Modelling, Poles and Zero's, Stepresponses

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Contents

- Modelling
  - Physical modelling
  - Black box
  - Grey box
- Descriptions
  - IPM's
  - Bond graph
  - Block diagram
  - Frequency characteristic
  - State space
  - Time response

Contents (2)

- Relations poles and zero's and step responses.
  - first order systems
  - second order systems
  - influence of zero's
  - initial and final value theorems.
- Relation between (step) responses and poles and zero's (identification)

Physical modelling

- if a clear idea of the process and the various physical parts is available
- during the design of a (control) system

    - based on physical laws and geometrical properties (dynamische systemen)

Black box modelling

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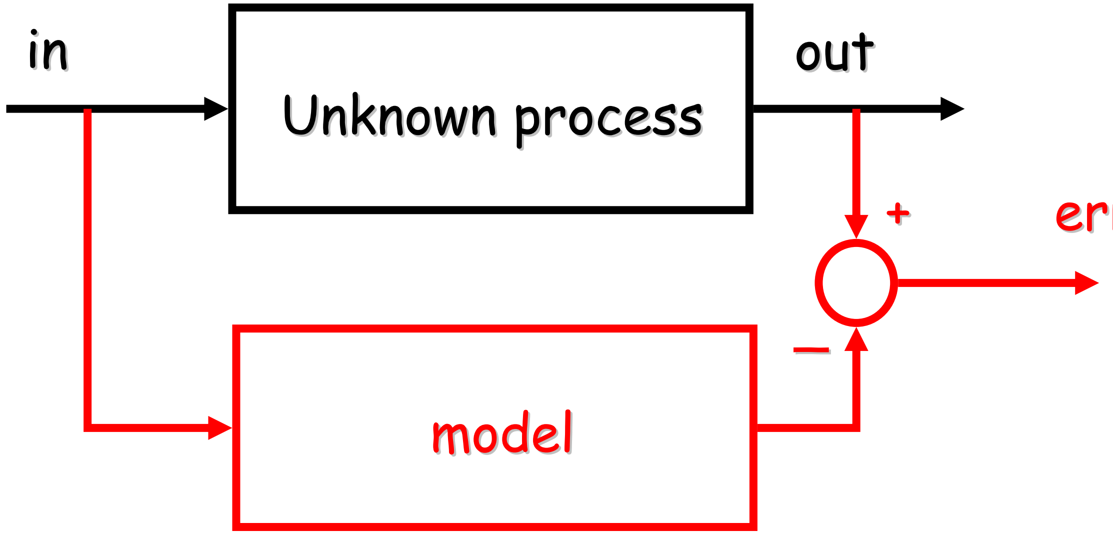
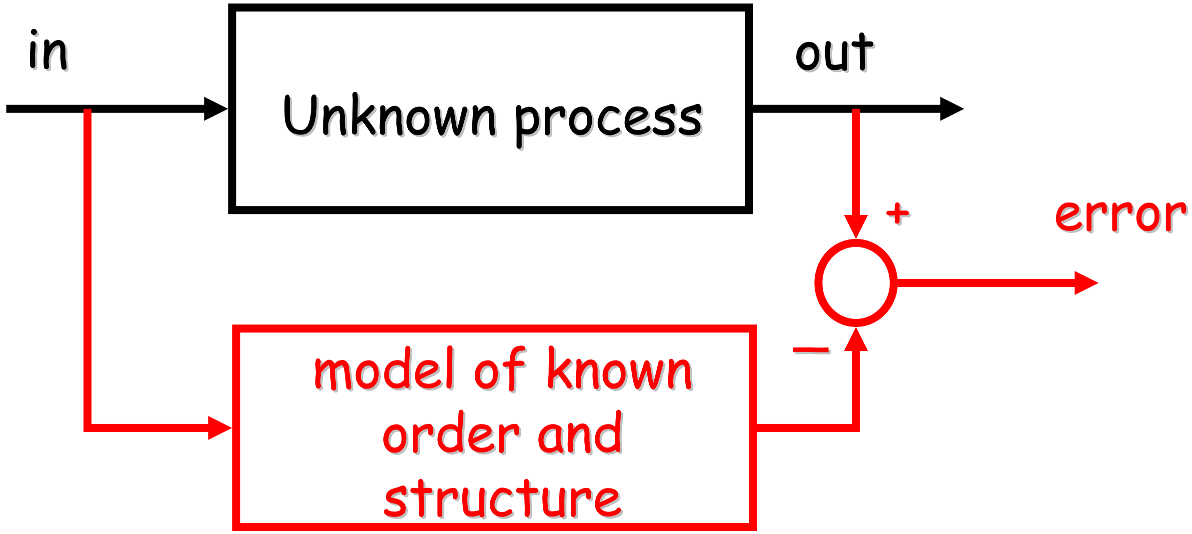
- When only measurements are available



