

Frequency Responses

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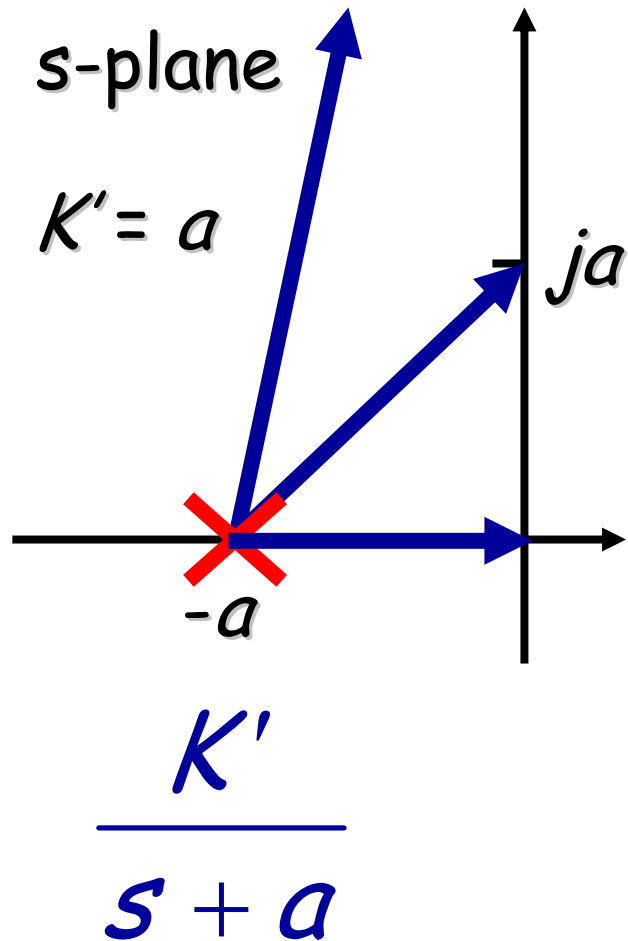
Relation $s \leftrightarrow j\omega$

$$s = \alpha + j\omega$$

$$\text{for } \alpha = 0 \quad s = j\omega$$

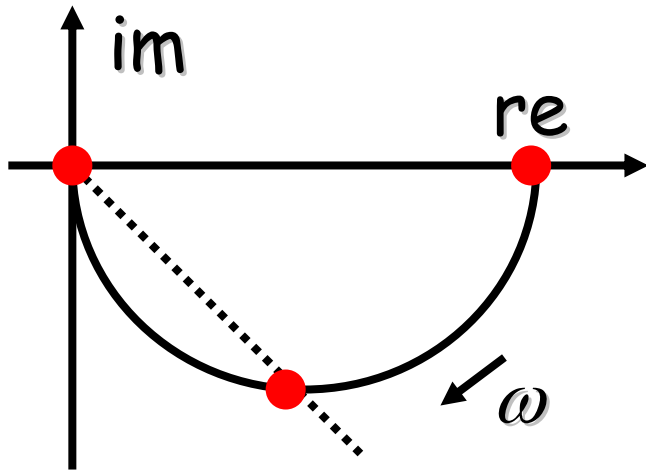
- Relation $s \leftrightarrow j\omega$
- Frequency responses:
 - Nyquist (polar plot)
 - Bode
 - Nichols

Relation $s \leftrightarrow j\omega$



ω	$ H(j\omega) $	$\arg(H(j\omega))$
0	$\frac{a}{a}$	0
a	$\frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$	$-\frac{\pi}{4}$
∞	0	$-\frac{\pi}{2}$

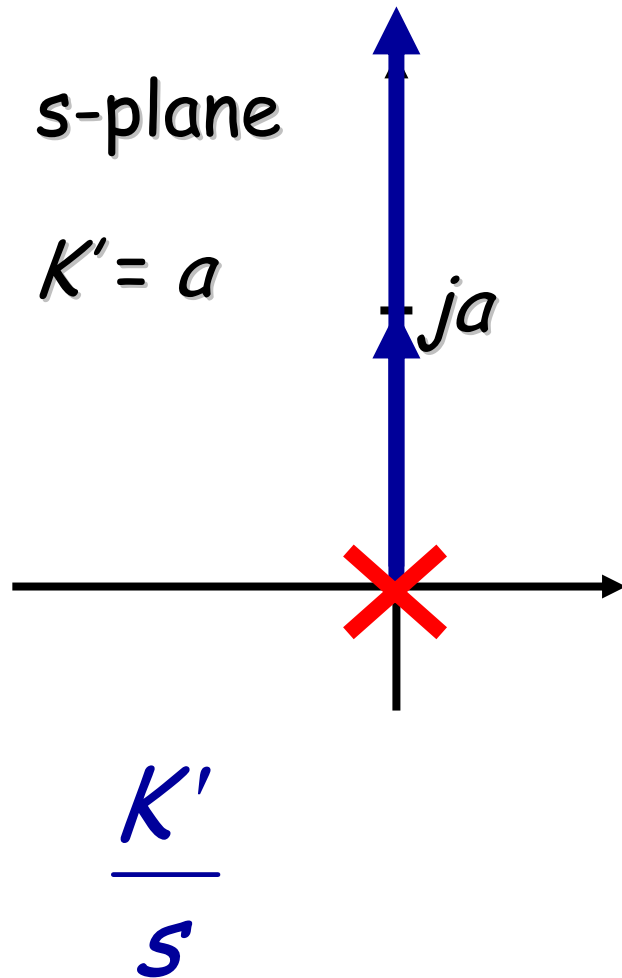
Nyquist plot ($a/(j\omega+a)$)



Nyquist diagram
Polar plot

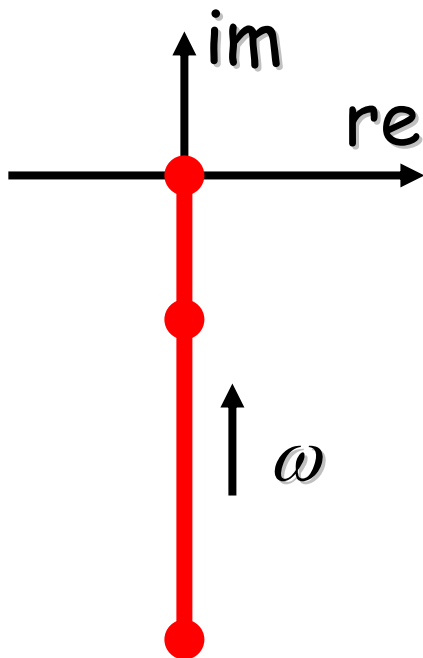
ω	$ H(j\omega) $	$\arg(H(j\omega))$
0	$\frac{a}{a} = 1$	0
a	$\frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$	$-\frac{\pi}{4}$
∞	0	$-\frac{\pi}{2}$

Relation $s \leftrightarrow j\omega$



ω	$ H(j\omega) $	$\arg(H(j\omega))$
0	$\frac{a}{0} \rightarrow \infty$	$-\frac{\pi}{2}$
a	$\frac{a}{a} = 1$	$-\frac{\pi}{2}$
∞	0	$-\frac{\pi}{2}$

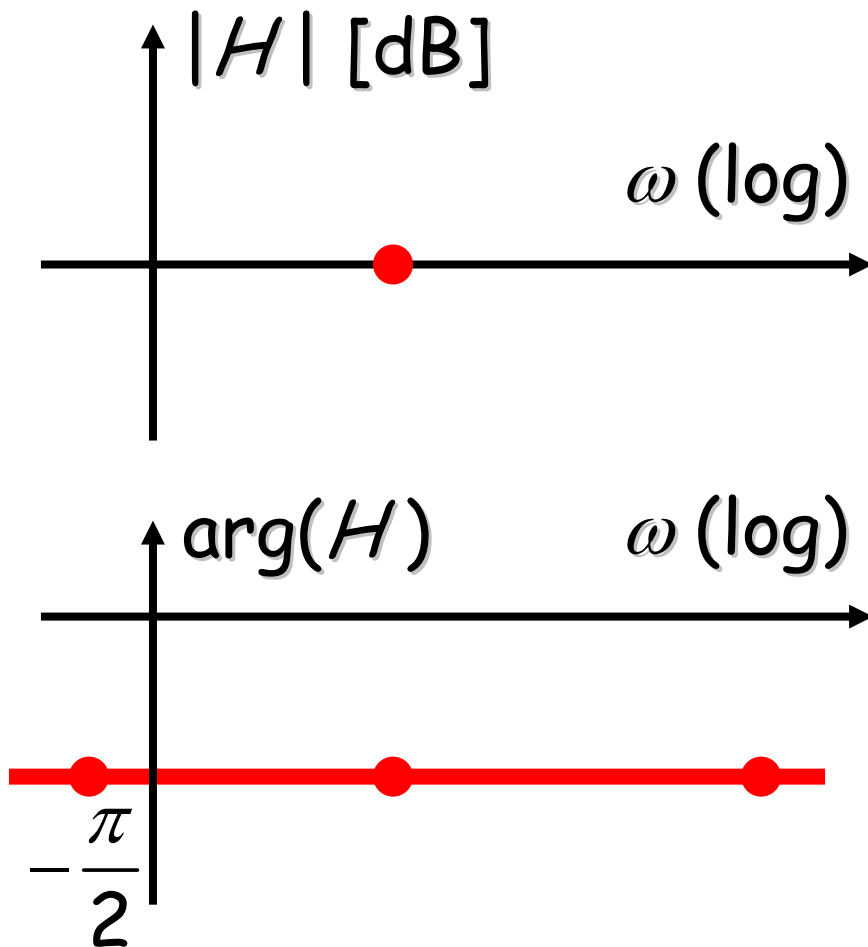
Nyquist plot



Nyquist diagram
Polar plot

ω	$ H(j\omega) $	$\arg(H(j\omega))$
0	$\frac{a}{0} \rightarrow \infty$	$-\frac{\pi}{2}$
a	$\frac{a}{a} = 1$	$-\frac{\pi}{2}$
∞	0	$-\frac{\pi}{2}$

Bode plot ($a/j\omega$)



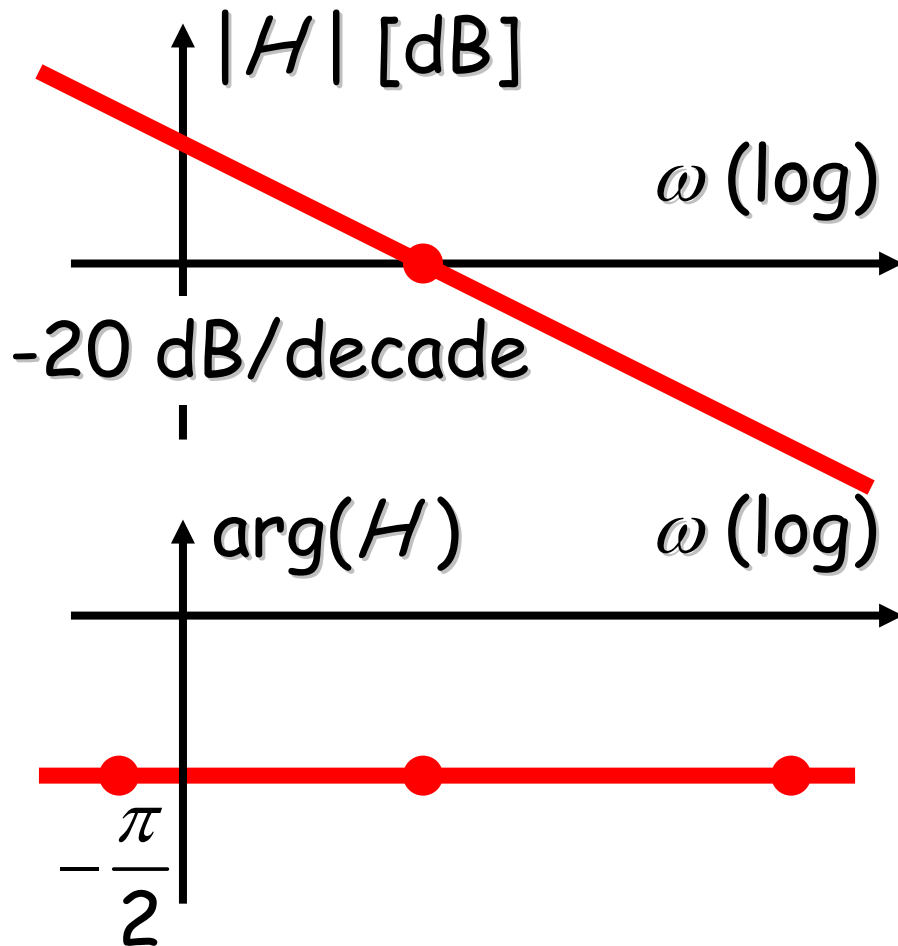
ω	$ H(j\omega) $	$\arg(H(j\omega))$
$\omega \rightarrow 0$	$\frac{a}{0} \rightarrow \infty$	$-\frac{\pi}{2}$
a	$\frac{a}{a} = 1$	$-\frac{\pi}{2}$
∞	0	$-\frac{\pi}{2}$

Bode diagram:

$20^{10} \log |H|$ versus $\omega(\log)$

$\arg(H)$ versus $\omega(\log)$

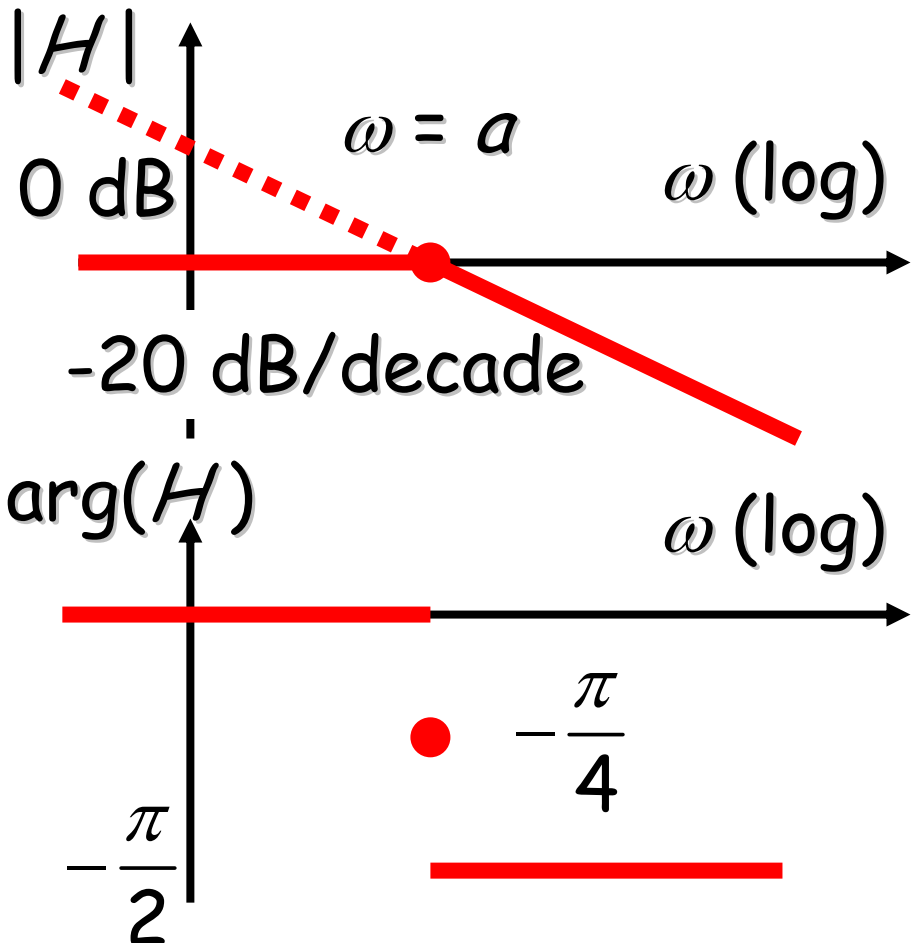
Bode plot ($a/j\omega$)



