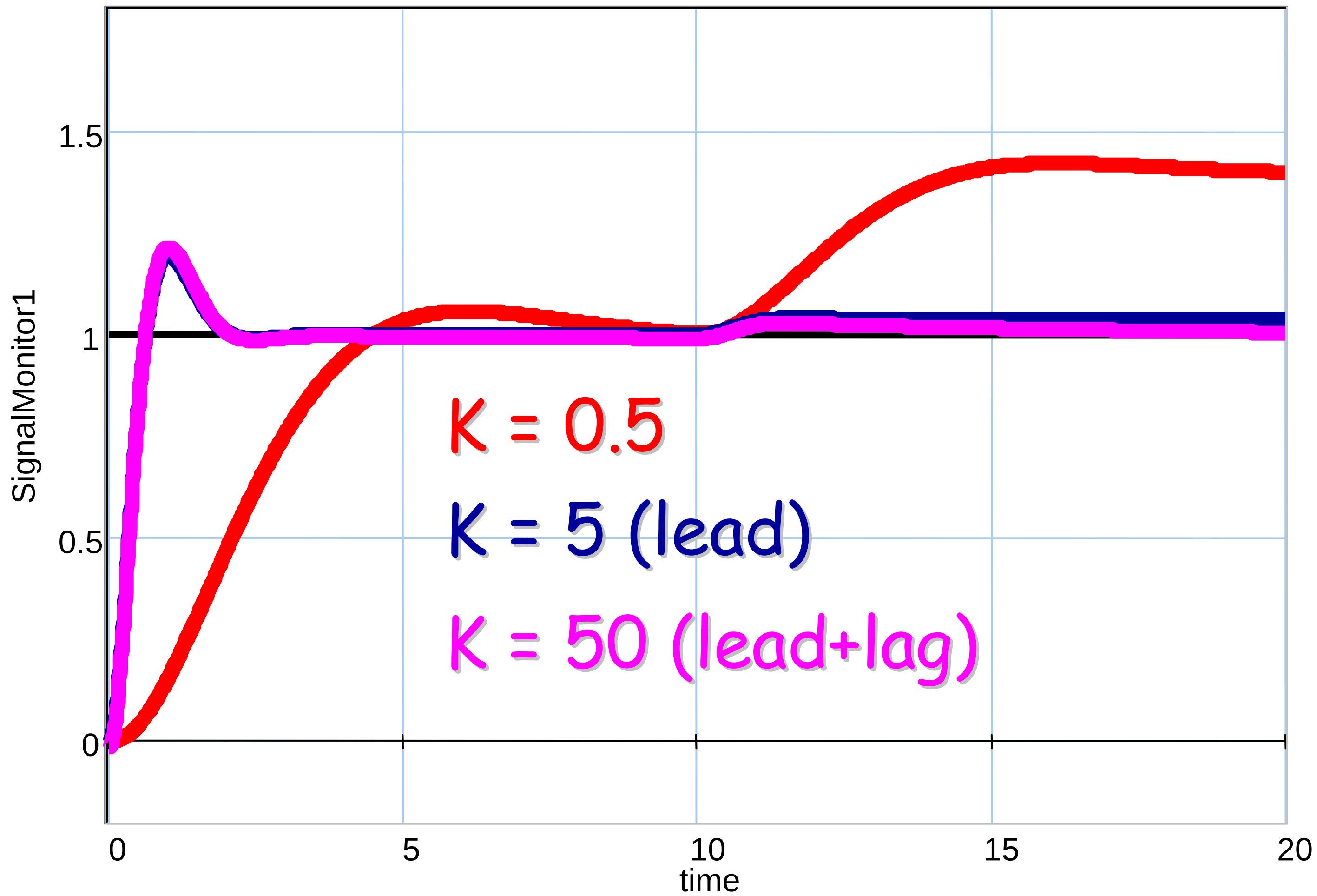
# Combination (Bode)