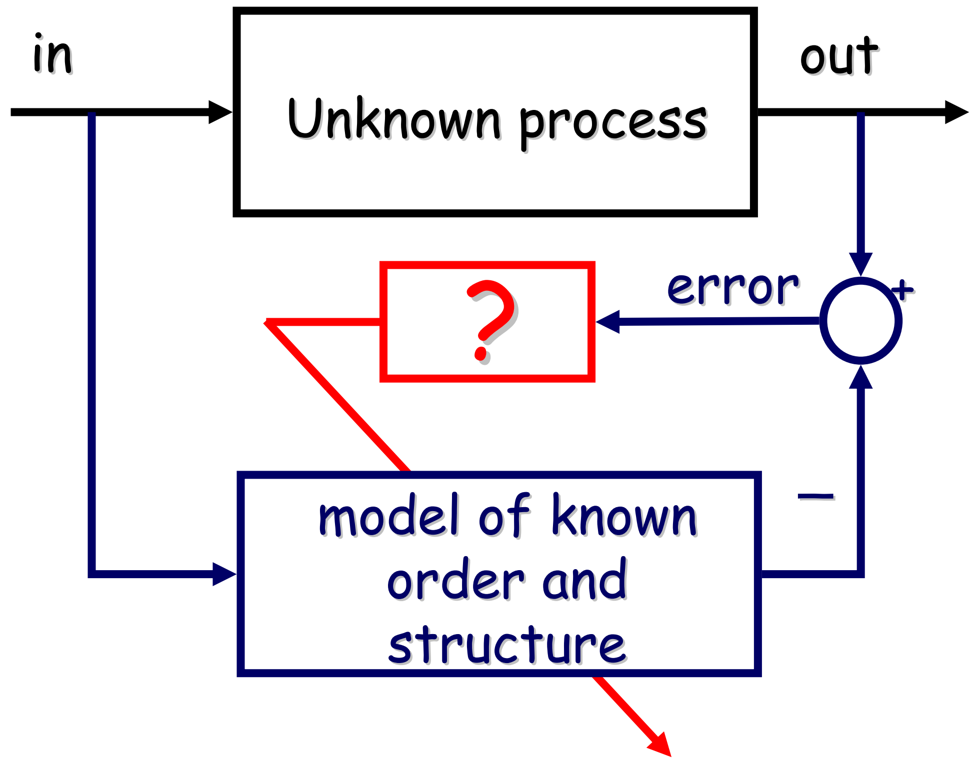
Grey box modelling

- Process structure and order is (assumed to be) known

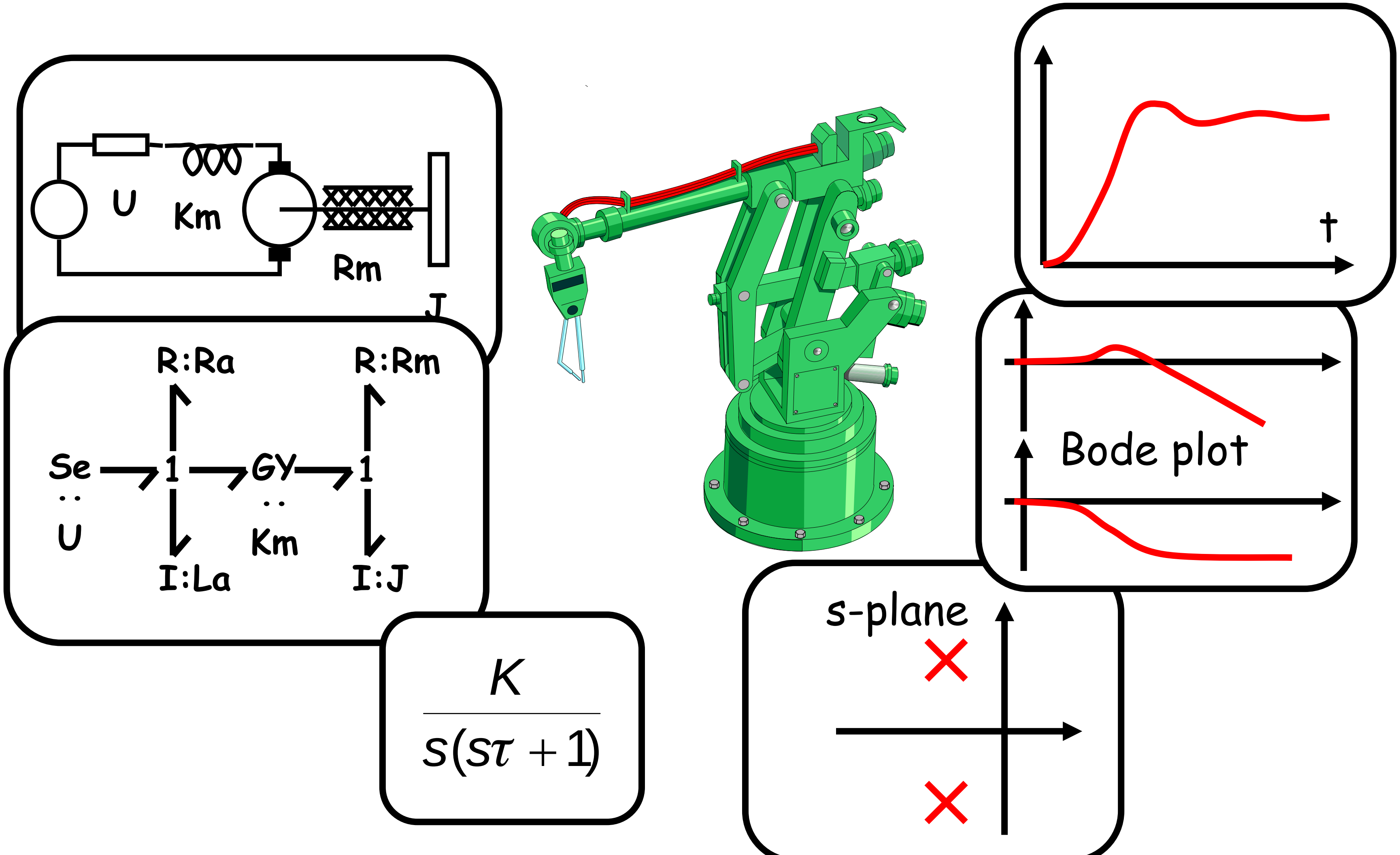