If ω ten times larger
 $a/j\omega$ ten times smaller:

straight line with a slope
of $-20^{10} \log(10) =$
 -20 dB/decade
or -6 dB/octave

Bode plot ($a/(j\omega+a)$)



$$H(j\omega) = \frac{a}{j\omega + a}$$

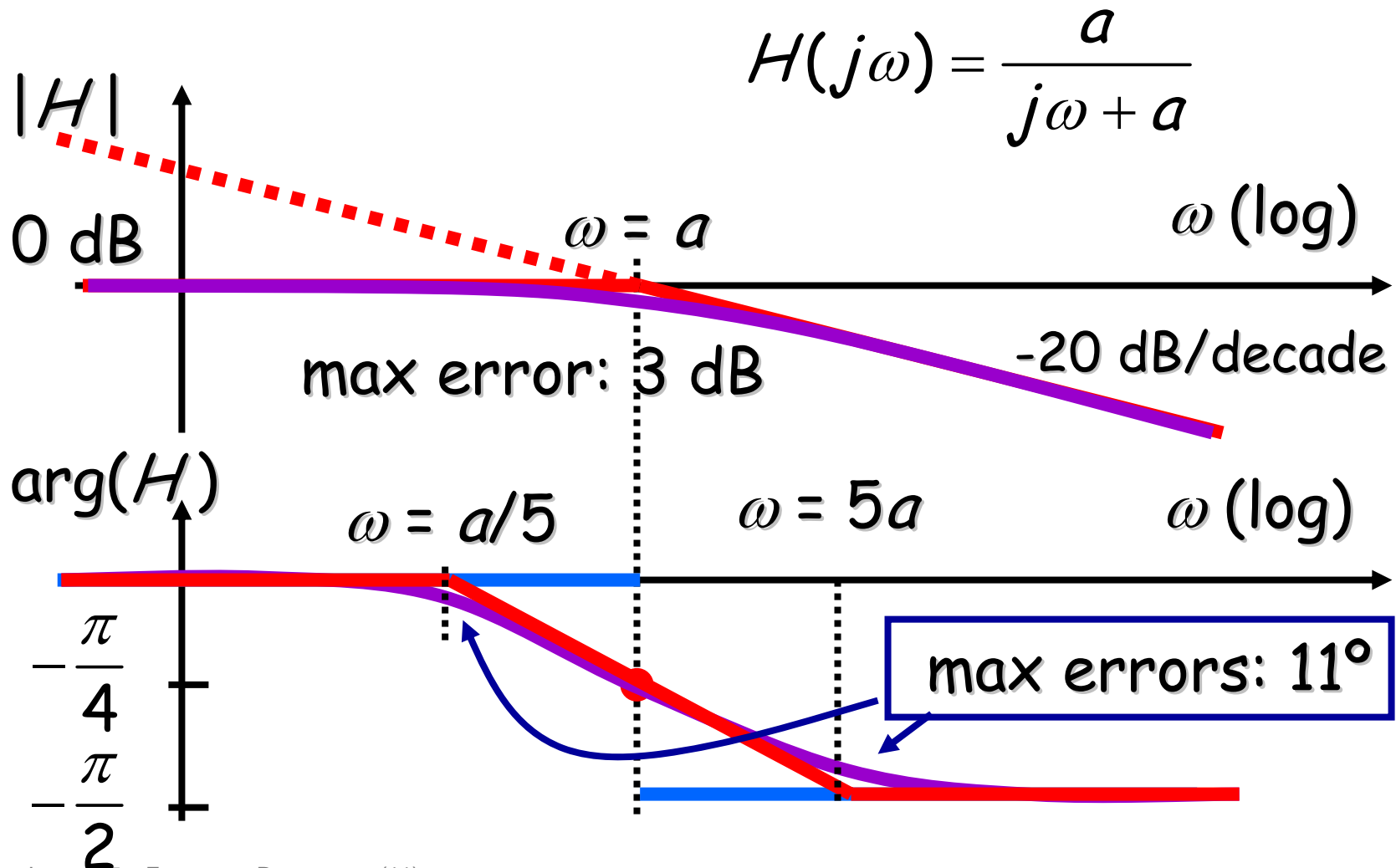
asymptotic
approximation:

$$\omega < a: H(j\omega) = \frac{a}{a}$$

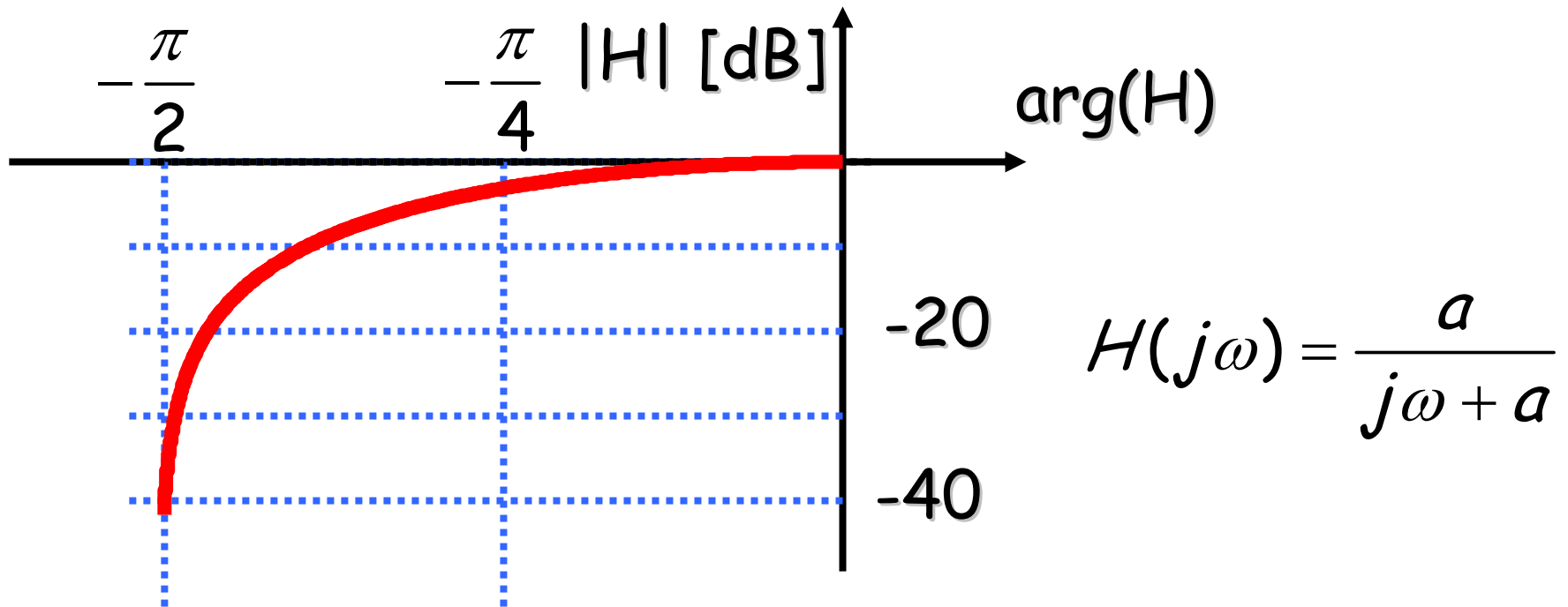
$$\omega > a: H(j\omega) = \frac{a}{j\omega}$$

$$\omega = a: H(j\omega) = \frac{a}{j\omega + a}$$

Bode plot ($a/(j\omega + a)$)



Nichols diagram



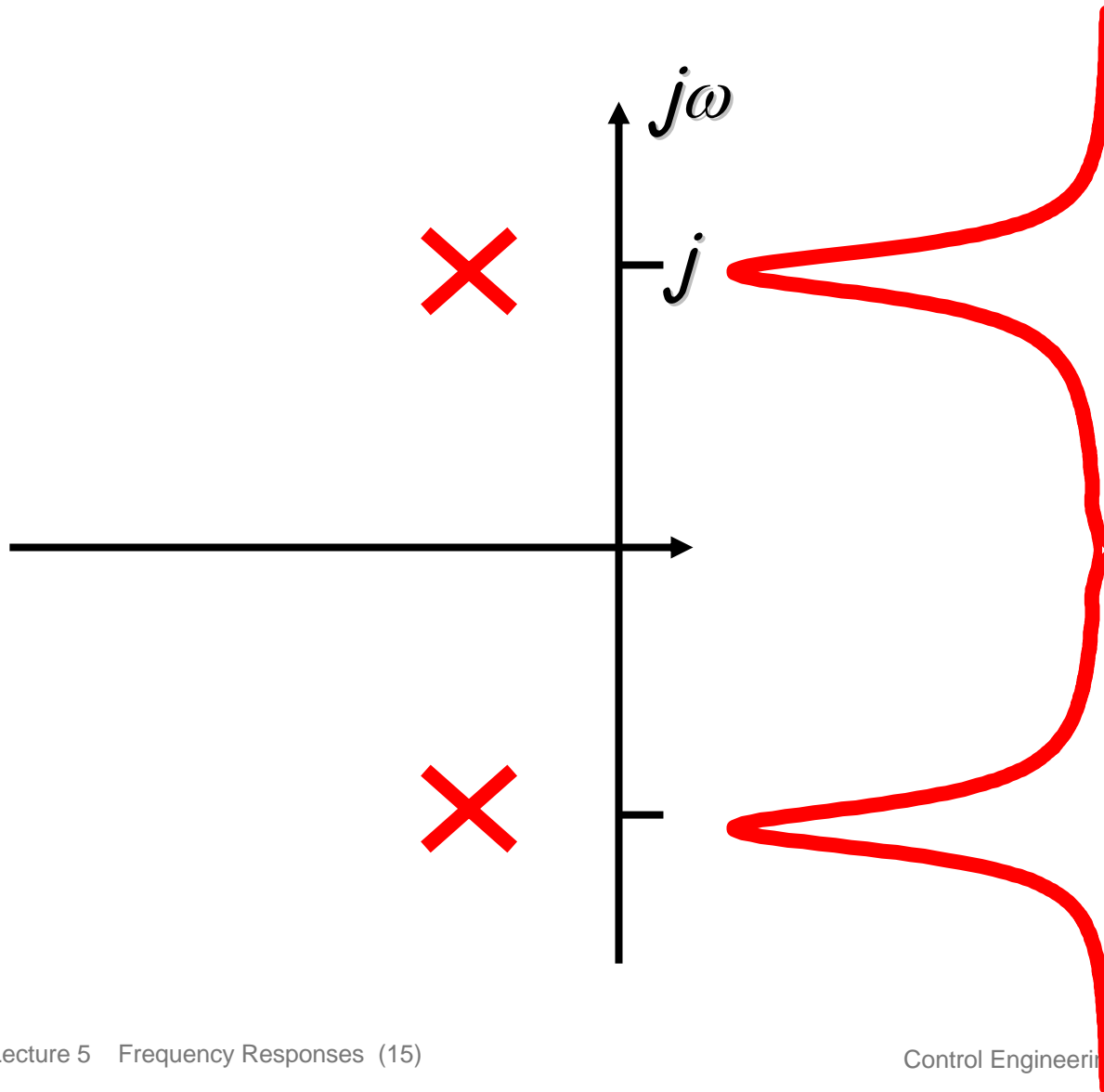
First-order system

- Nyquist
- Bode
- Nichols

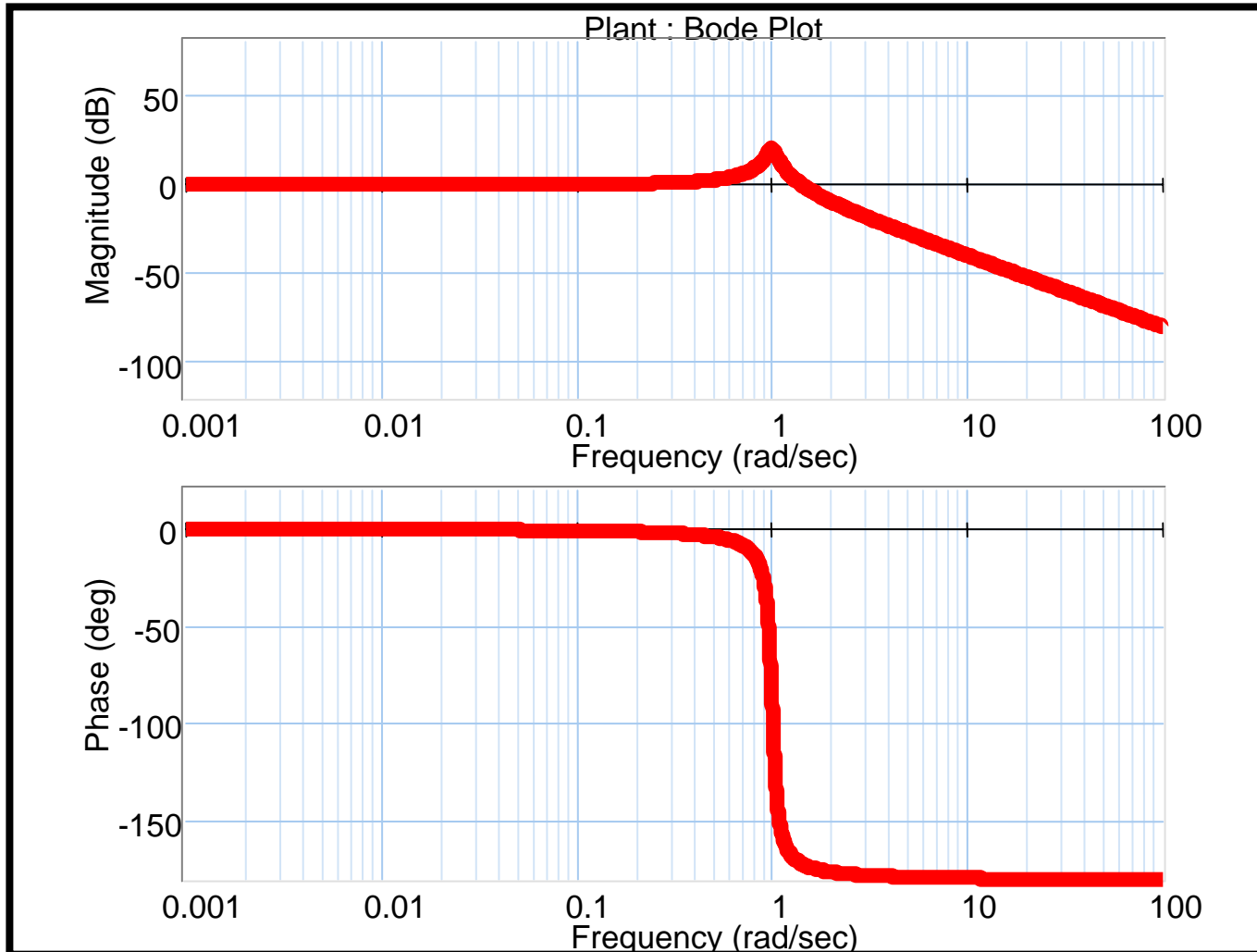
20-sim
demo

- s -plane can be seen as a rubber membrane
- poles are needles under the membrane
- zeros are push pins in the membrane
- push pins in infinity
- $H(j\omega)$ is a cross cut through the membrane at the $j\omega$ axis