Lead

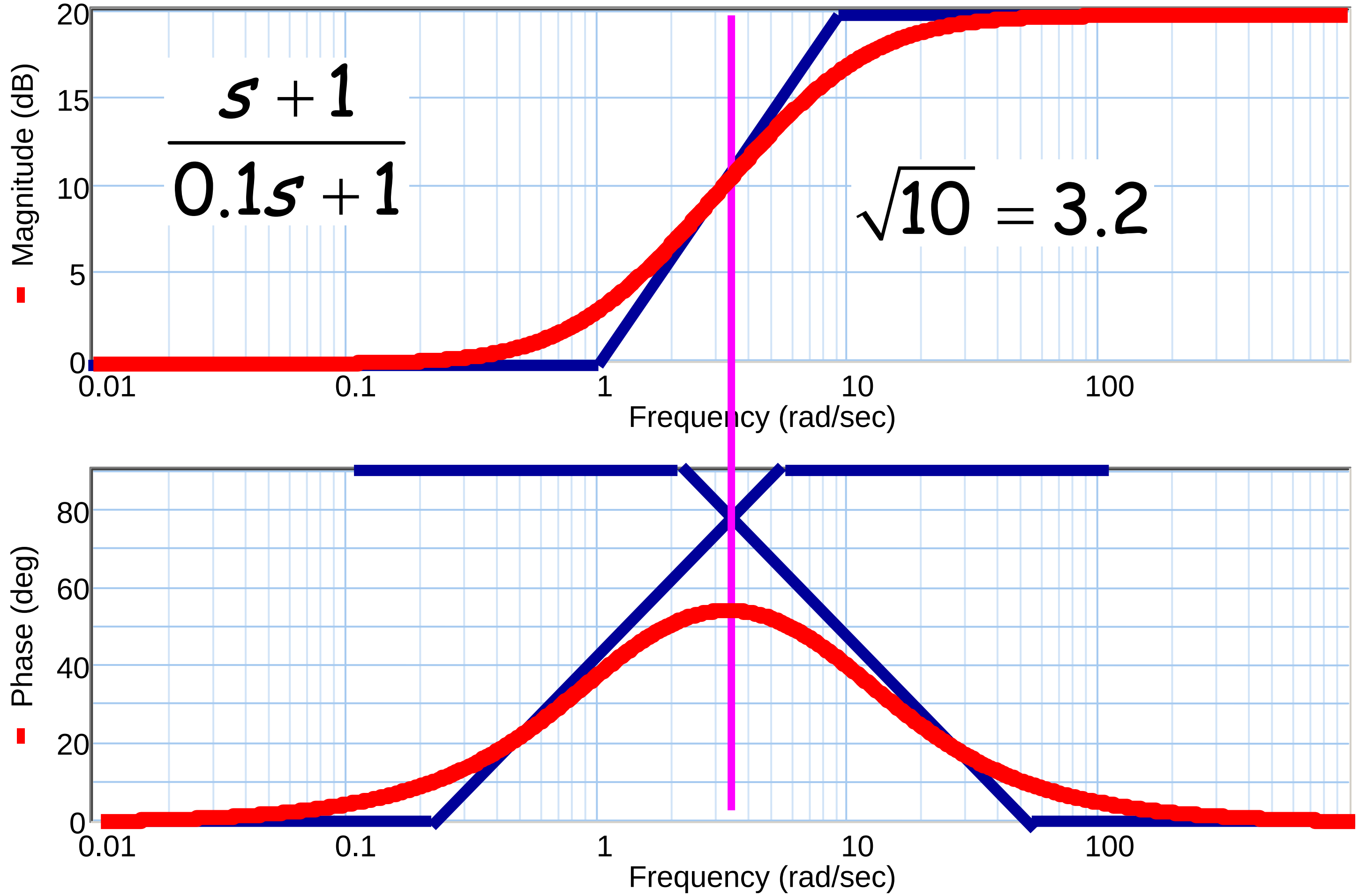
Lead
+
Lag



# Combination (responses)



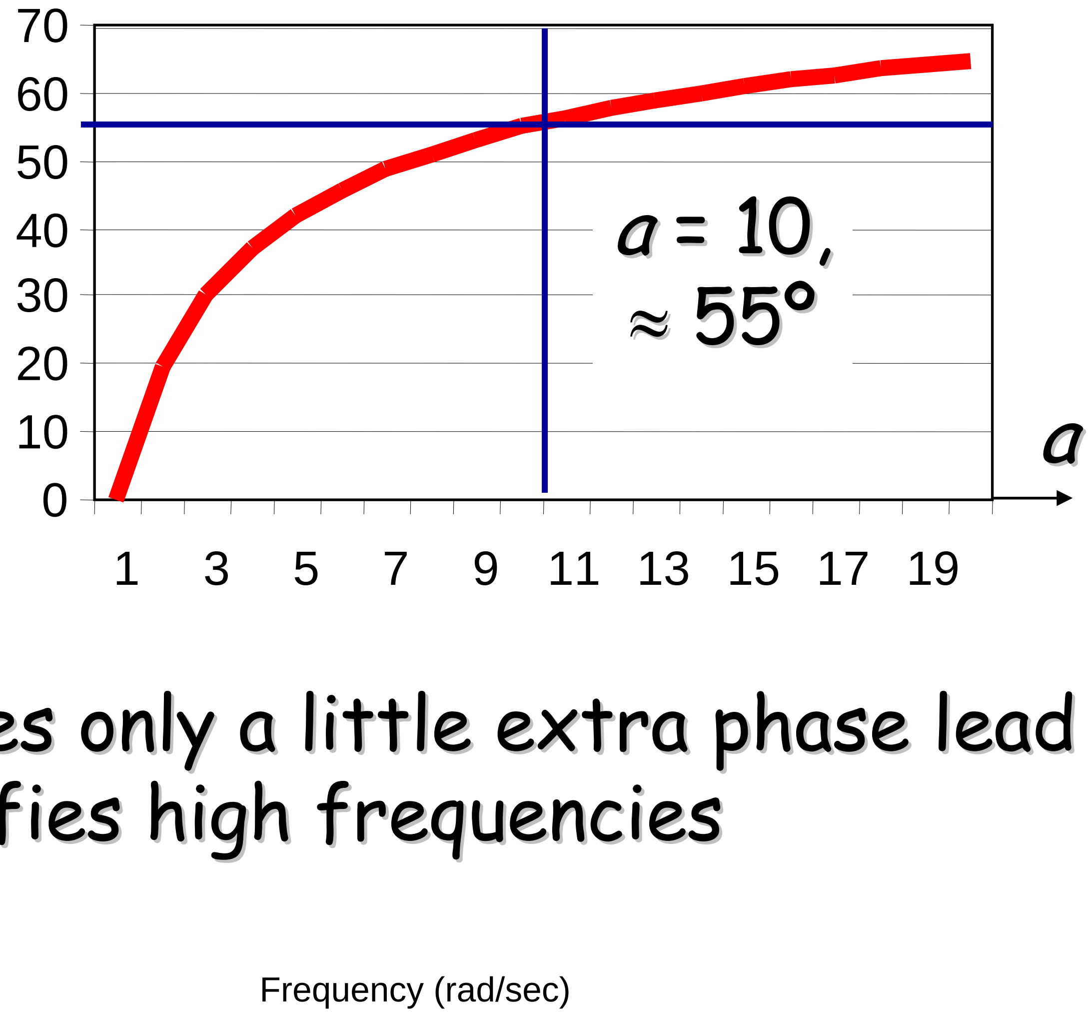
# Accurate Design



# Accurate Design

$\varphi_{max}$ [degree]

$\varphi_{max}$ as a function of $a$




$a > 10$ gives only a little extra phase lead but amplifies high frequencies

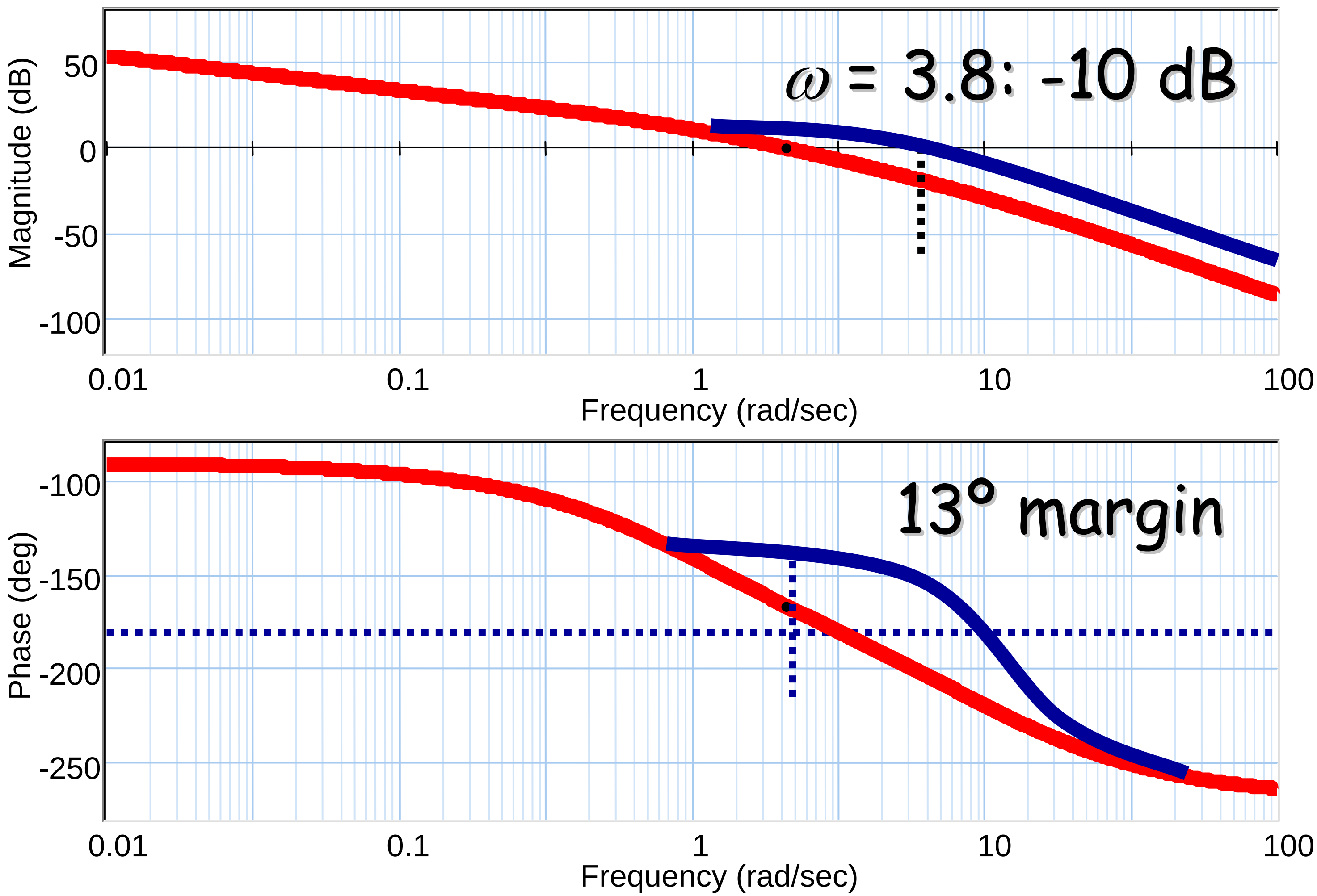
# Rules (1)