Grey box modelling

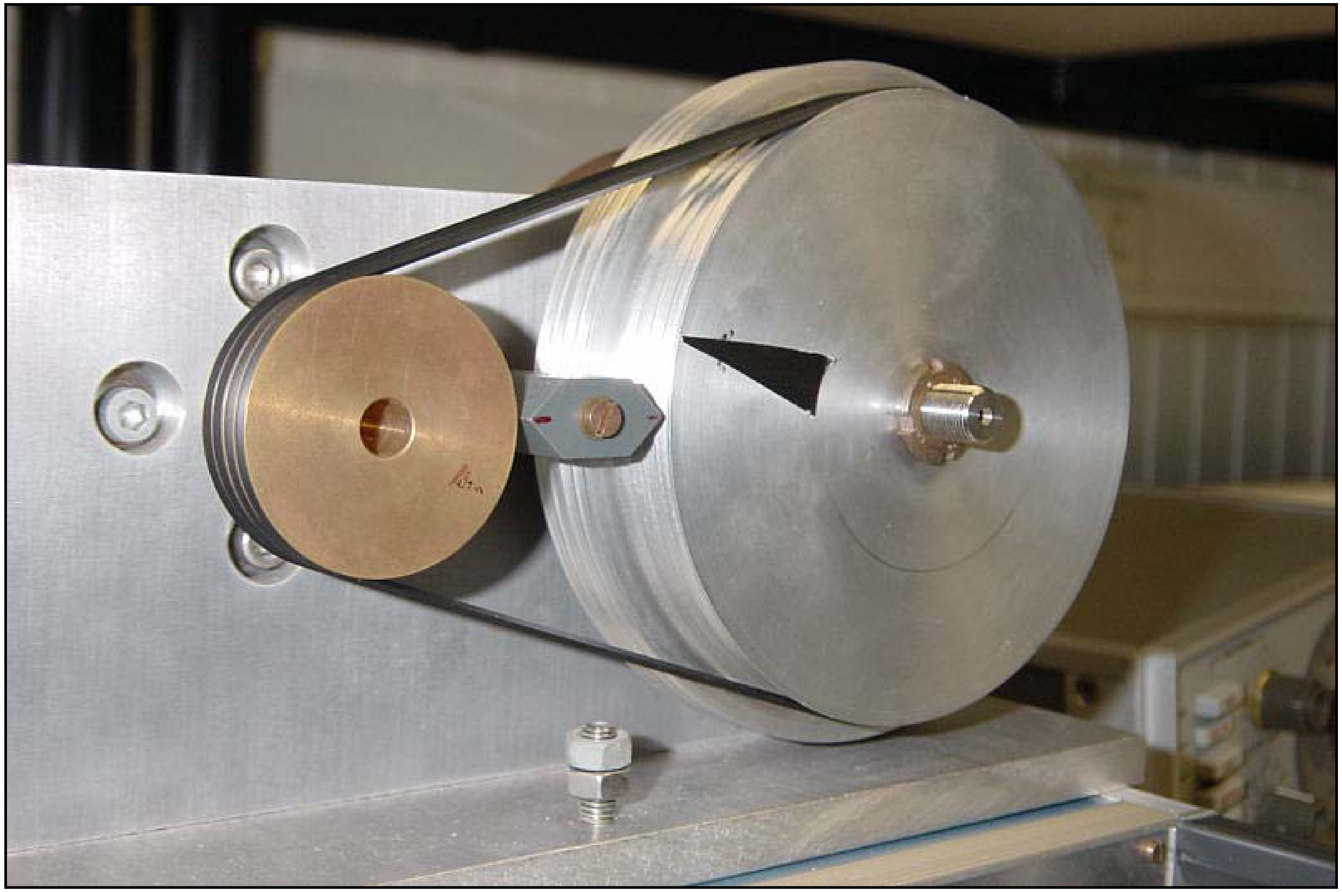
- Process structure and order is (assumed to be) known