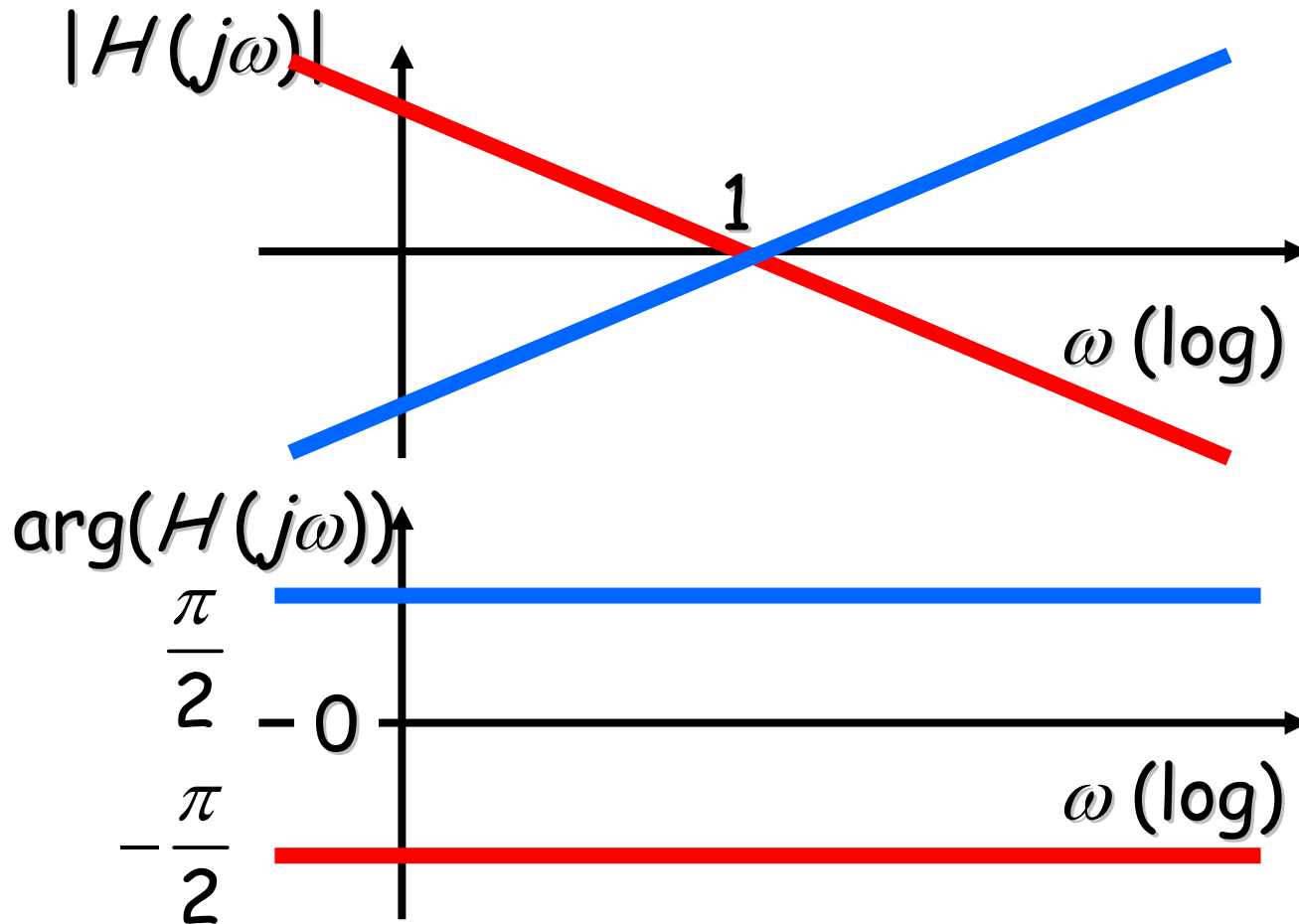
Complex poles



Bode plot (complex poles)



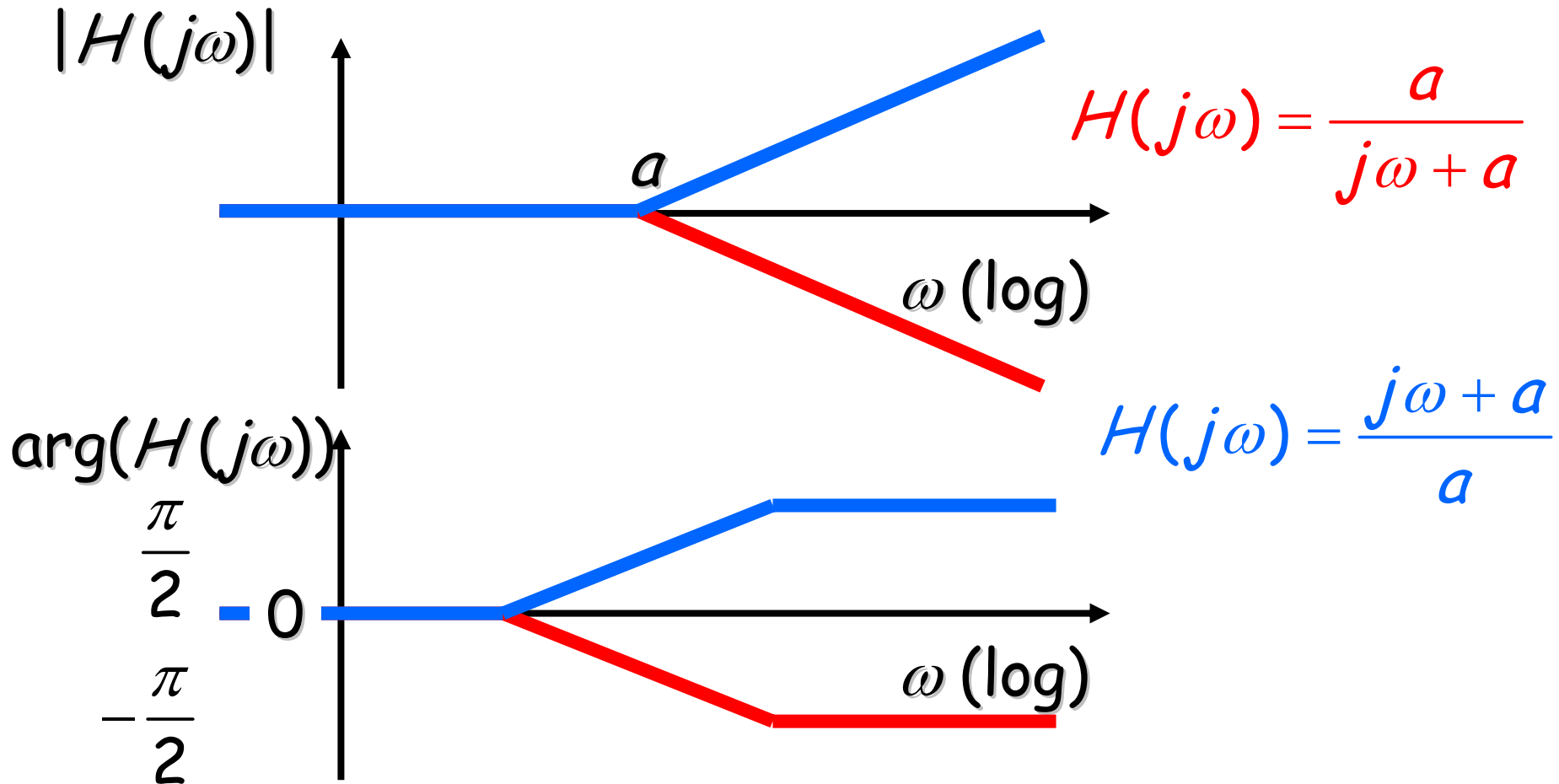
Basic elements ($j\omega$)



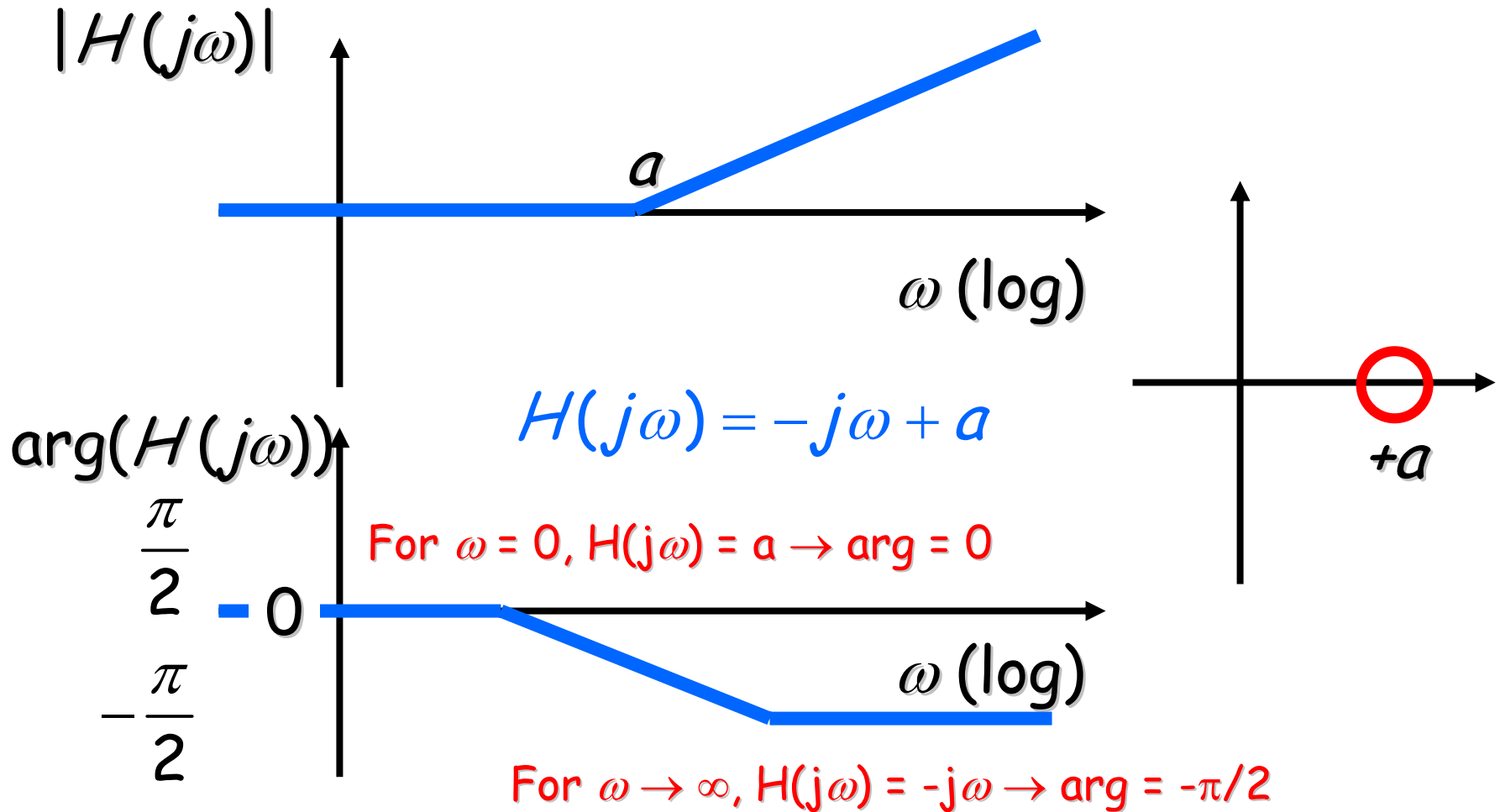
$$H(j\omega) = \frac{1}{j\omega}$$

$$H(j\omega) = j\omega$$

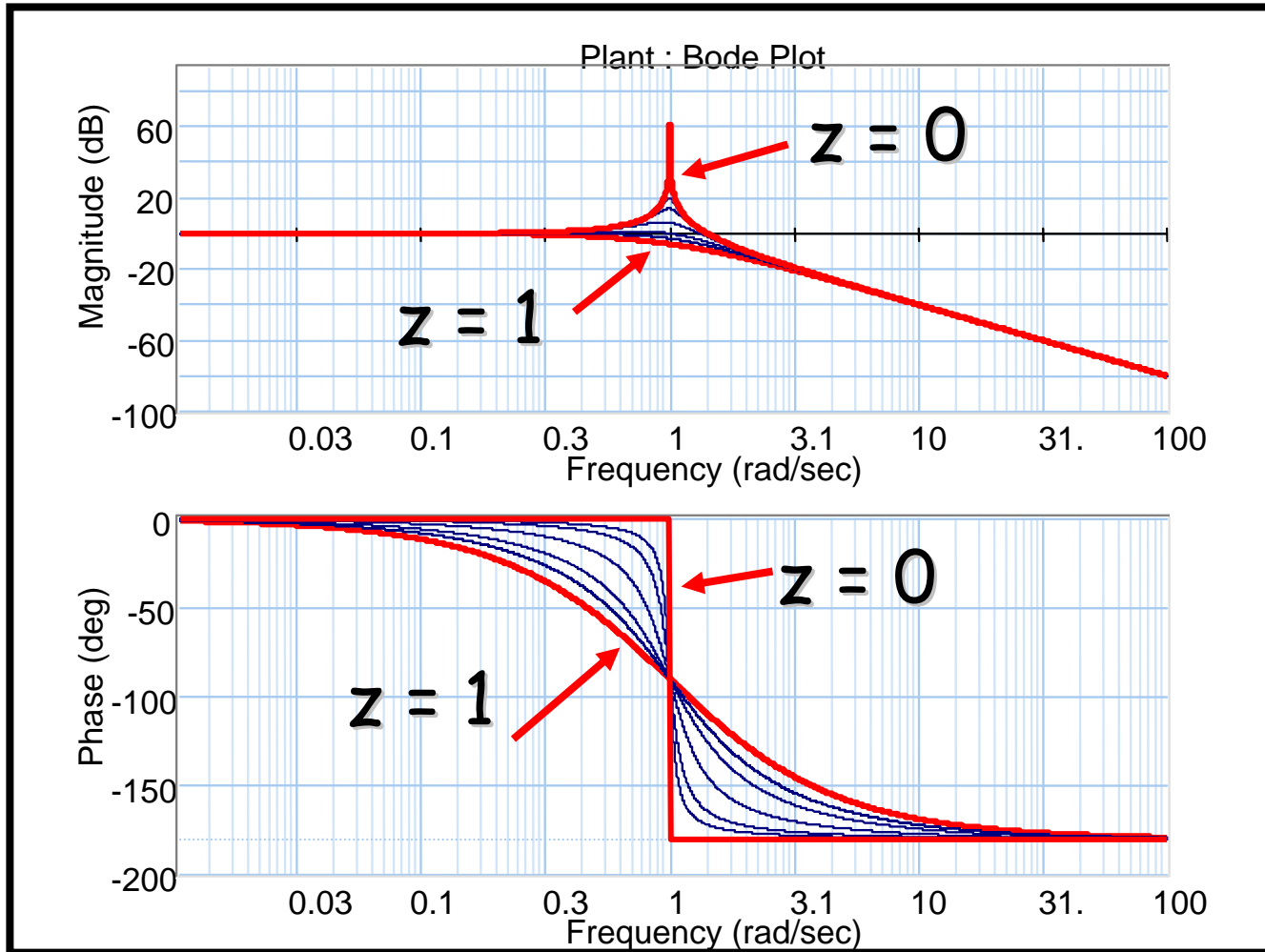
Basic elements ($j\omega+a$)