- Choose $a = 10$ by default
- Draw the bode plot for the desired gain
- The lead network gives 10 dB extra gain at $\varphi_{max}$
- We want $\varphi_{max}$ at the new zero crossing of the modulus

# Accurate Design



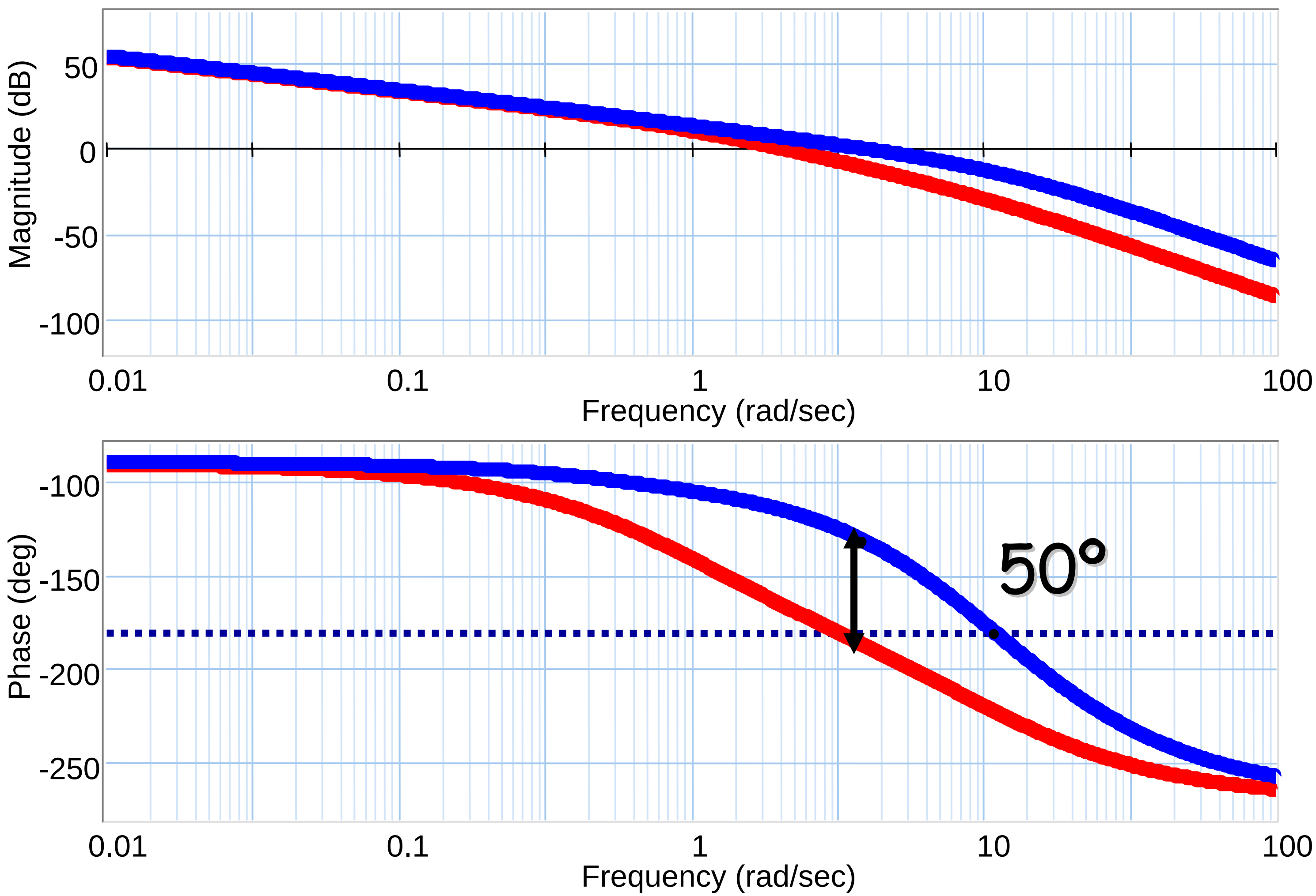


# Rules (2)

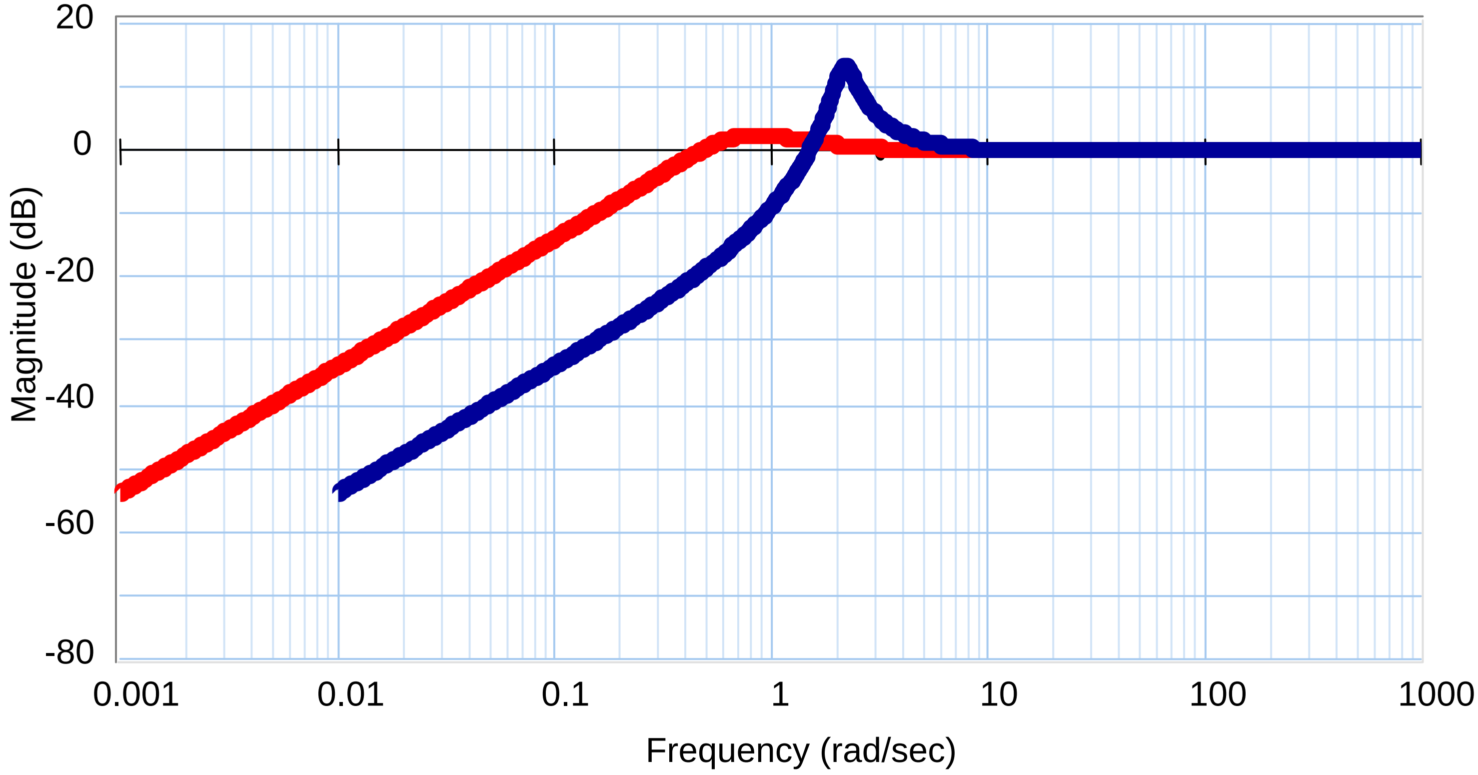


- We want $\varphi_{max}$ at the new zero crossing of the modulus $\omega = \omega_b$
- This implies that
  - zero should be located in $\omega = \omega_b / \sqrt{10}$
  - pole should be located in $\omega = \omega_b \cdot \sqrt{10}$
- with $\omega_b = 3.8$ it follows that
  - $\omega_b / \sqrt{10} = 1.2$
  - $\omega_b \cdot \sqrt{10} = 12$