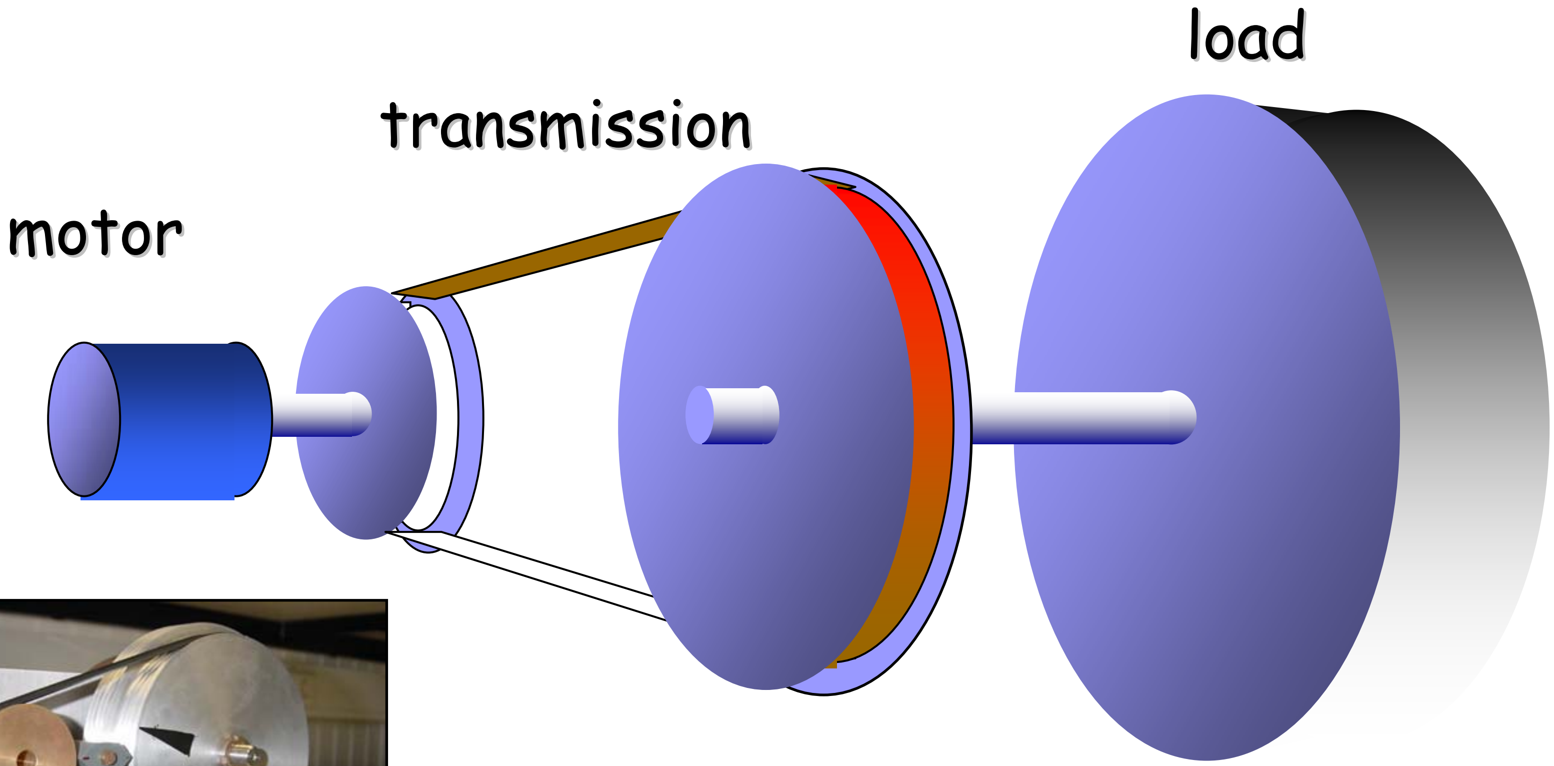
Multiple views

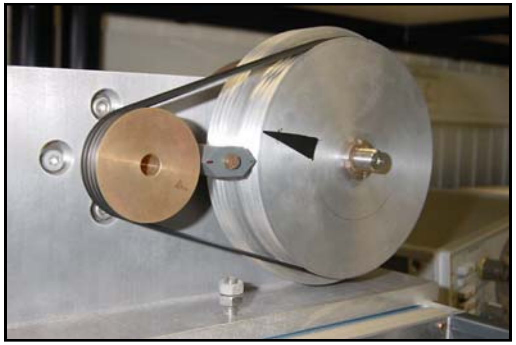


Process



Process

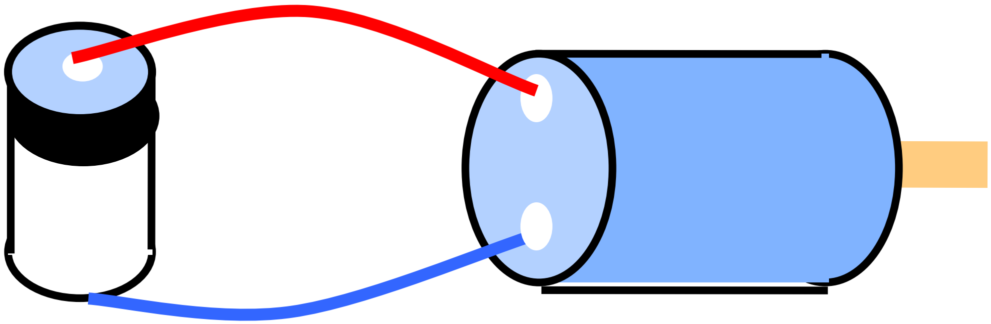




Physical modelling

- Starts with considering the components in the process
- Model the process with a suffiently level of detail
- i.e make a competent model

Simple DC-motor



Ideal Physical Model

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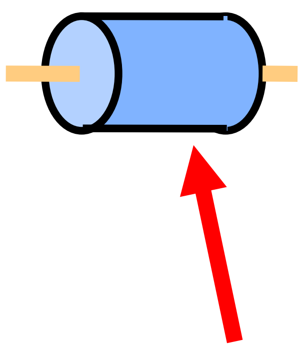
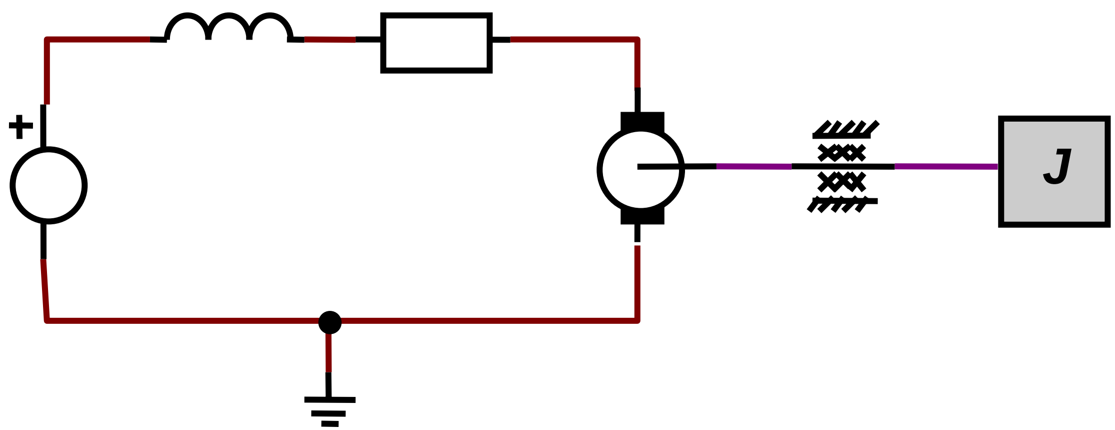