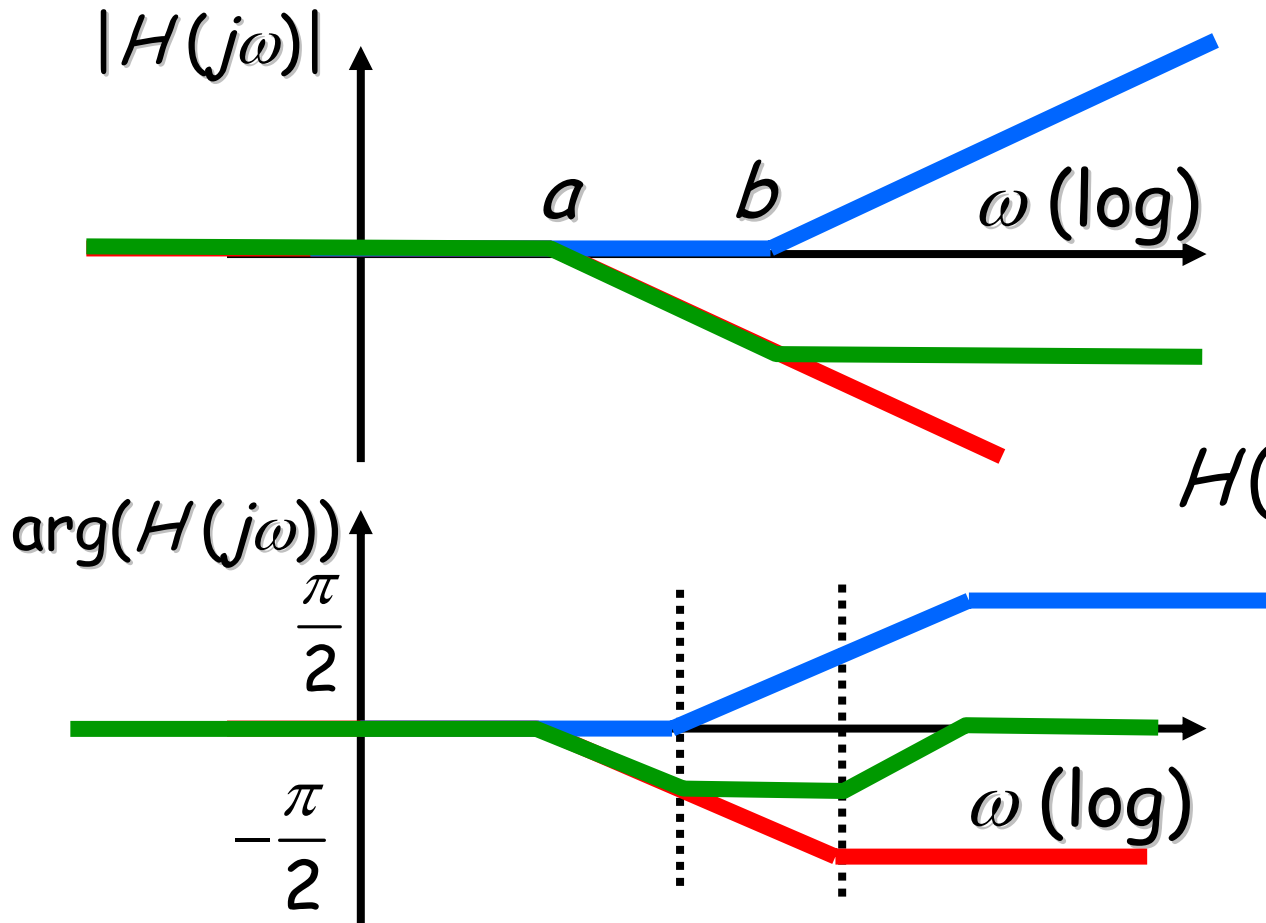
Basic elements $(-j\omega+a)$



Basic elements (2nd order)



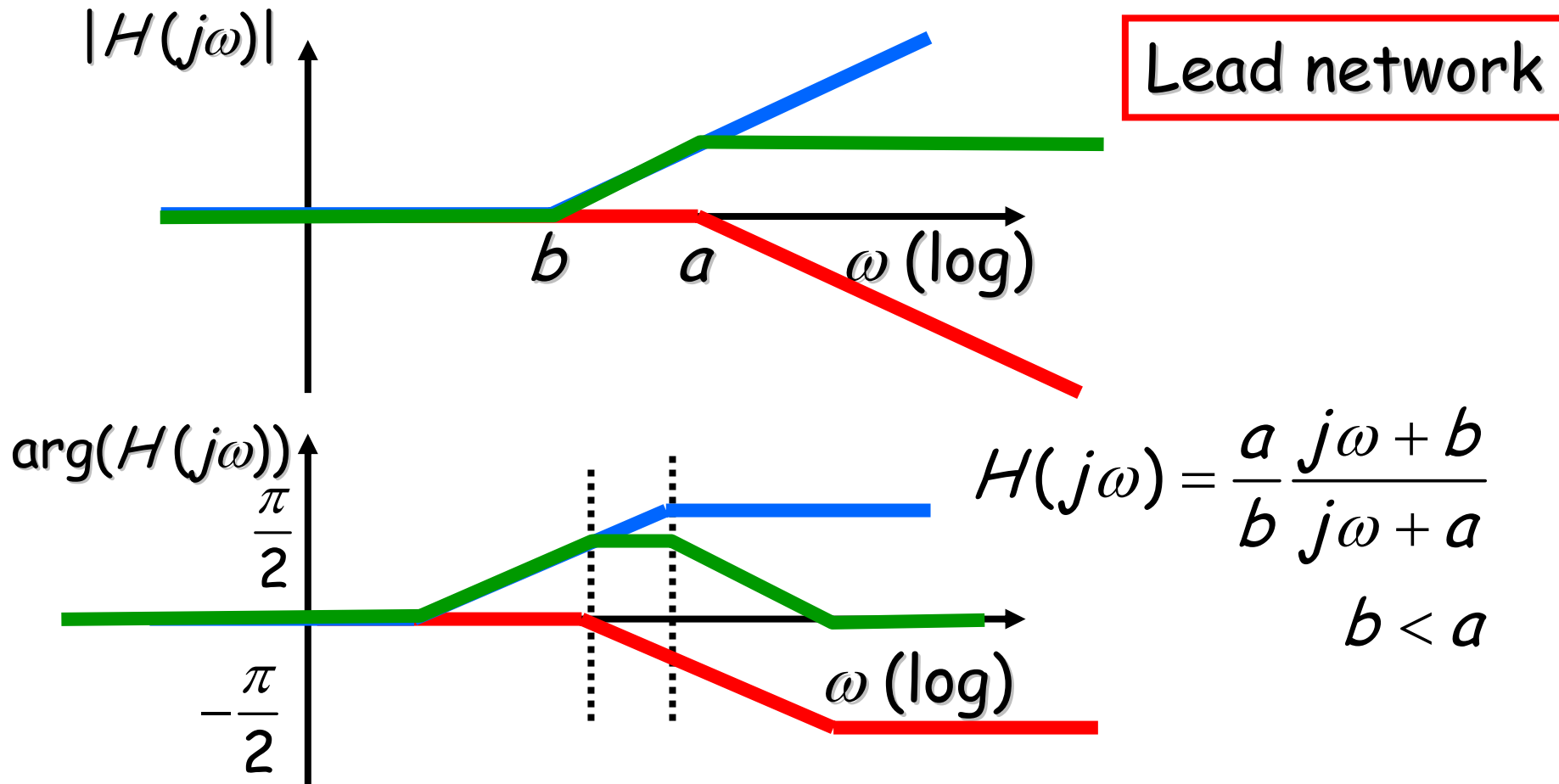
Combinations



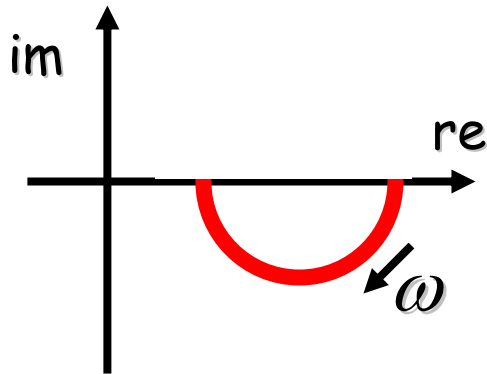
Lag network

$$H(j\omega) = \frac{a}{b} \frac{j\omega + b}{j\omega + a}$$

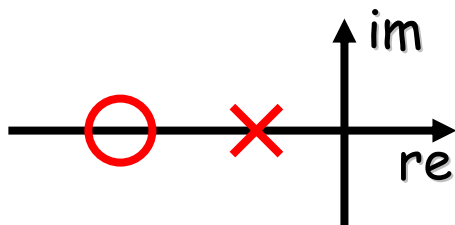
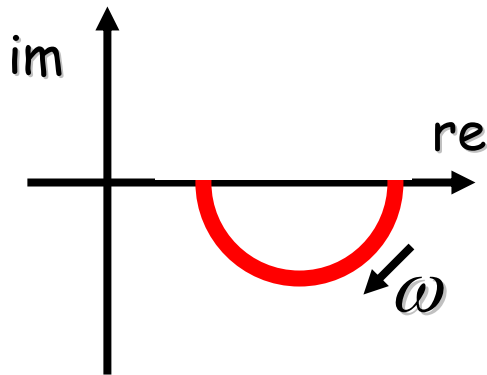
$a < b$



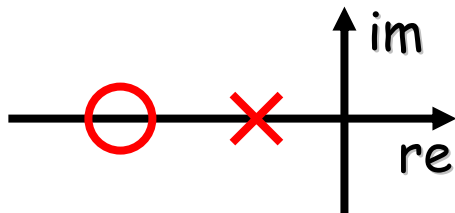
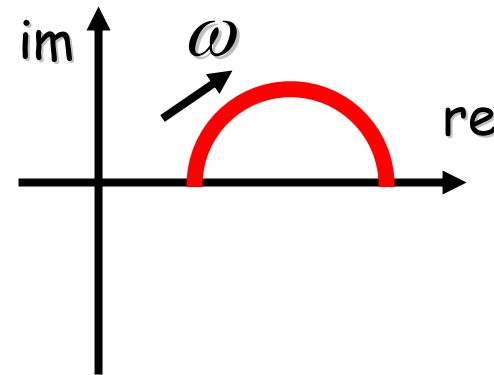
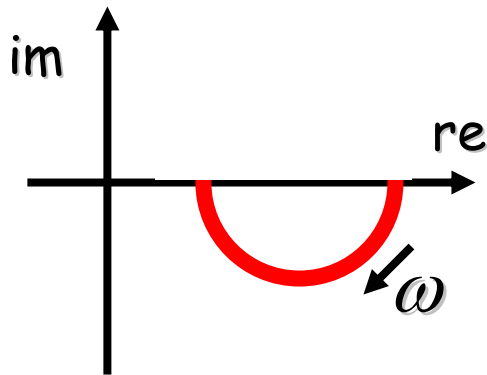
- Relation $s \leftrightarrow j\omega$ for lag and lead networks



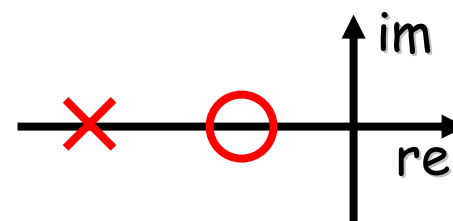
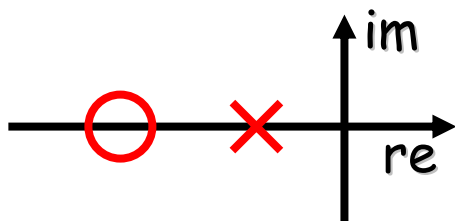
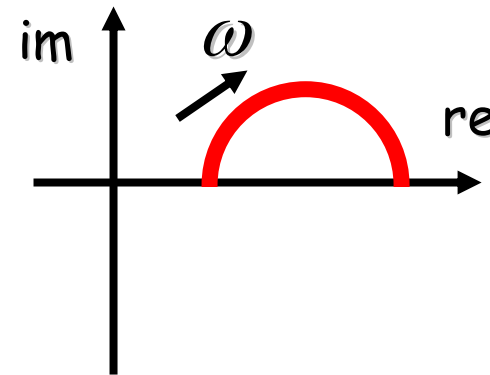
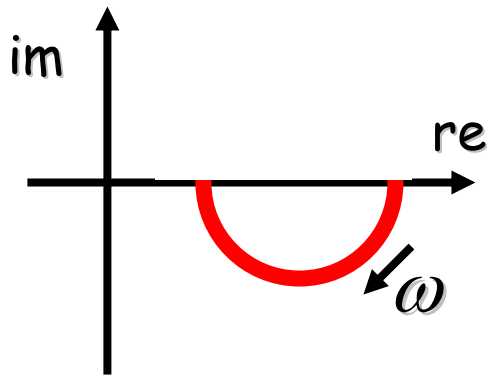
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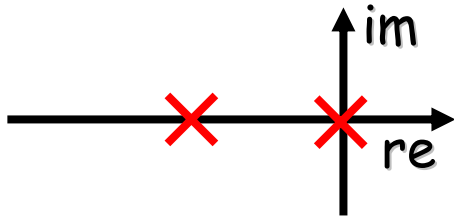
- Relation $s \leftrightarrow j\omega$ for lag and lead networks



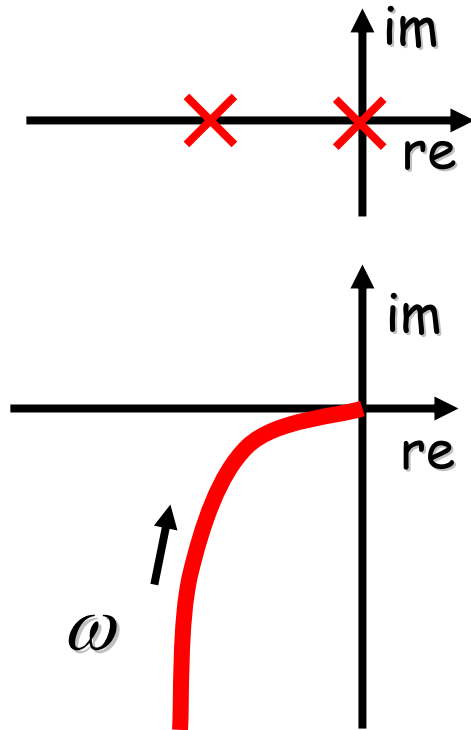
- Relation $s \leftrightarrow j\omega$ for lag and lead networks