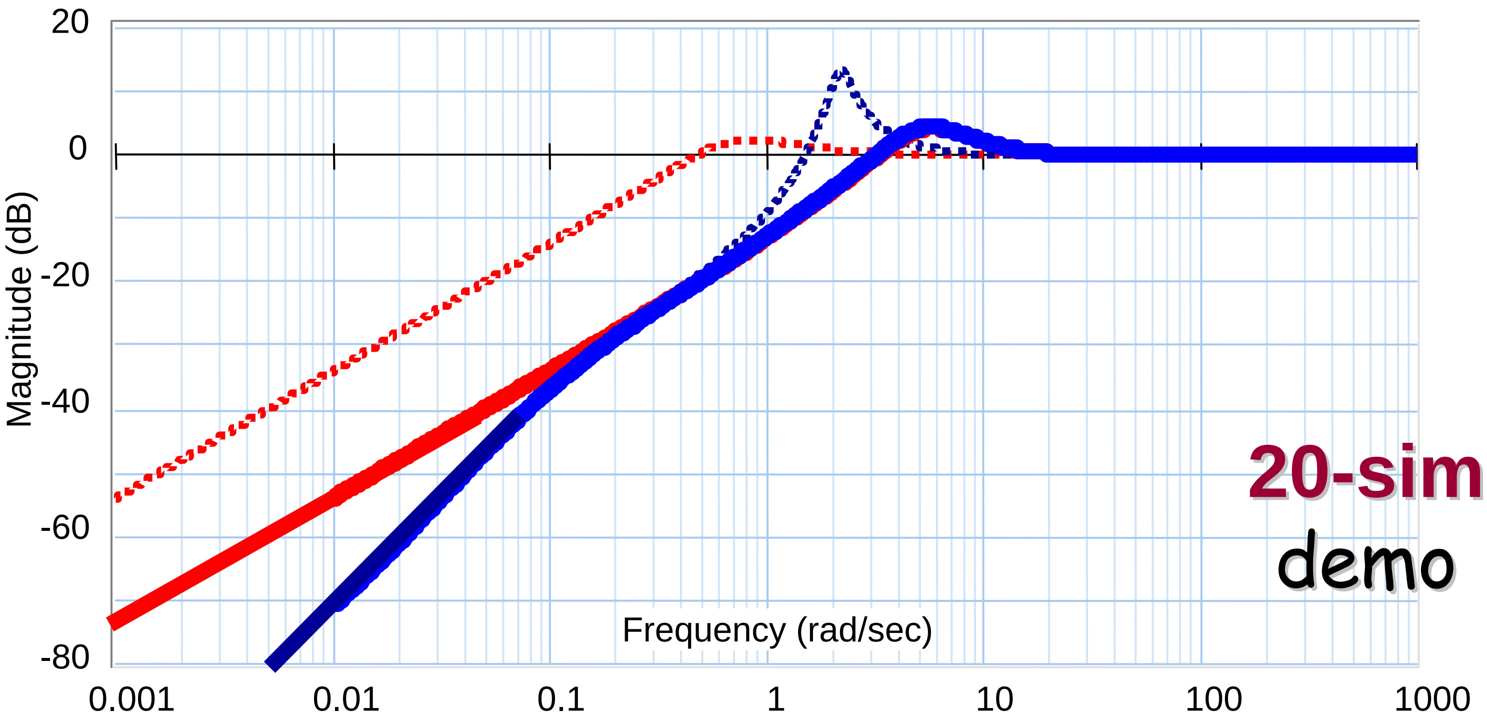
# Result


50
0
-50
-100
Magnitude (dB)
0.01
0.1
1
10
100
Frequency (rad/sec)
-100
-150
-200
-250
Phase (deg)
50°
0.01
0.1
1
10
100
Frequency (rad/sec)