component

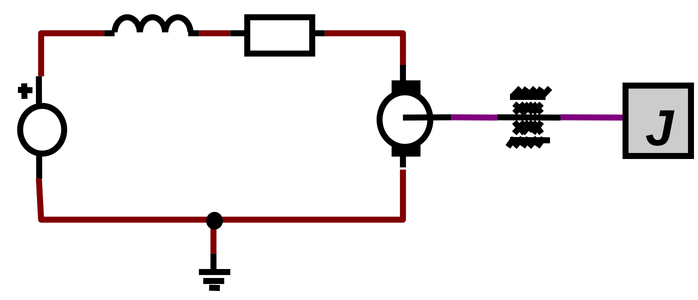


Ideal elements

IPM → equations

$$U - L_a \frac{di}{dt} - K_m\omega - R_a i_a = 0$$

$$K_m i - J\frac{d\omega}{dt} - R_m\omega = 0$$



$$L_a \frac{di}{dt} + R_a i_a = U - K_m\omega$$

$$J\frac{d\omega}{dt} + R_m\omega = K_m i$$

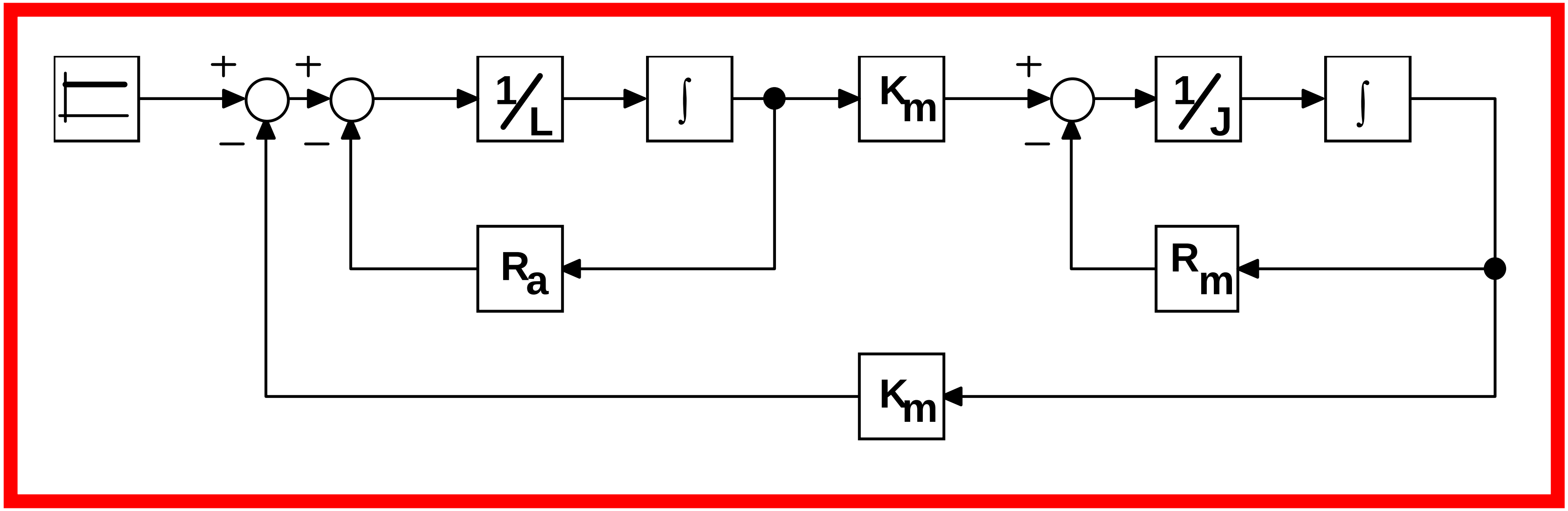
$$i_a = \frac{1}{L_a}\int\left(U - K_m\omega - R_a i_a\right) dt$$