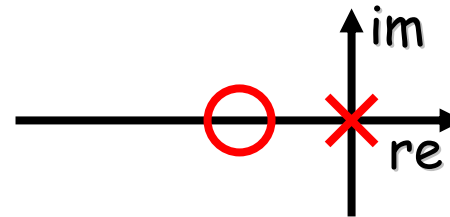
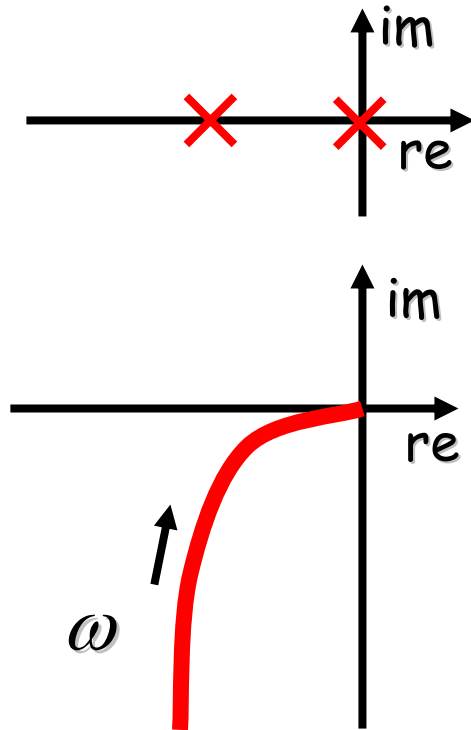
Relation $s \leftrightarrow j\omega$



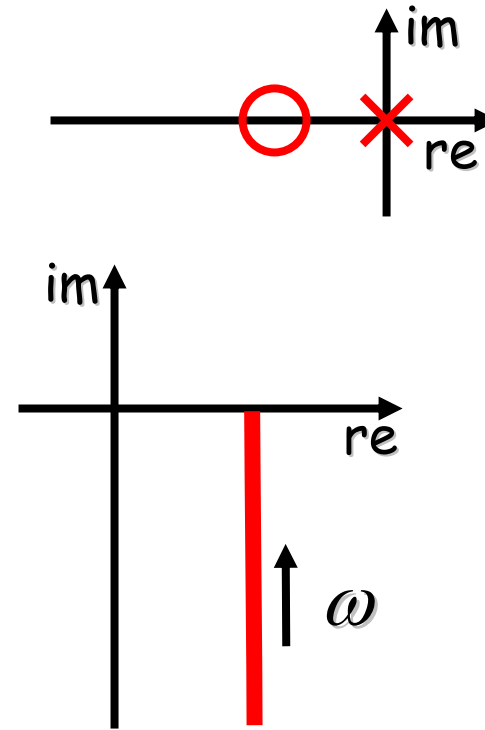
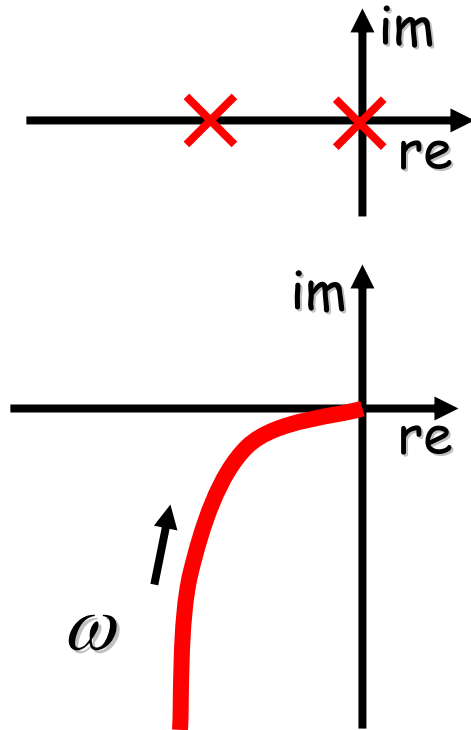
Relation $s \leftrightarrow j\omega$



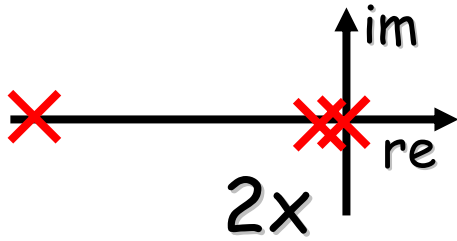
Relation $s \leftrightarrow j\omega$



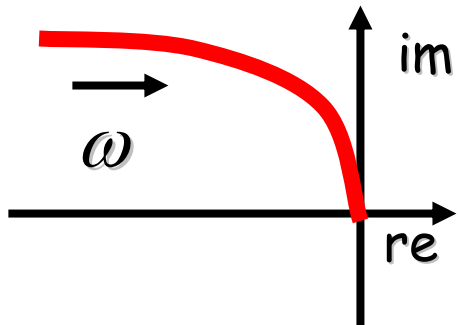
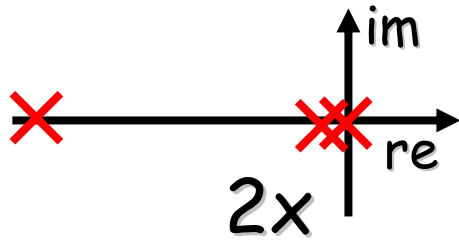
Relation $s \leftrightarrow j\omega$



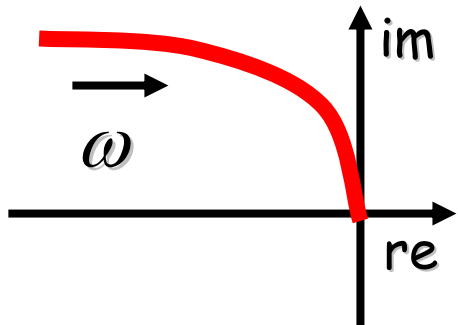
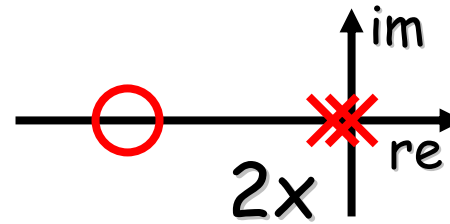
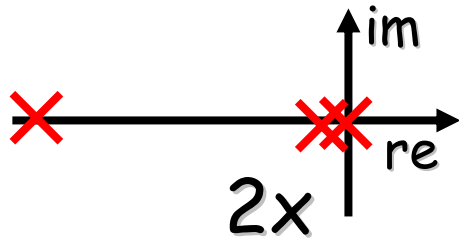
Relation $s \leftrightarrow j\omega$



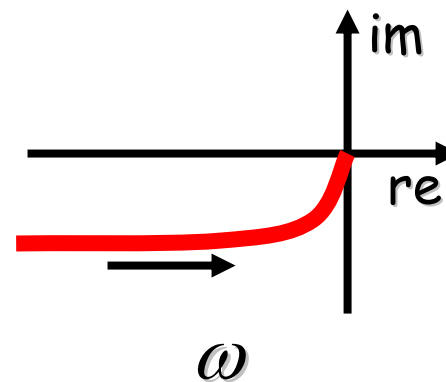
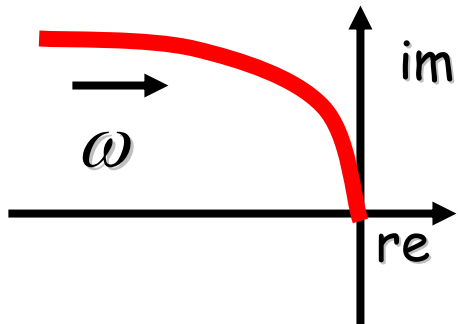
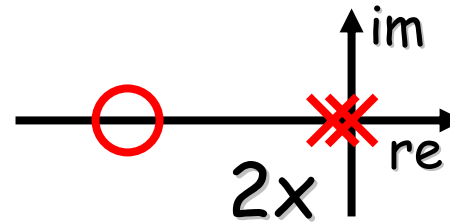
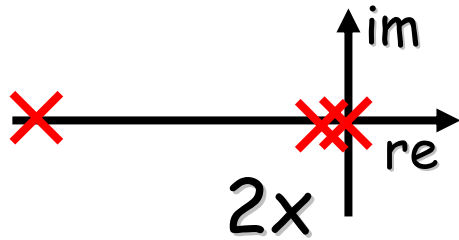
Relation $s \leftrightarrow j\omega$



Relation $s \leftrightarrow j\omega$



Relation $s \leftrightarrow j\omega$



- lag network

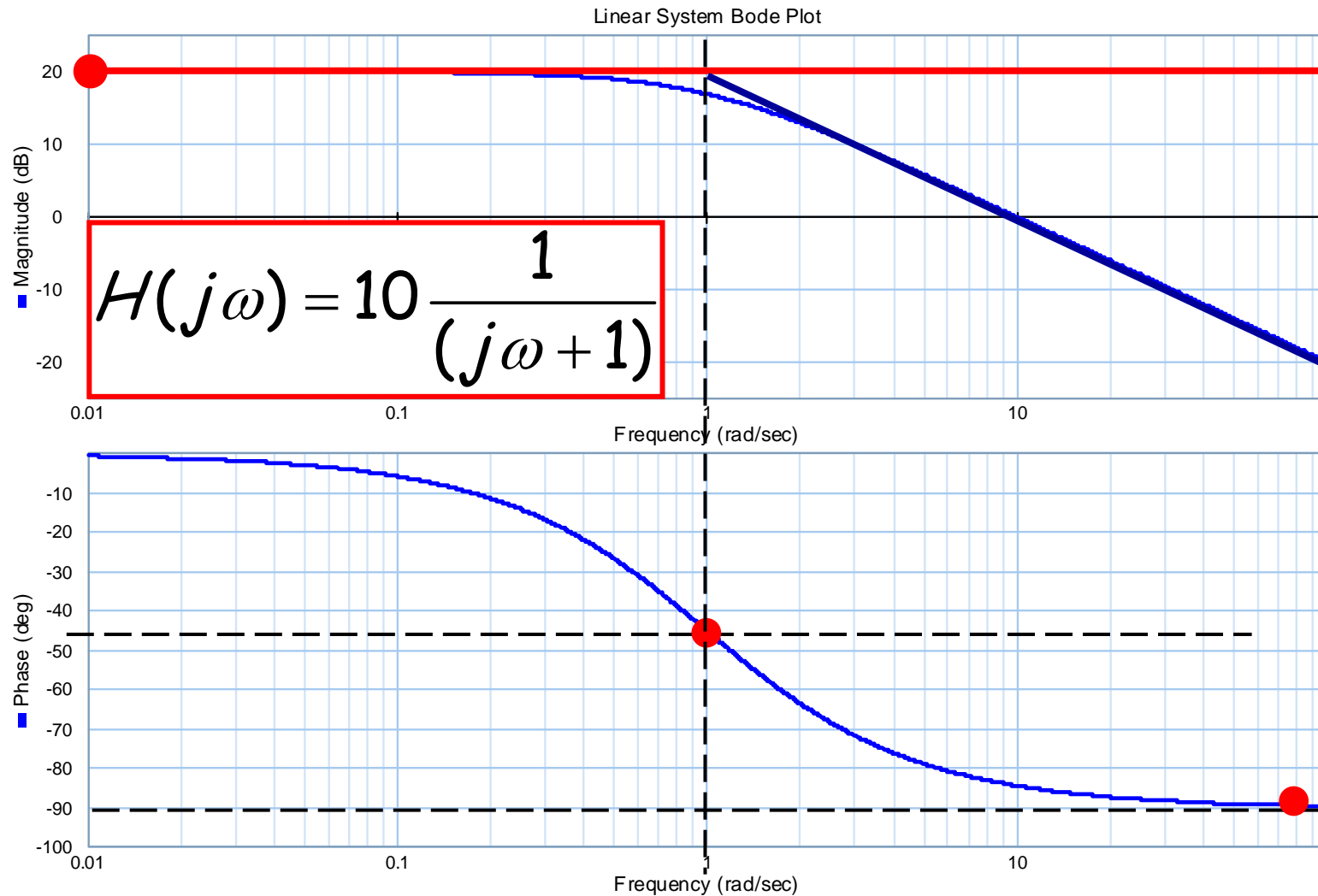
- bode
- nyquist
- nichols

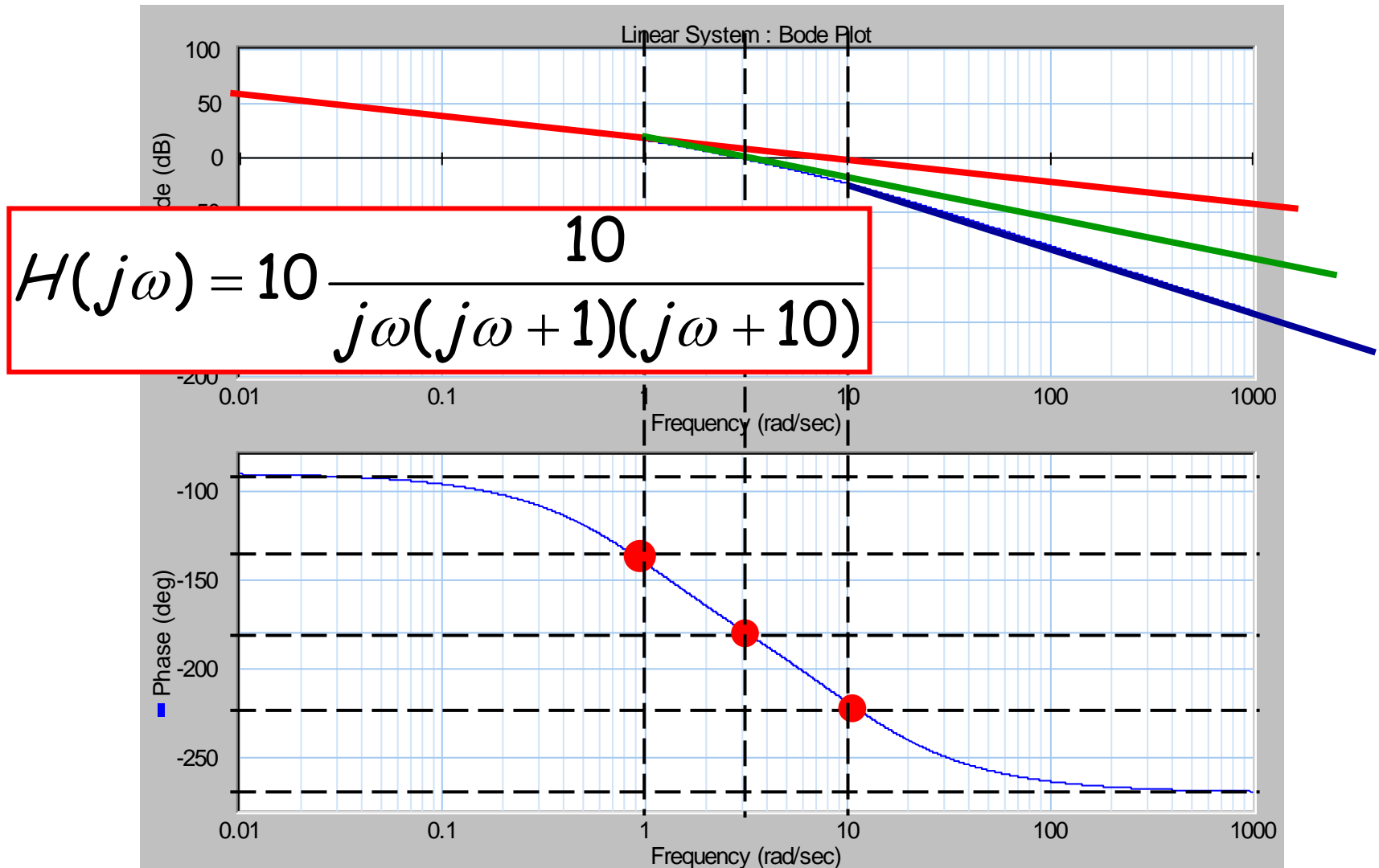
20-sim
demo

- lead network

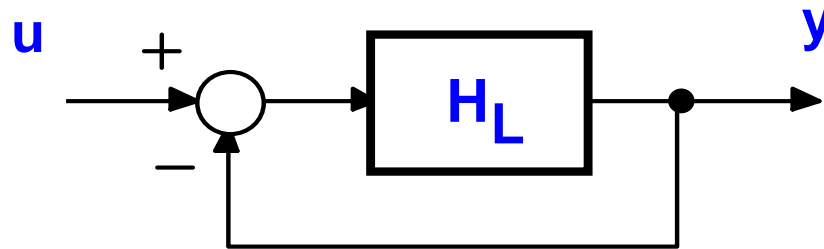
- bode
- nyquist
- nichols

20-sim
demo





We consider the following feedback system



The system is on the border of instability when:

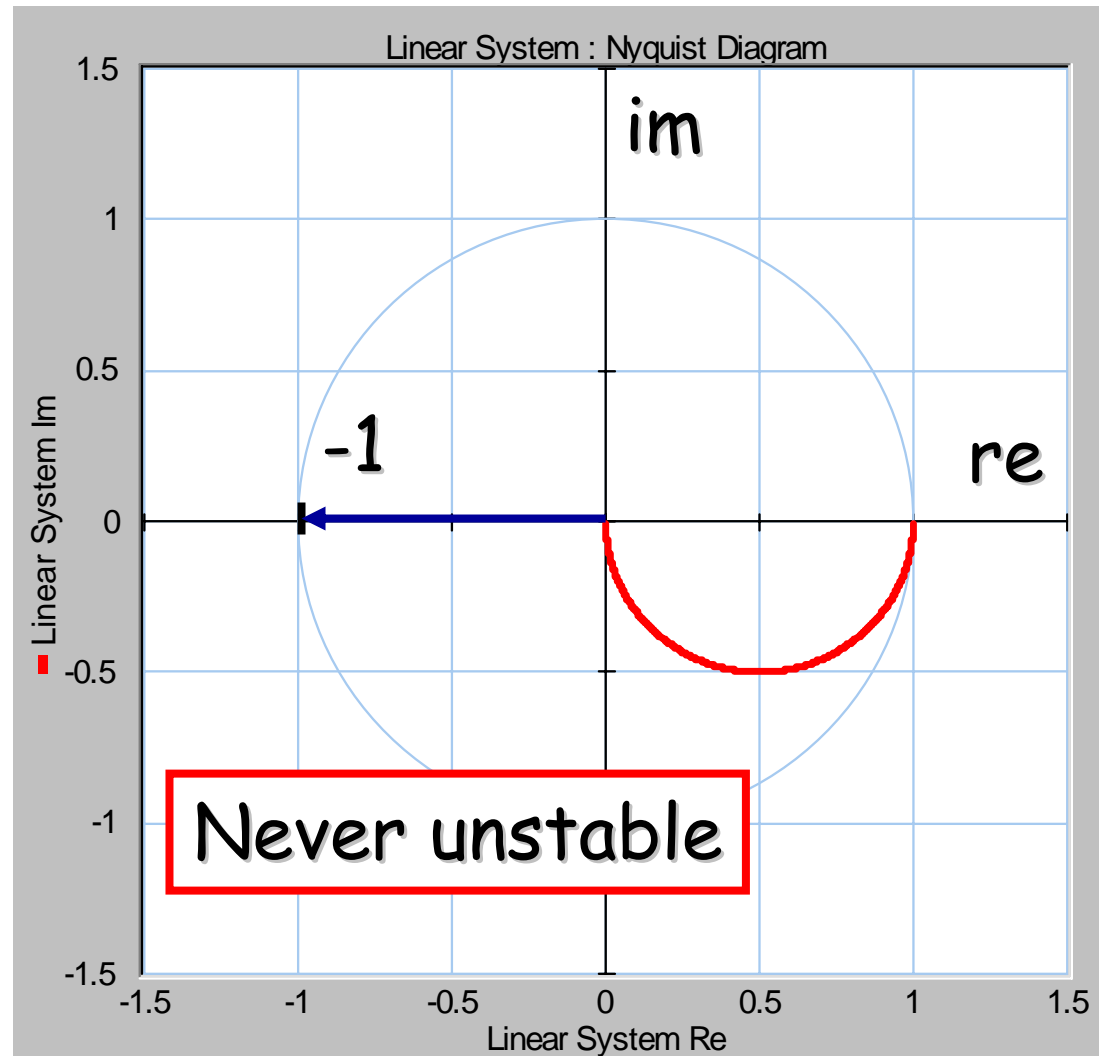
$$|H_L(j\omega)| = 1 \text{ and } \arg(H_L(j\omega)) = -\pi$$

$$\text{or } H_L(j\omega) = 1e^{-j\pi}$$

$$H(j\omega) = \frac{1}{(j\omega + 1)}$$

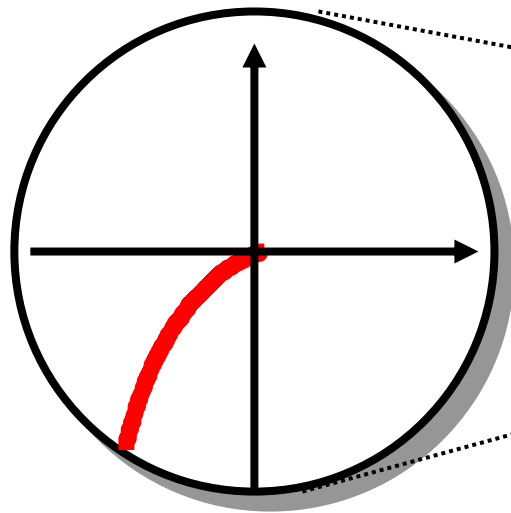
$$\text{vector} = 1e^{-j\pi}$$

if $H(j\omega) = 1e^{-j\pi}$
closed loop system
on border of stability

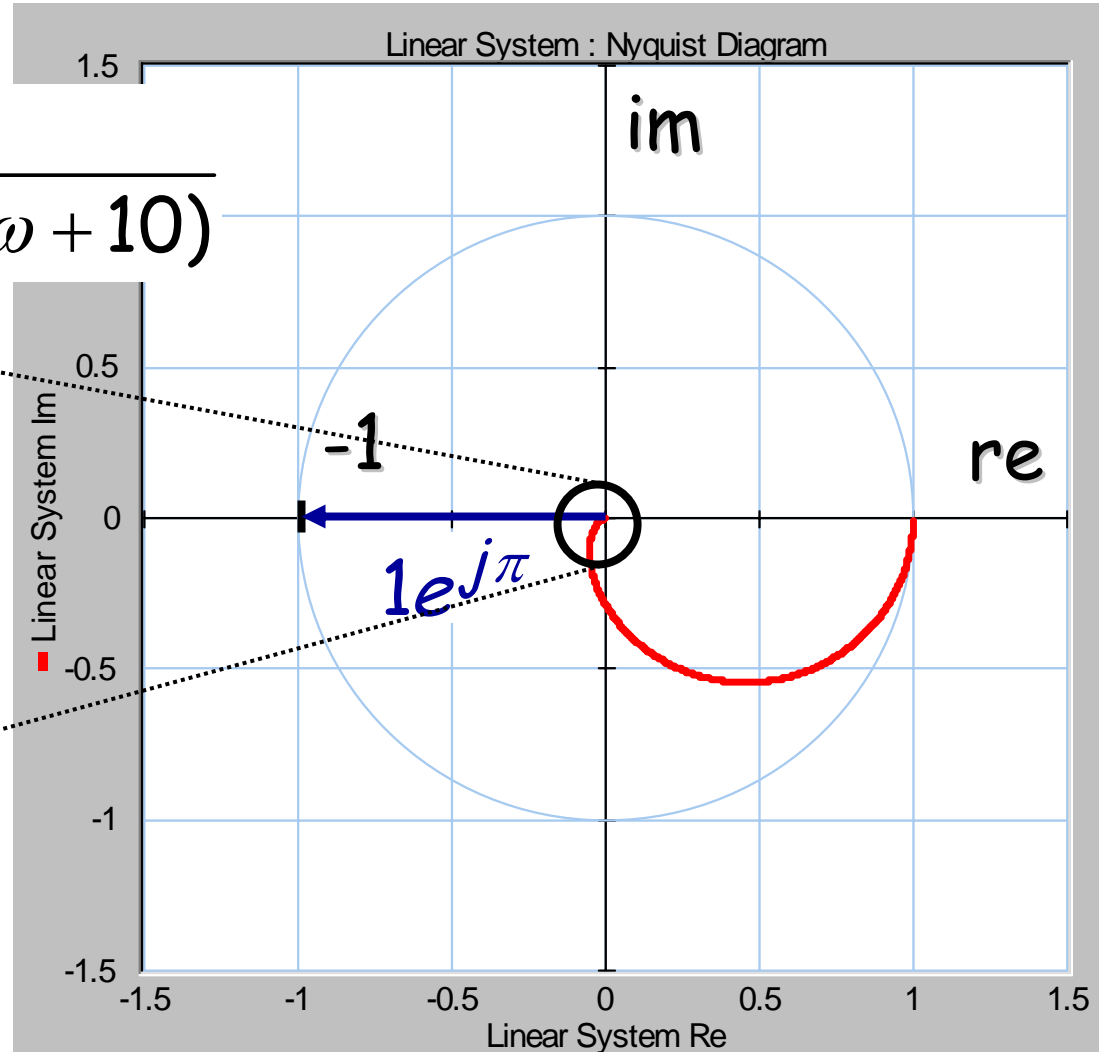


Stability

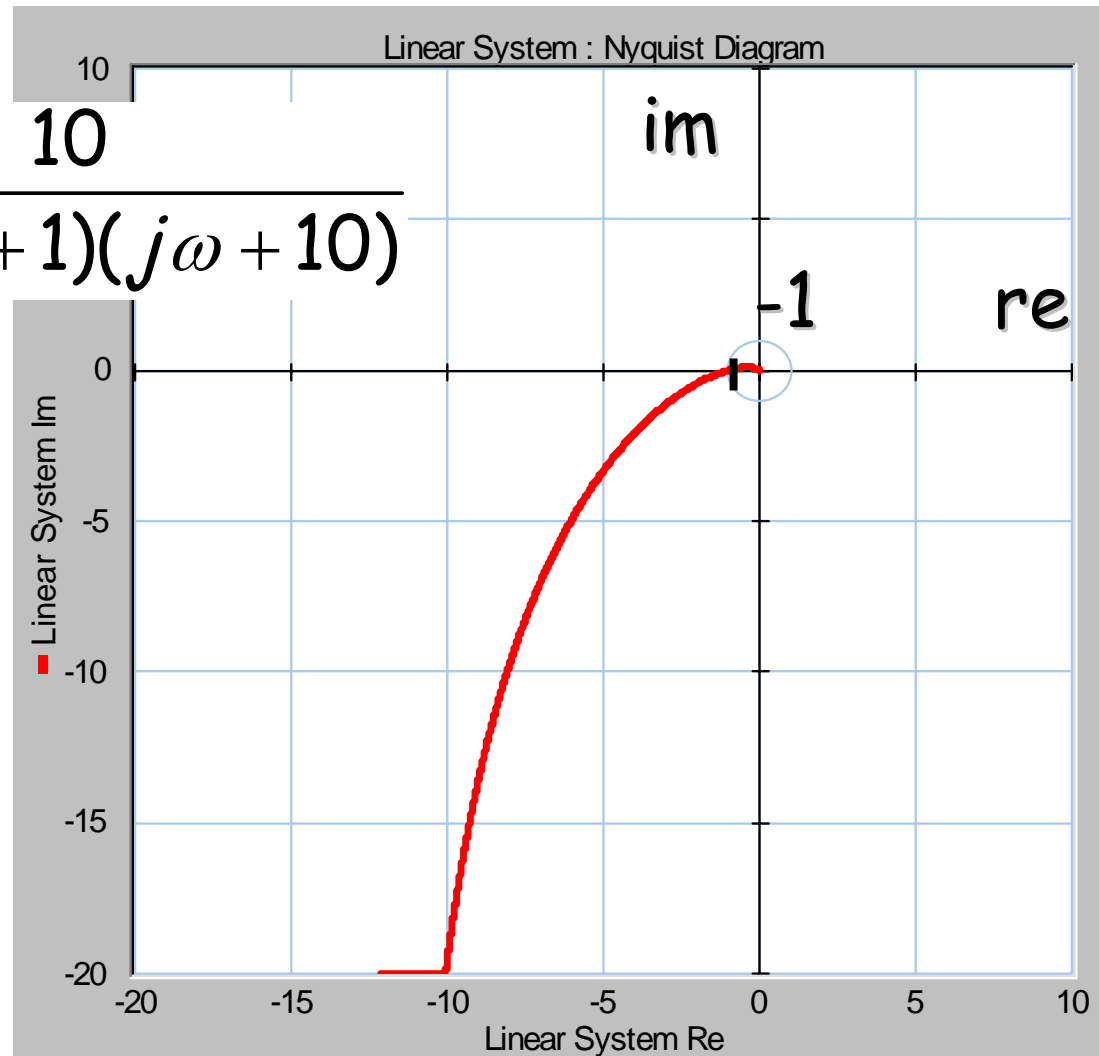
$$H(j\omega) = \frac{10}{(j\omega + 1)(j\omega + 10)}$$



Never unstable



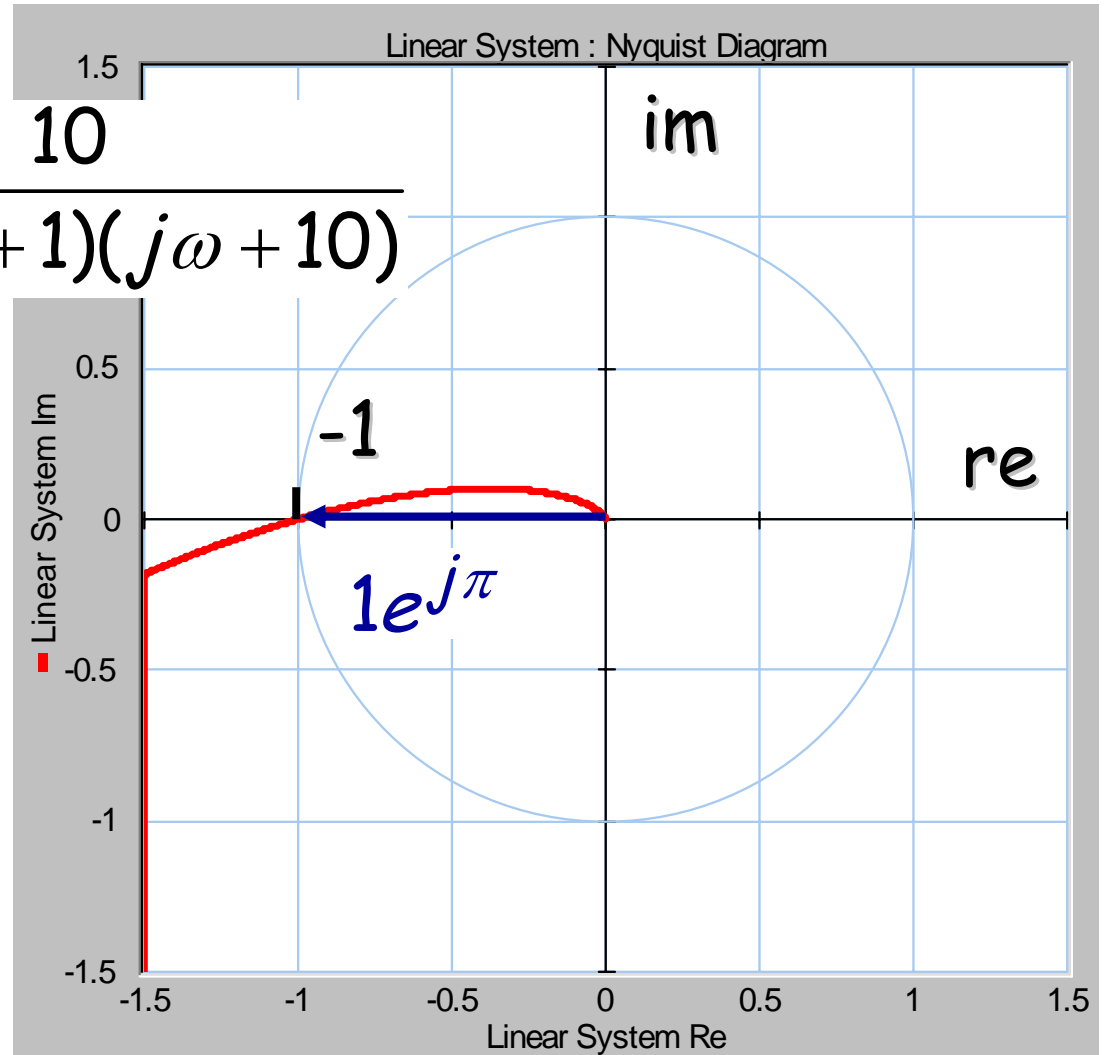
$$H(j\omega) = 11 \frac{10}{j\omega(j\omega + 1)(j\omega + 10)}$$



$$H(j\omega) = 11 \frac{10}{j\omega(j\omega + 1)(j\omega + 10)}$$

Border of
instability

Compare
Root loci (38)