# Sensitivity

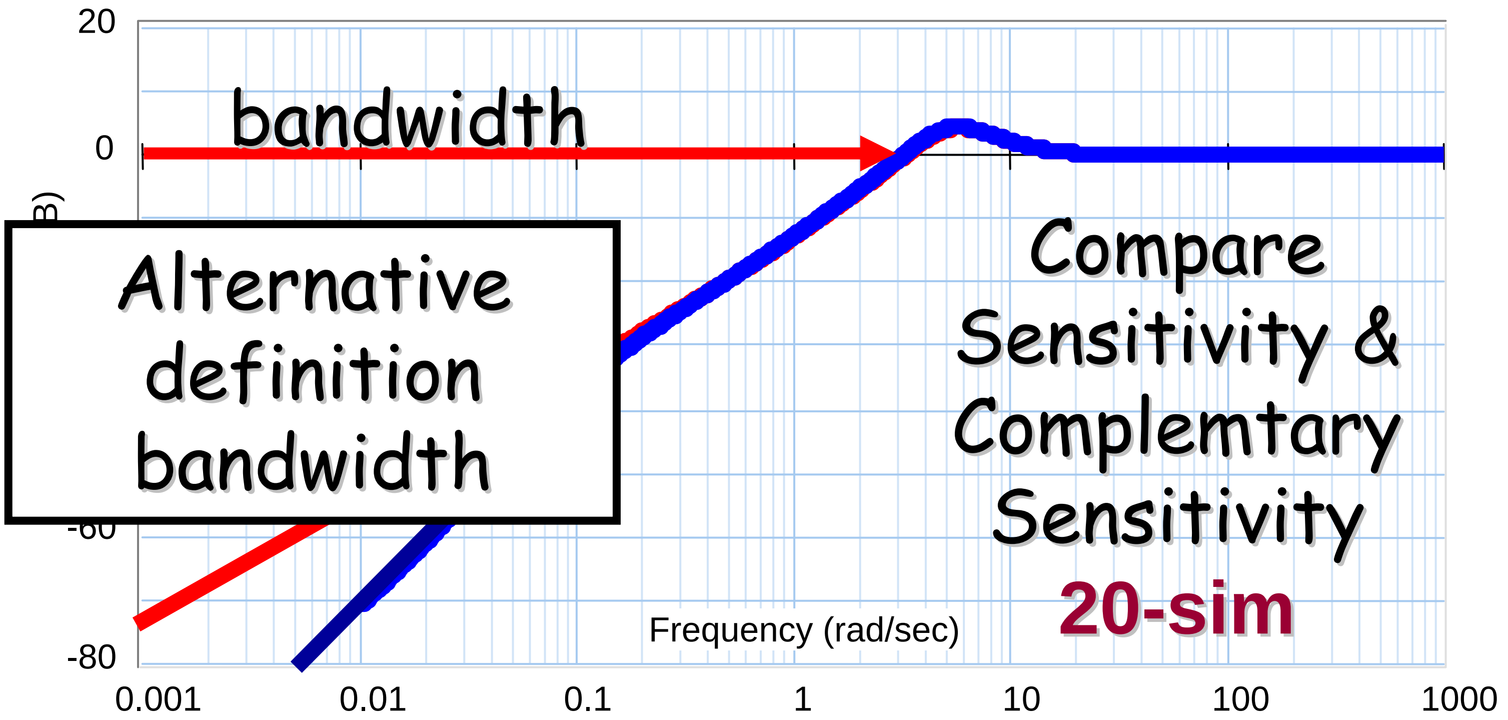



K = 0.5 K = 5



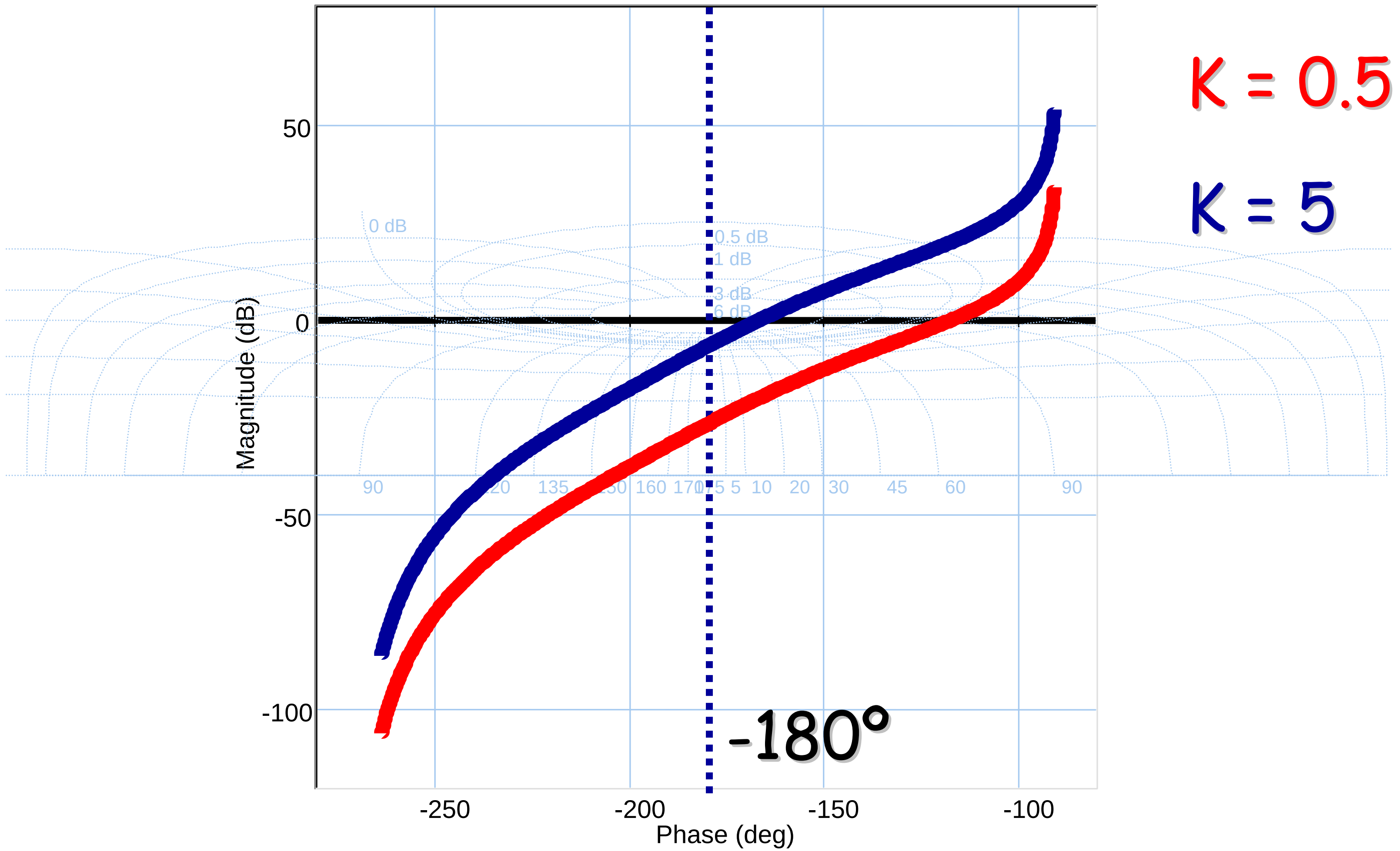

K = 0.5  K = 5  $5\frac{12}{1.2}\frac{s+1.2}{s+12}$  $10\frac{0.1}{0.01}\frac{s+0.1}{s+0.01}$