$$\omega = \frac{1}{J}\int\left(K_m i - R_m\omega\right) dt$$

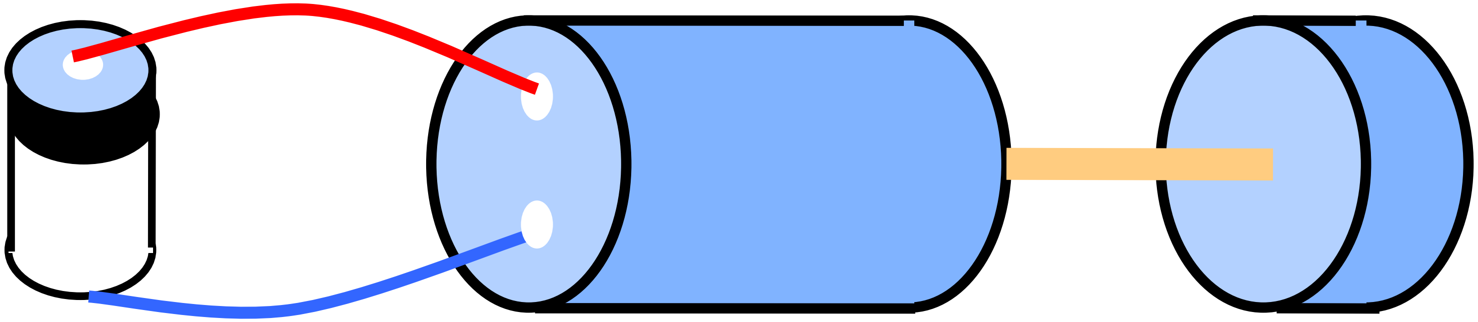
equations → block diagram

$$i_a = \frac{1}{L_a}\int\left(U - K_m\omega - R_a i_a\right)\,\mathrm{d}t$$

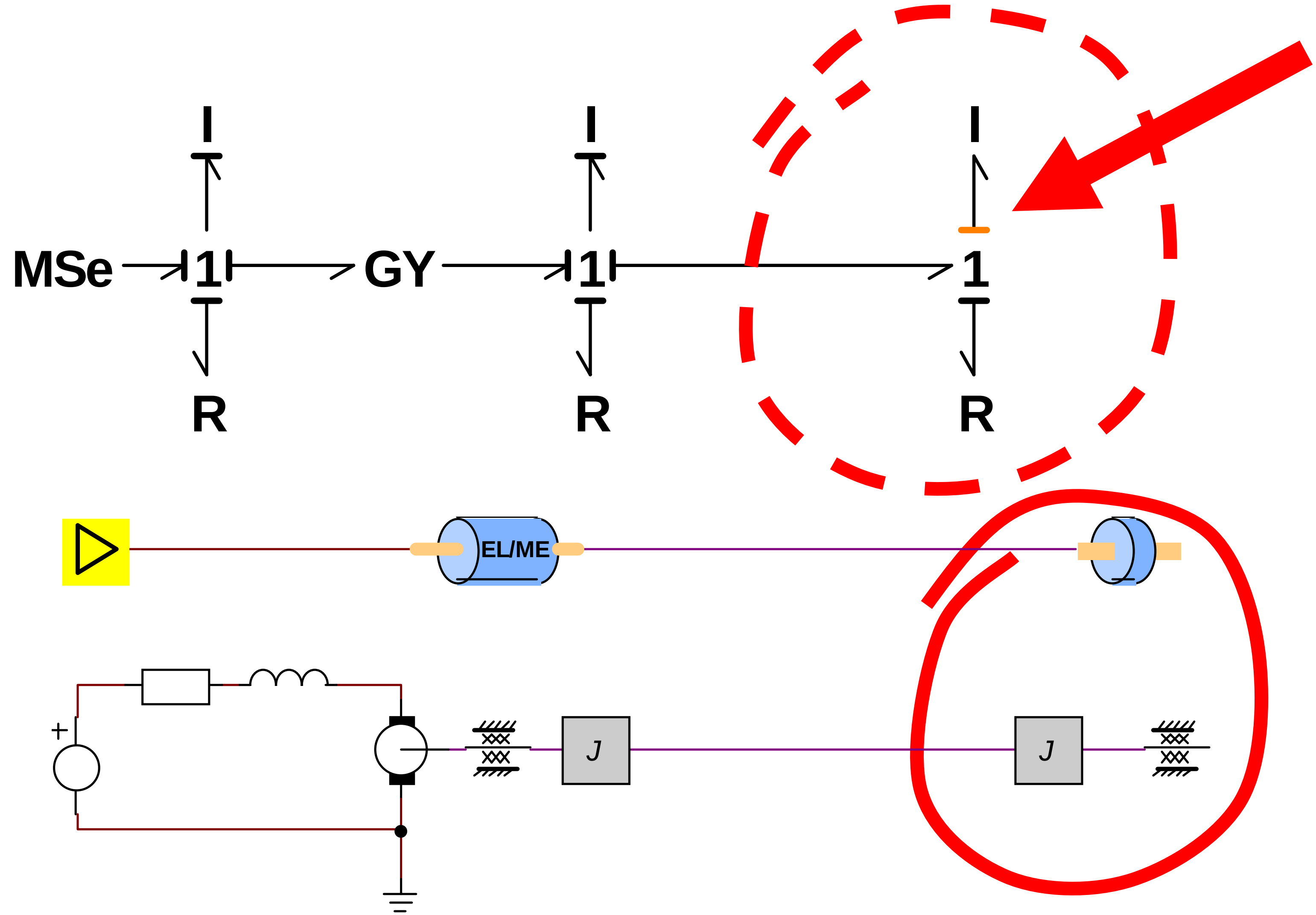
$$\omega = \frac{1}{J}\int\left(K_m i - R_m\omega\right)\,\mathrm{d}t$$