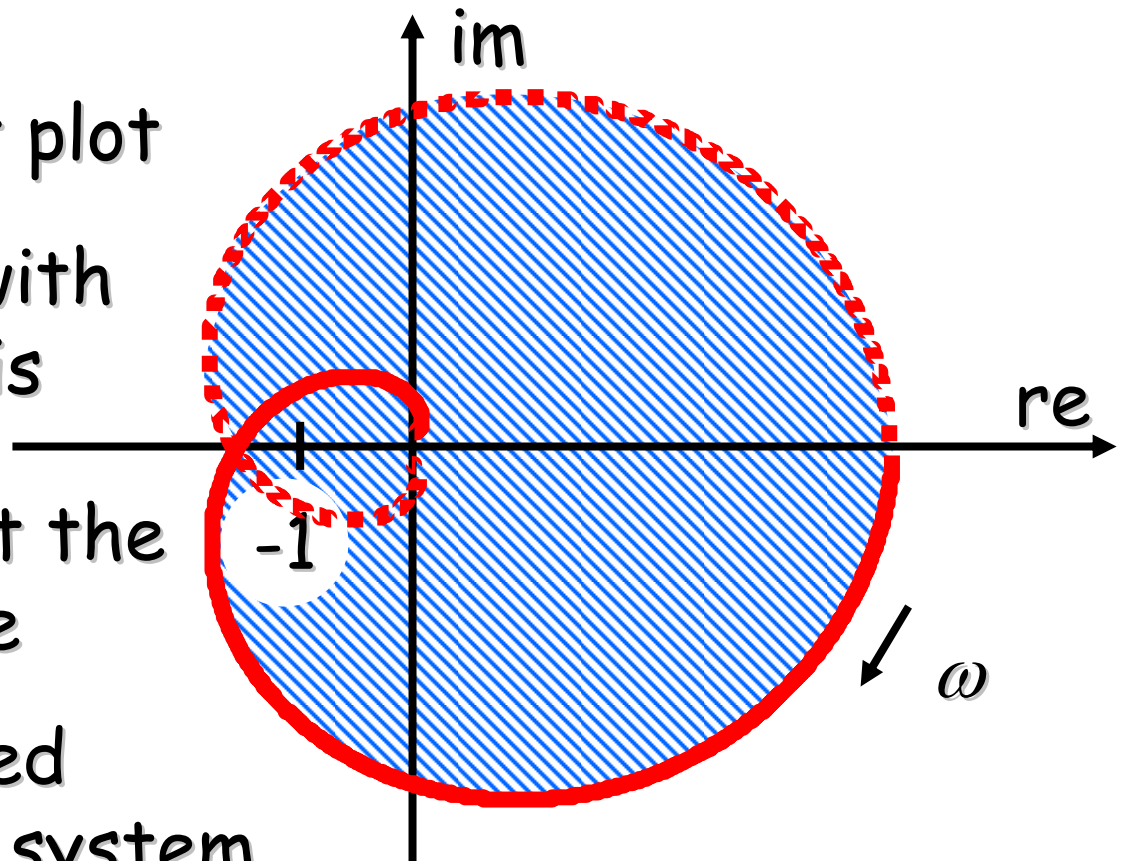
-1 Stability criterion

Draw the Nyquist plot

Mirror the plot with respect to re-axis

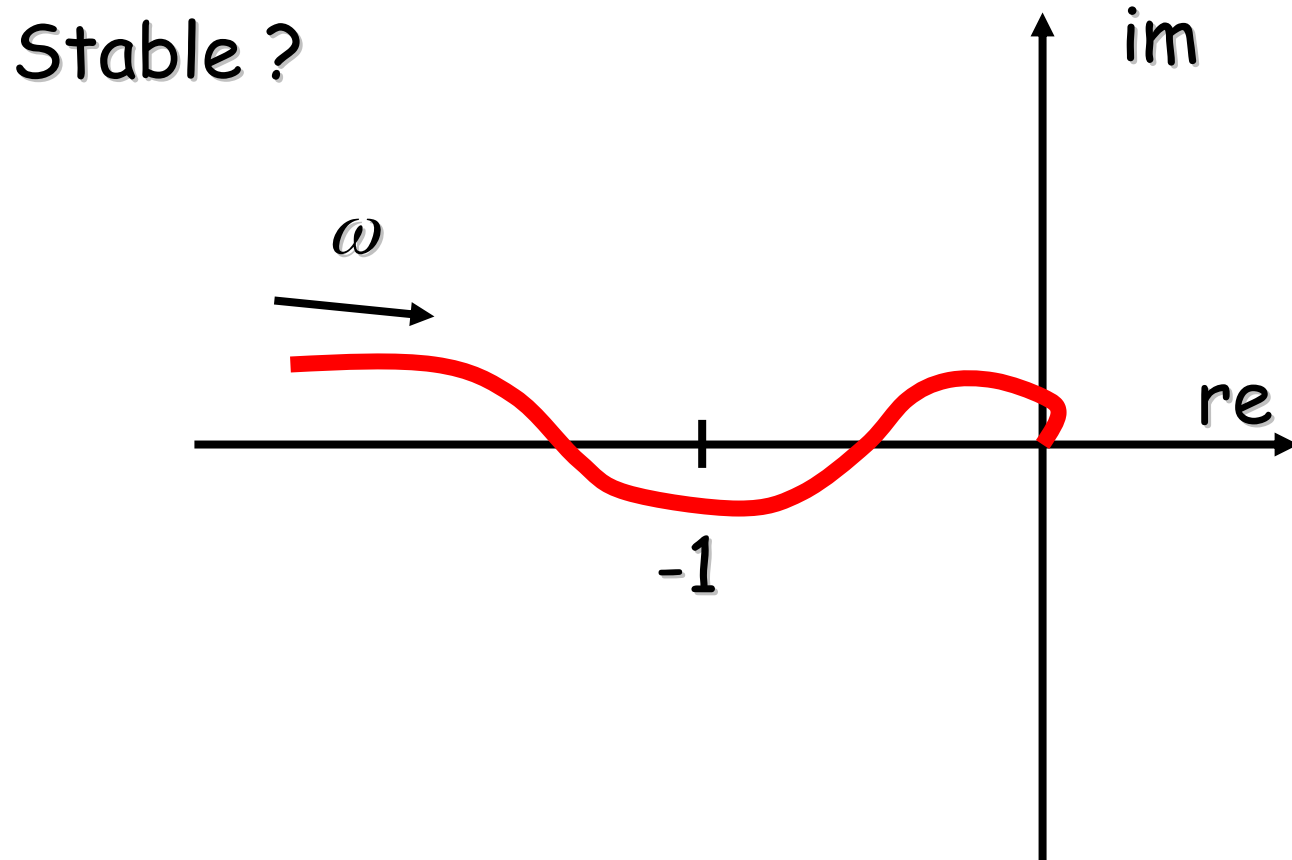
Shade the area at the right of the curve