# Sensitivity

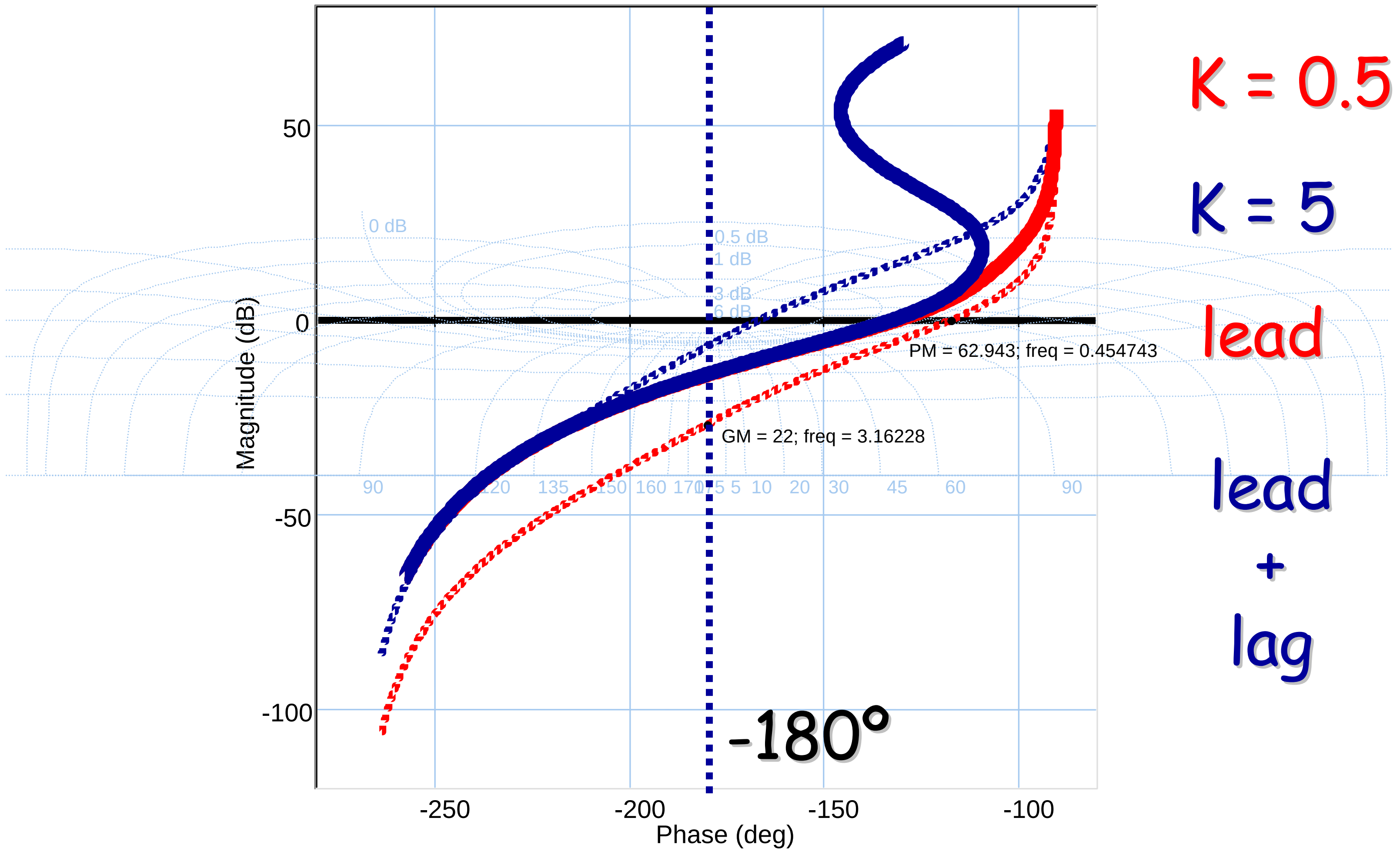


# Nichols



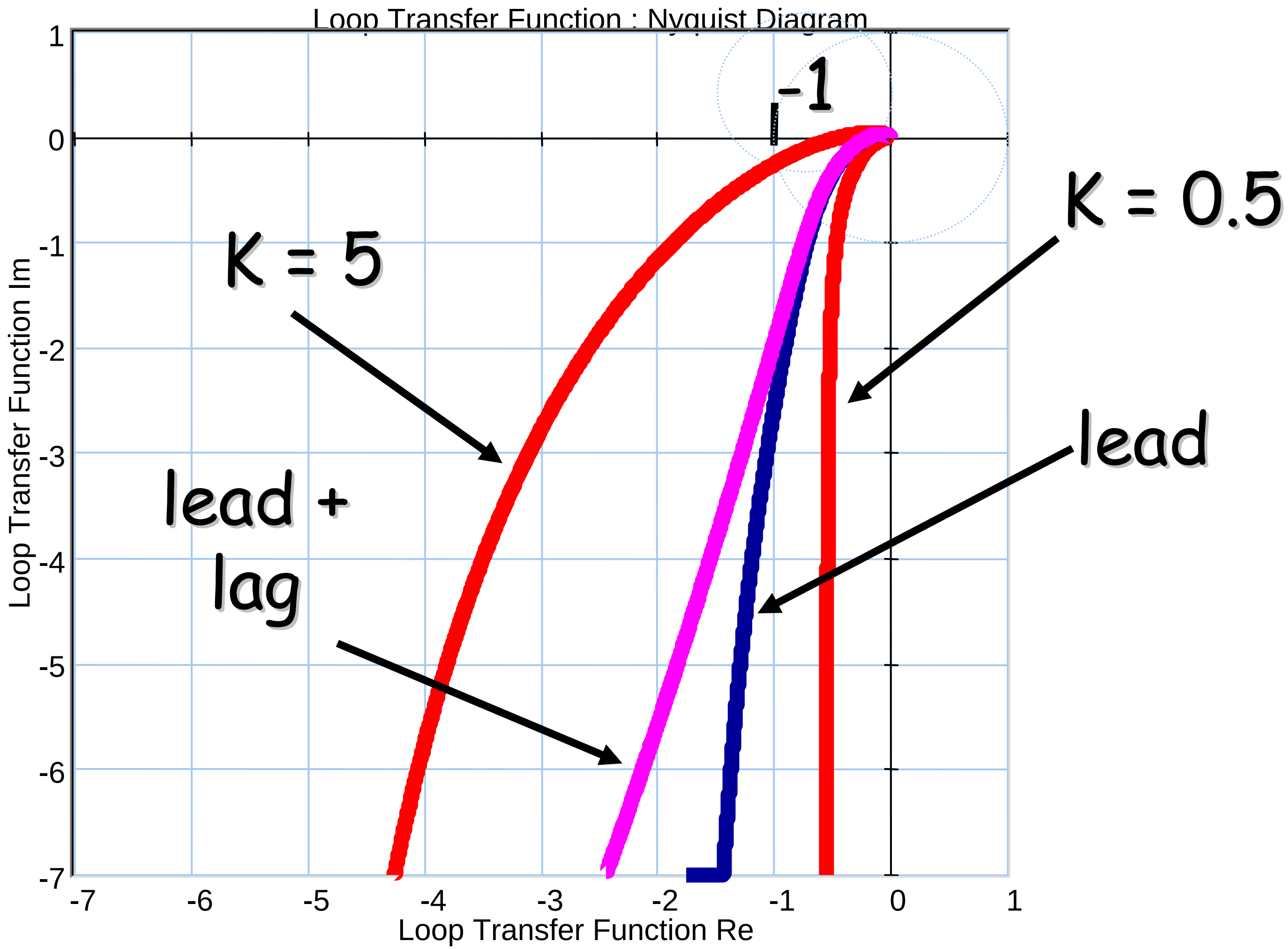

K = 0.5
K = 5
-180°
Magnitude (dB)
Phase (deg)
50
0
-50
-100
-250
-200
-150
-100
0 dB
0.5 dB
1 dB
3 dB
6 dB
90
135
160
5
10
20
30
45
60
90

# Nichols


50
0
-50
-100
Magnitude (dB)
-250
-200
-150
-100
Phase (deg)
0 dB
0.5 dB
1 dB
3 dB
6 dB
PM = 62.943; freq = 0.454743
GM = 22; freq = 3.16228
-180°
K = 0.5
K = 5
lead
lead
+
lag

# Nyquist



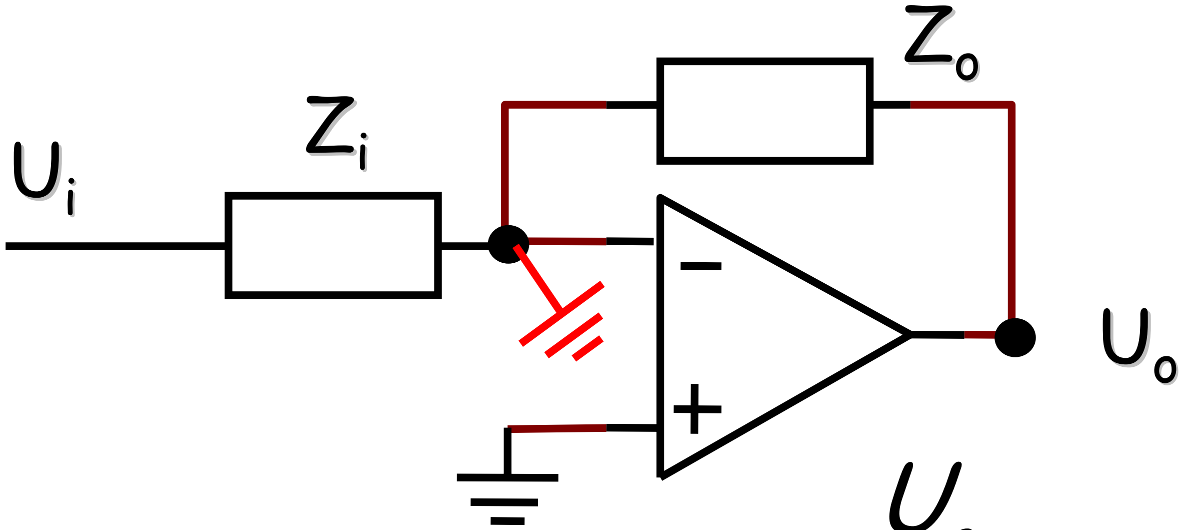

Loop Transfer Function : Nyquist Diagram
Loop Transfer Function Im
Loop Transfer Function Re
1
0
-1
-2
-3
-4
-5
-6
-7
-7
-6
-5
-4
-3
-2
-1
0
1
-1
K = 5
K = 0.5
lead
lead +
lag