DC-motor + Load



Add the load



Add flexible axis

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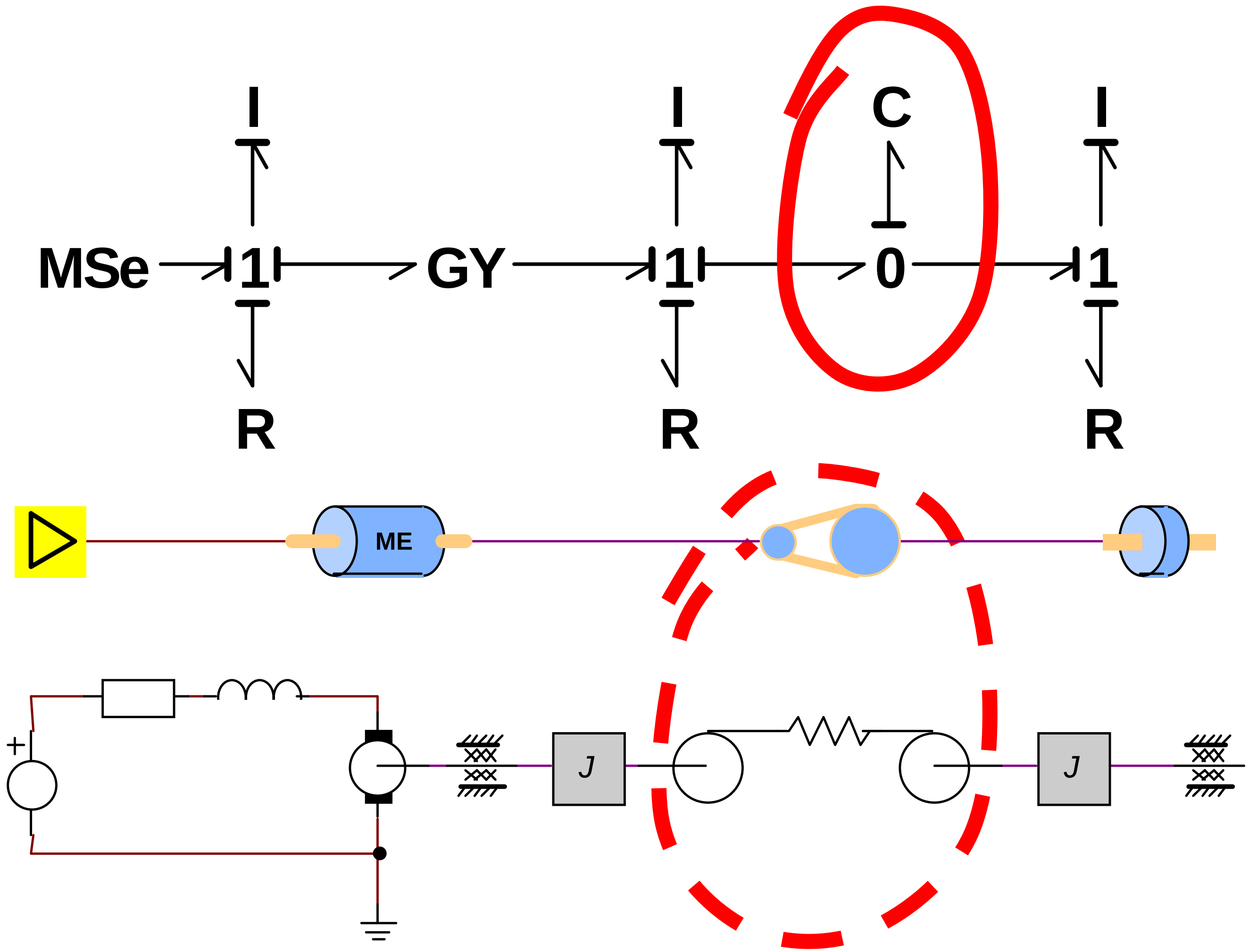

IPM → equations

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$$U - L_a \frac{di}{dt} - K_m \omega_1 - R_a i_a = 0$$

$$K_m i_a - J_1 \frac{d\omega_1}{dt} - R_m \omega \ldots$$

$$\frac{1}{C}\left(\int (\omega_1 - \omega_1)\, dt\right) - \ldots$$

$$i_a = \frac{1}{L_a}\left(\int \left(U - K_m \omega_1 - R_a i_a\right) dt\right)$$

$$\omega_1 = \frac{1}{J_1}\left(\int \left(K_m i_a - R_m \omega_1 - T_{spring}\right) dt\right)$$

$$T_{spring} = \frac{1}{C}\left(\int \left(\omega_1 - \omega_2\right) dt\right)$$

$$\omega_2 = \frac{1}{J_2}\left(\int \left(T_{spring} - R_m \omega_2\right) dt\right)$$

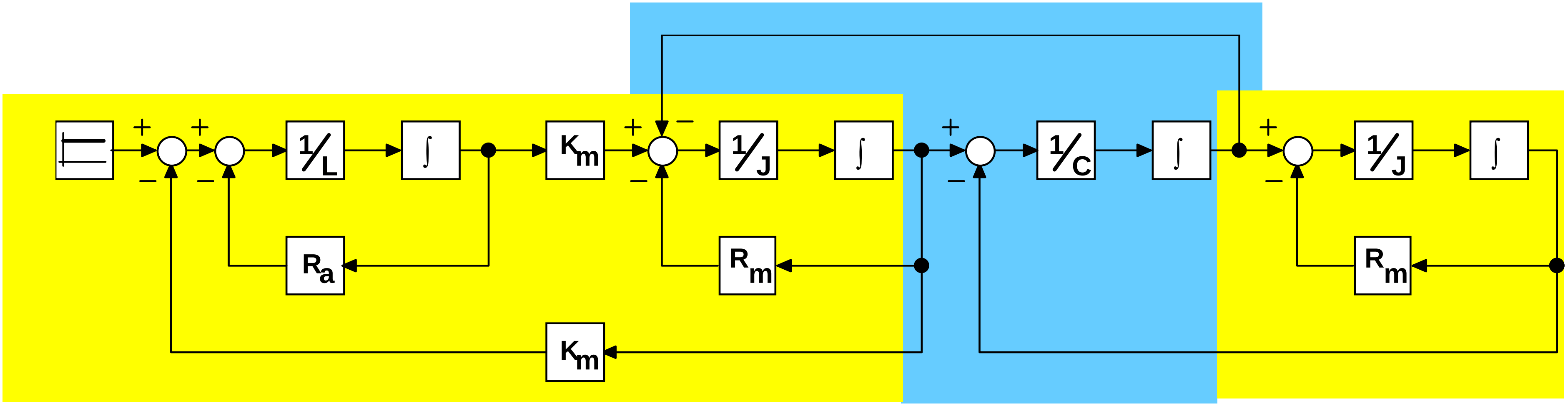
Block Diagram

$$i_a = \frac{1}{L_a}\left(\int \left(U - K_m\omega_1 - R_a i_a\right) dt\right)$$