If -1 in the shaded area:
closed loop system
UNSTABLE

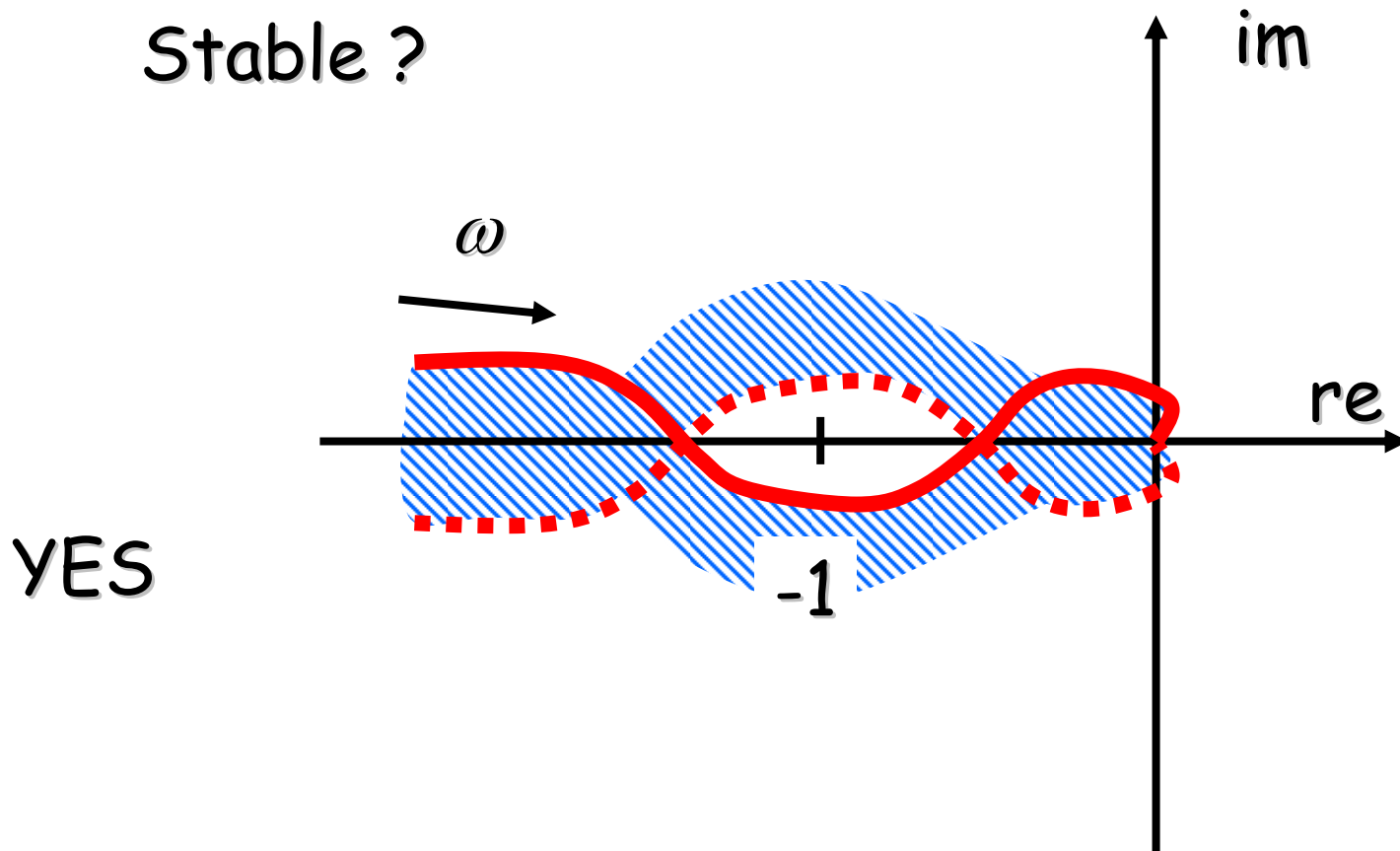


Linear System Re

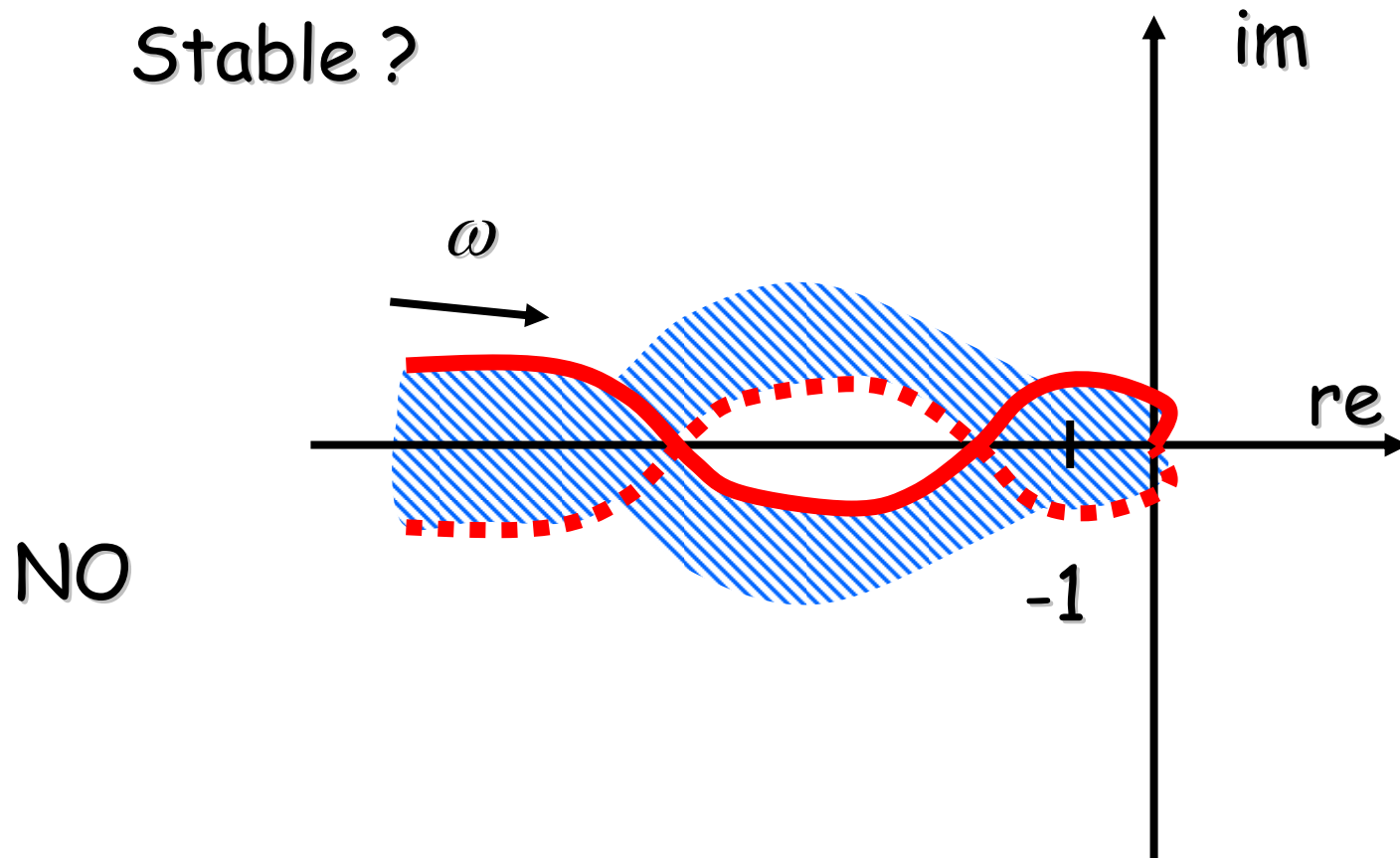
Conditionally Stable



Conditionally Stable

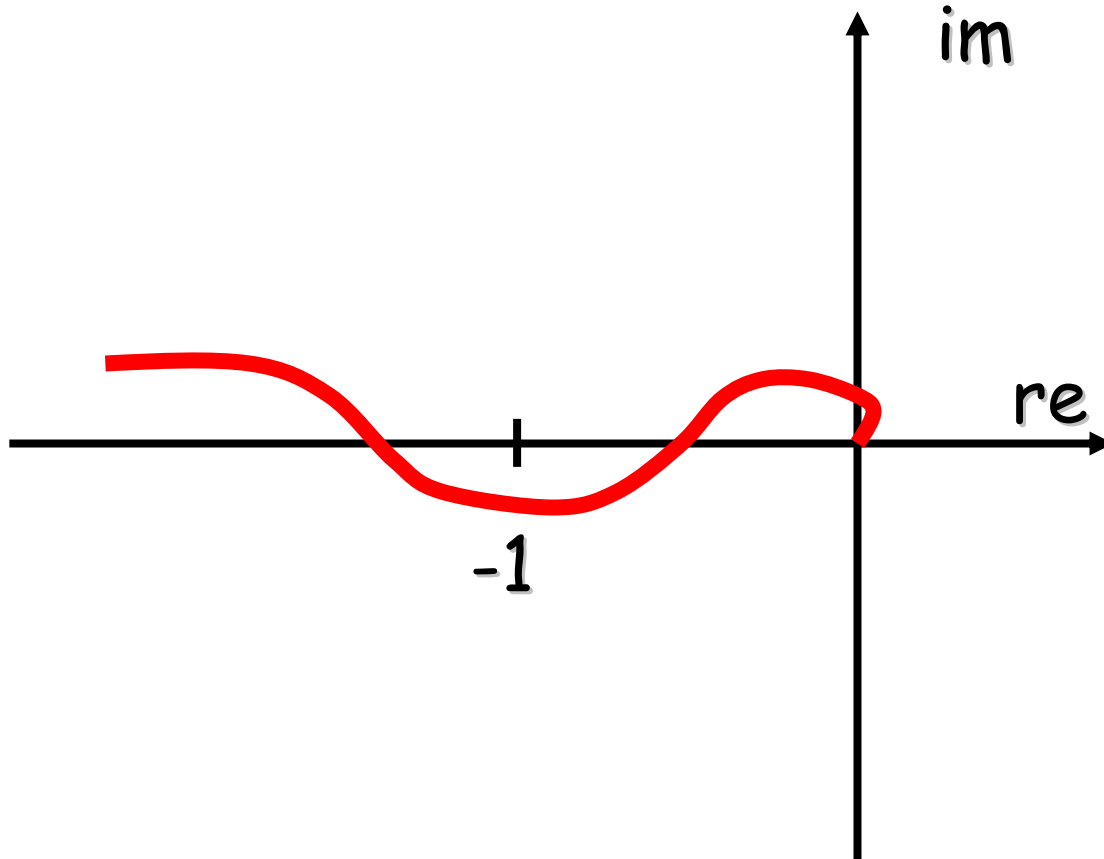


Conditionally Stable



- The stability of the **CLOSED** system depends on the fact whether the Nyquist plot of the **OPEN** system encircles -1
- If the Nyquist plot of the **CLOSED** system encircles -1 , this tells nothing about the stability of the system !

Exercise



Find the
poles and zero's

Draw the
root locus

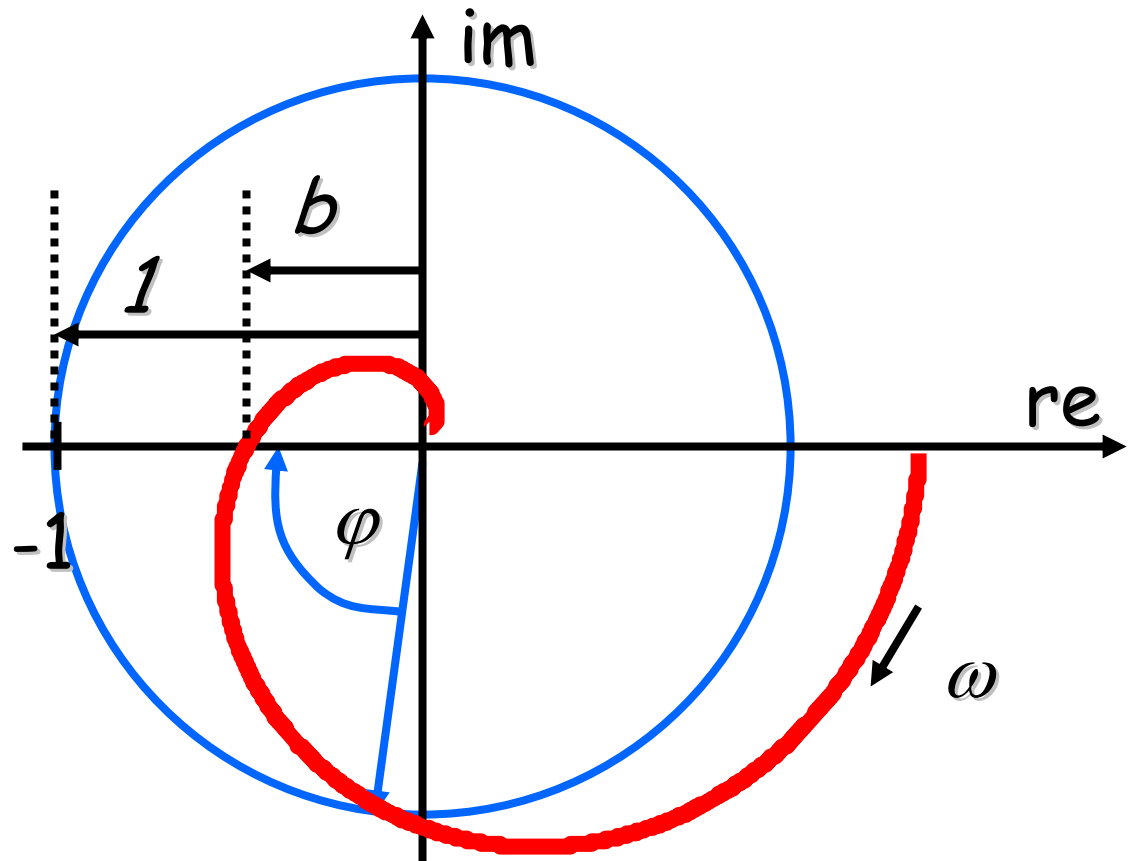
Gain & Phase margins

Gain margin:

$$1/b$$

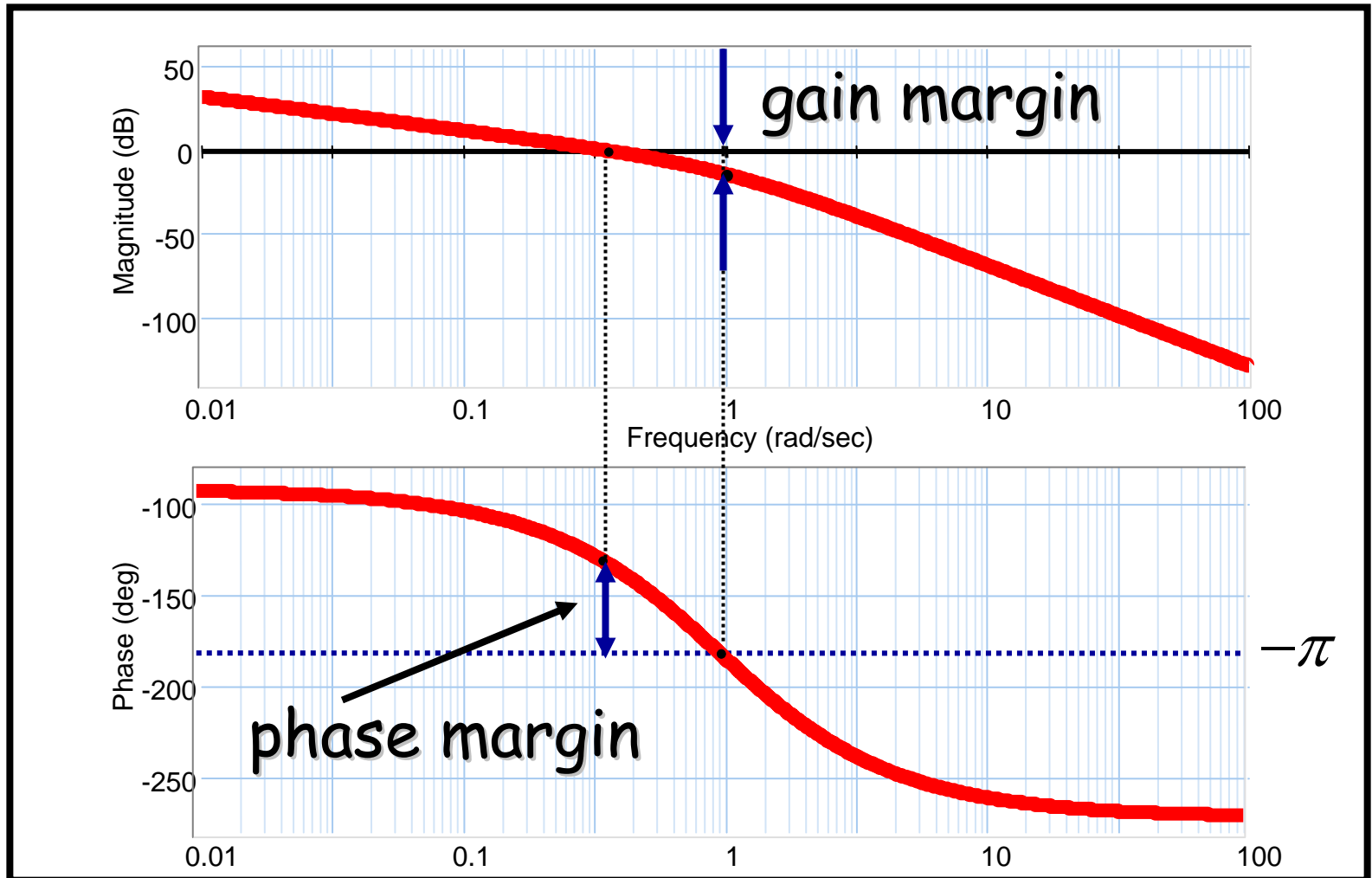
Phase margin:

$$\varphi$$



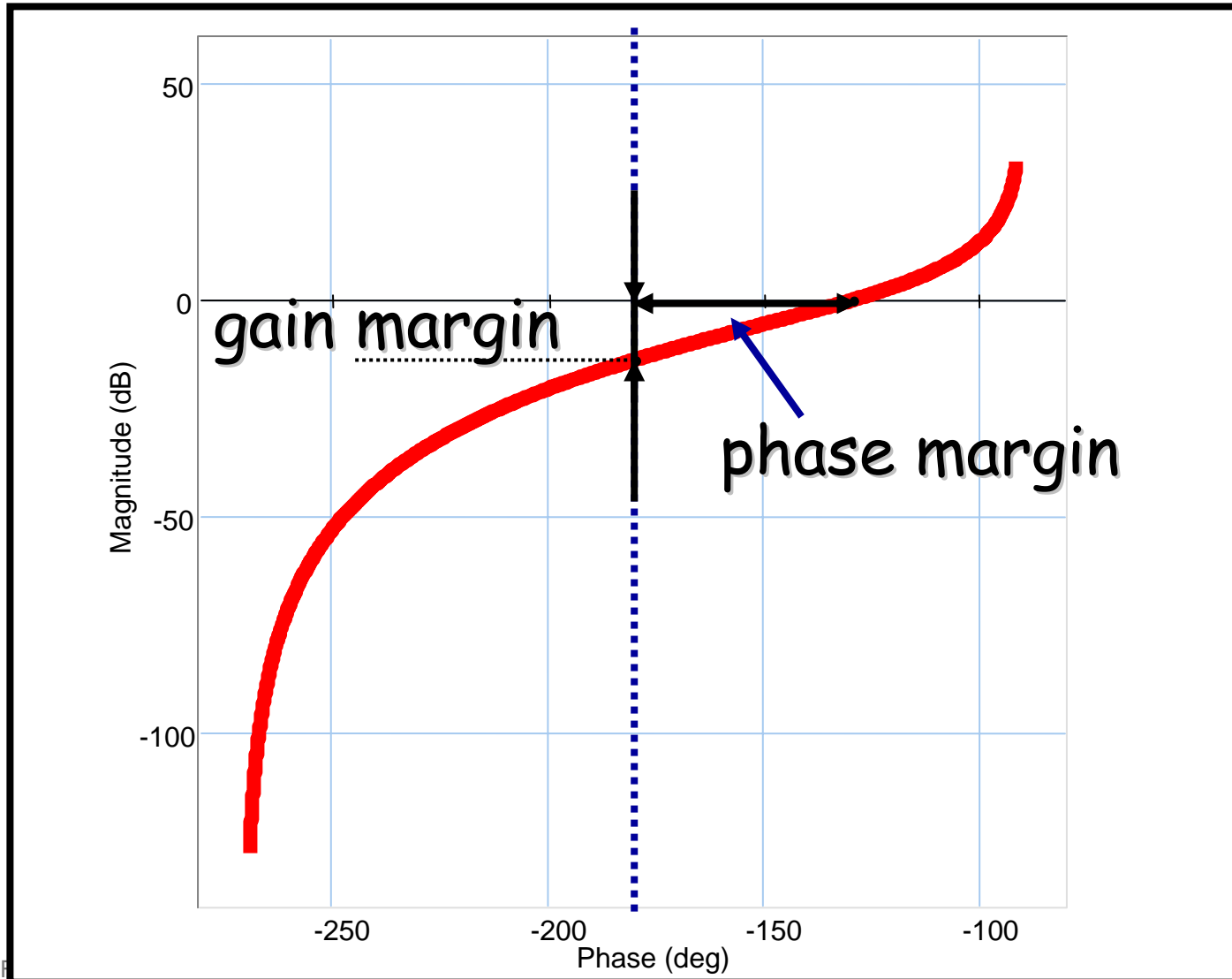
Linear System Re

Gain & Phase margins (Bode)



- Gain margin determines how much the gains may vary, before the system becomes unstable
- Phase margin influences transient behaviour (damping ratio, overshoot)
- Second order system:
- $z \approx \text{phase margin (in degrees)} / 100$

Gain & Phase margins (Nichols)



- Investigate the influence of gain and phase margins on the step response of the close loop system for various second- and third-order systems

Don't mix up the

- s -plane with its real (α) and imaginary ($j\omega$) axes and
- The complex plane used to draw the Nyquist (polar) plot of $H(j\omega)$