# Conclusions



- Compensation networks can improve the dynamic performance (transients) and/or the accuracy
- Lead networks: located in high-frequency region
- lag networks: located in low-frequency region

# Op amp realisation




$K \rightarrow \infty$

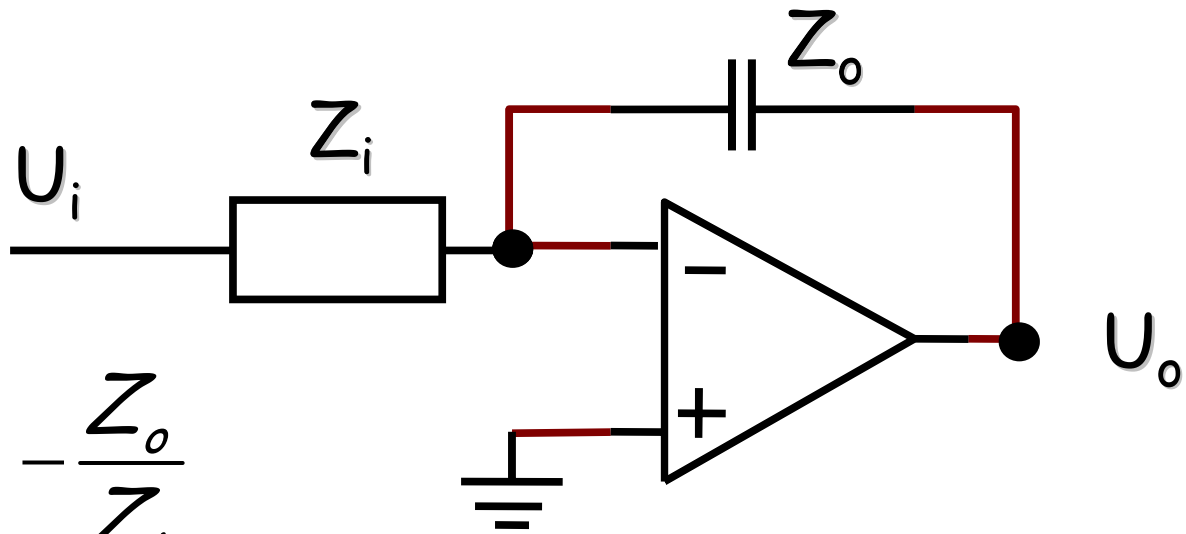
$R_{in} \rightarrow \infty$

$R_{out} \rightarrow 0$

$$\frac{U_o}{Z_o} = -\frac{U_i}{Z_i}$$

$$\frac{U_o}{U_i} = -\frac{Z_o}{Z_i}$$

# Op amp realisation




$$\frac{U_o}{U_i} = -\frac{Z_o}{Z_i}$$

$$Z_o = \frac{1}{j\omega C}$$

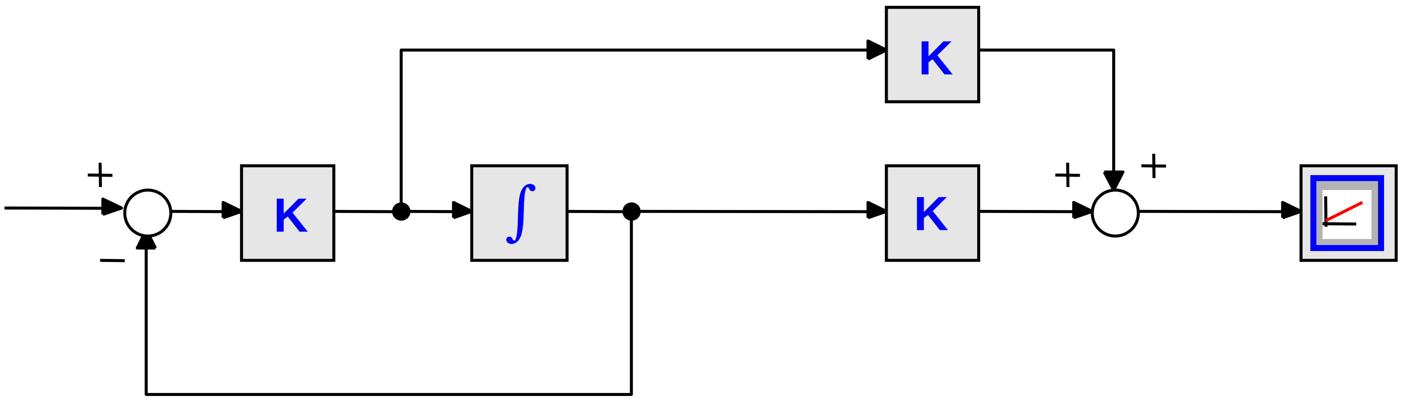
$$R_i = R$$

$$\frac{U_o}{U_i} = -\frac{1}{j\omega RC} = -\frac{1}{sRC} = -K\frac{1}{s}$$

# Lead/Lag network

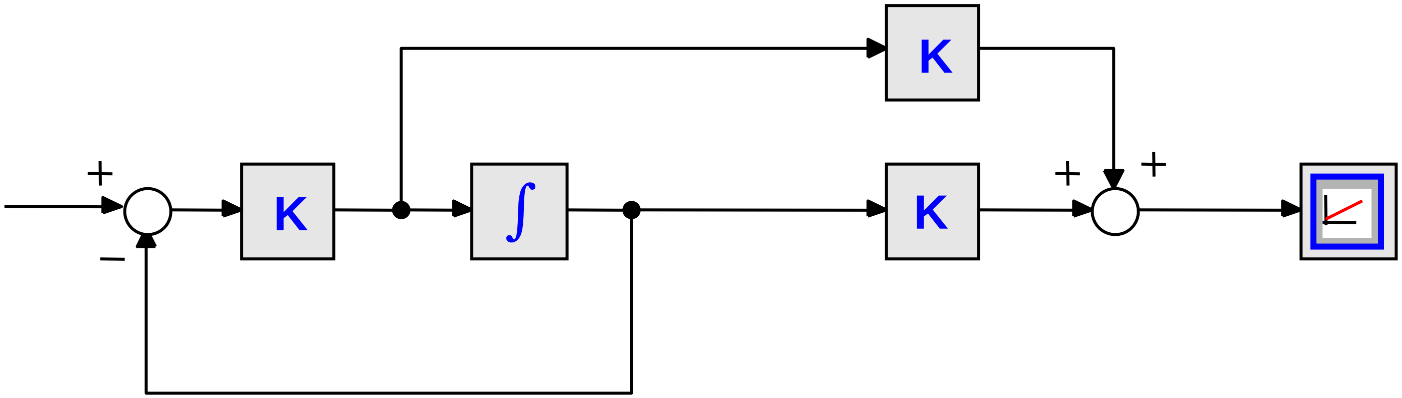


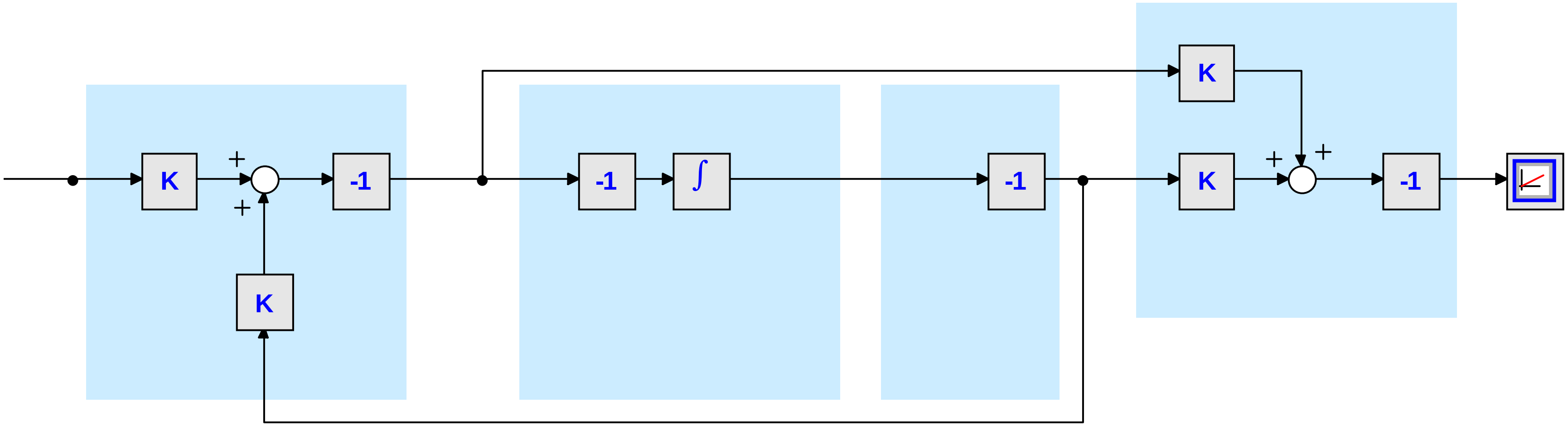
$$K\frac{as\tau+1}{s\tau+1}=K\left(\frac{1}{s\tau+1}+s\frac{a\tau}{s\tau+1}\right)$$



# Lead/Lag network

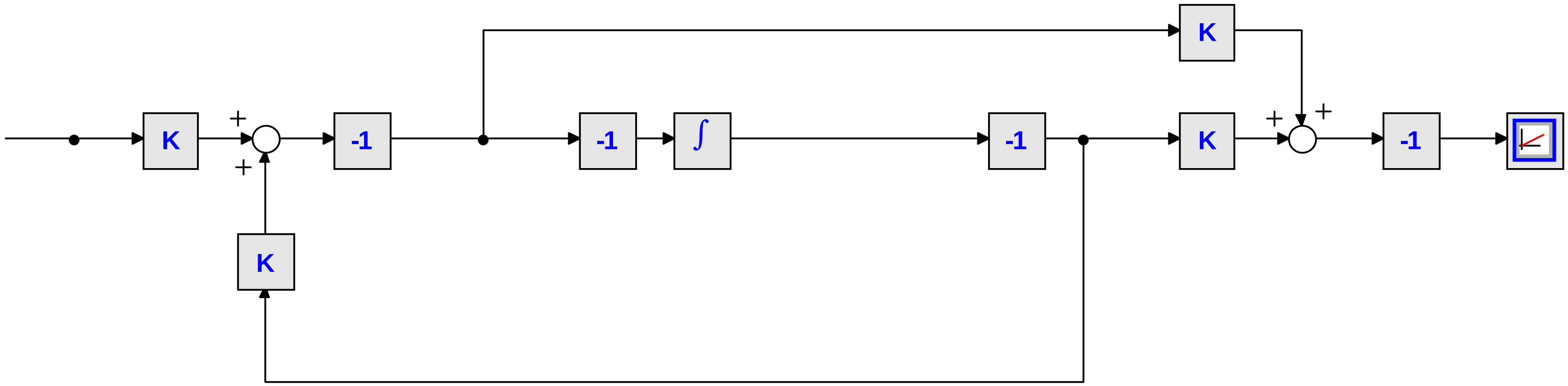


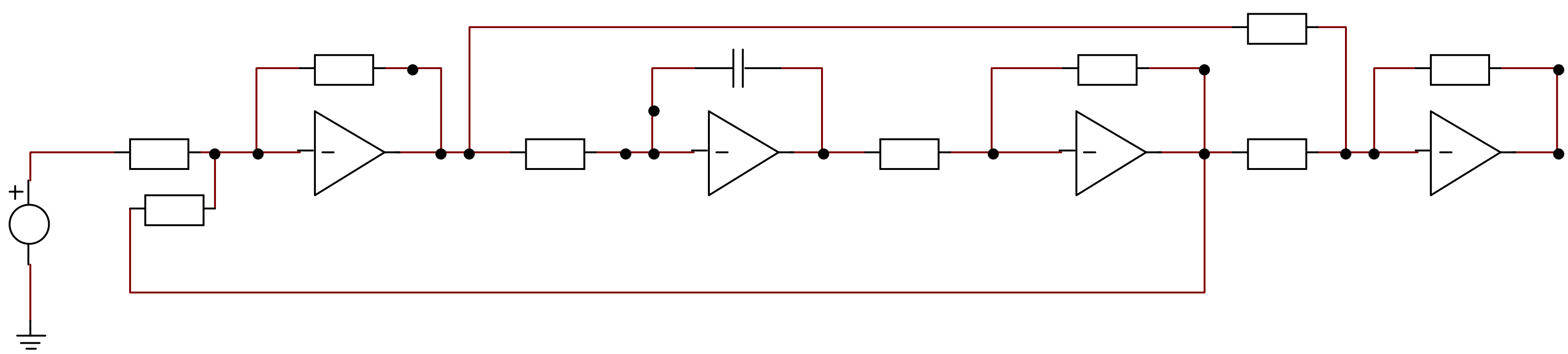


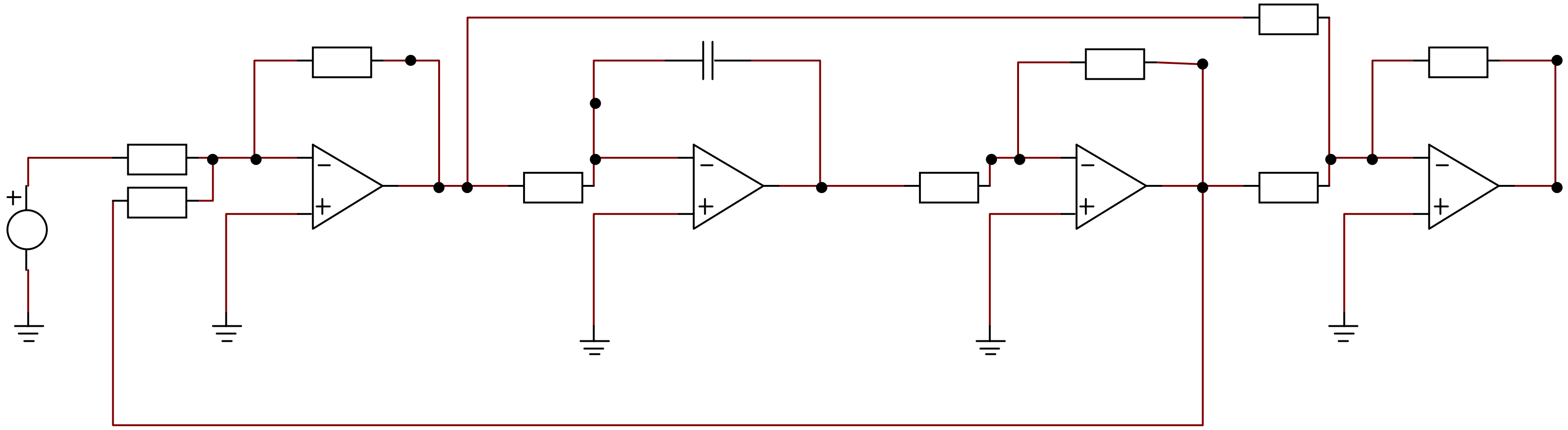
# Lead/Lag network





# Lead/Lag network



**20 sim**

opamp demo