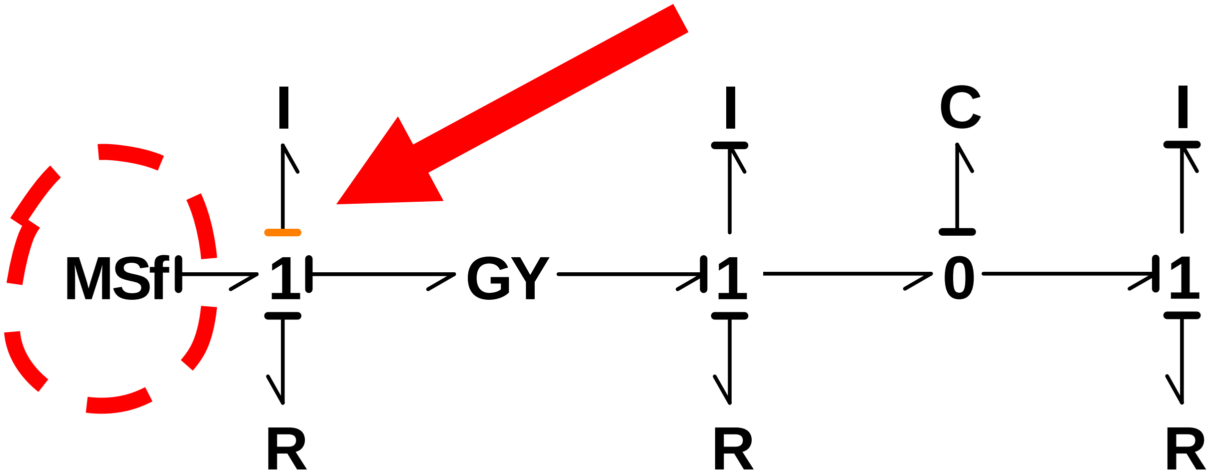
$$\omega_1 = \frac{1}{J_1}\left(\int \left(K_m i_a - R_m\omega_1 - T_{spring}\right) dt\right)$$

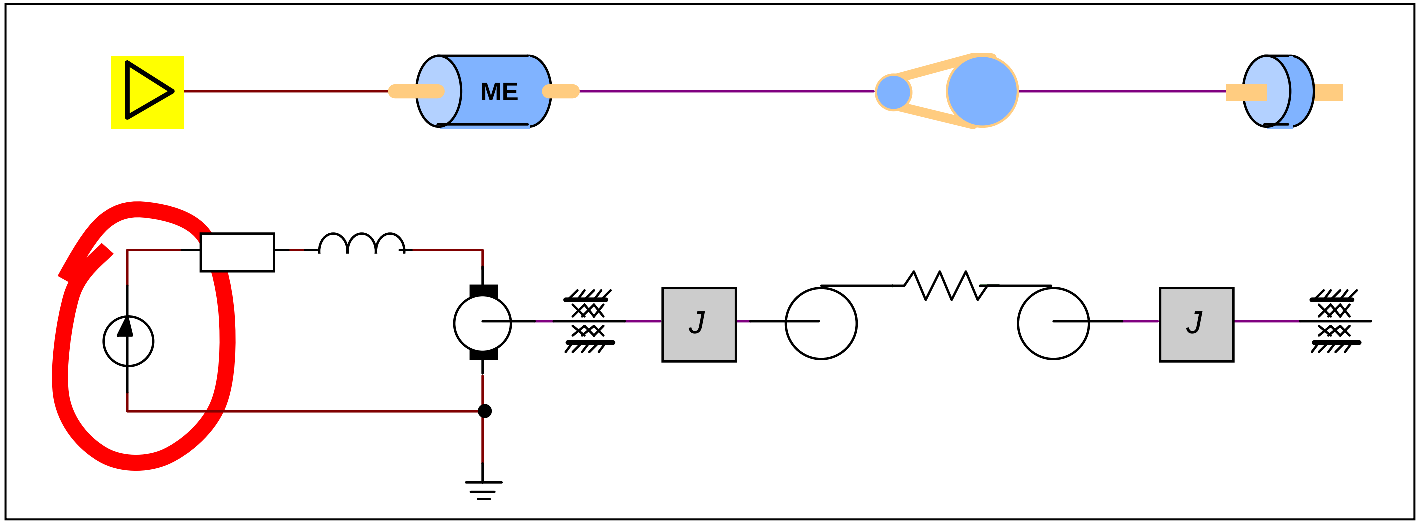
$$T_{spring} = \frac{1}{C}\left(\int \left(\omega_1 - \omega_2\right) dt\right)$$

$$\omega_2 = \frac{1}{J_2}\left(\int \left(T_{spring} - R_m\omega_2\right) dt\right)$$



Use current amplifier





No electrical time constant

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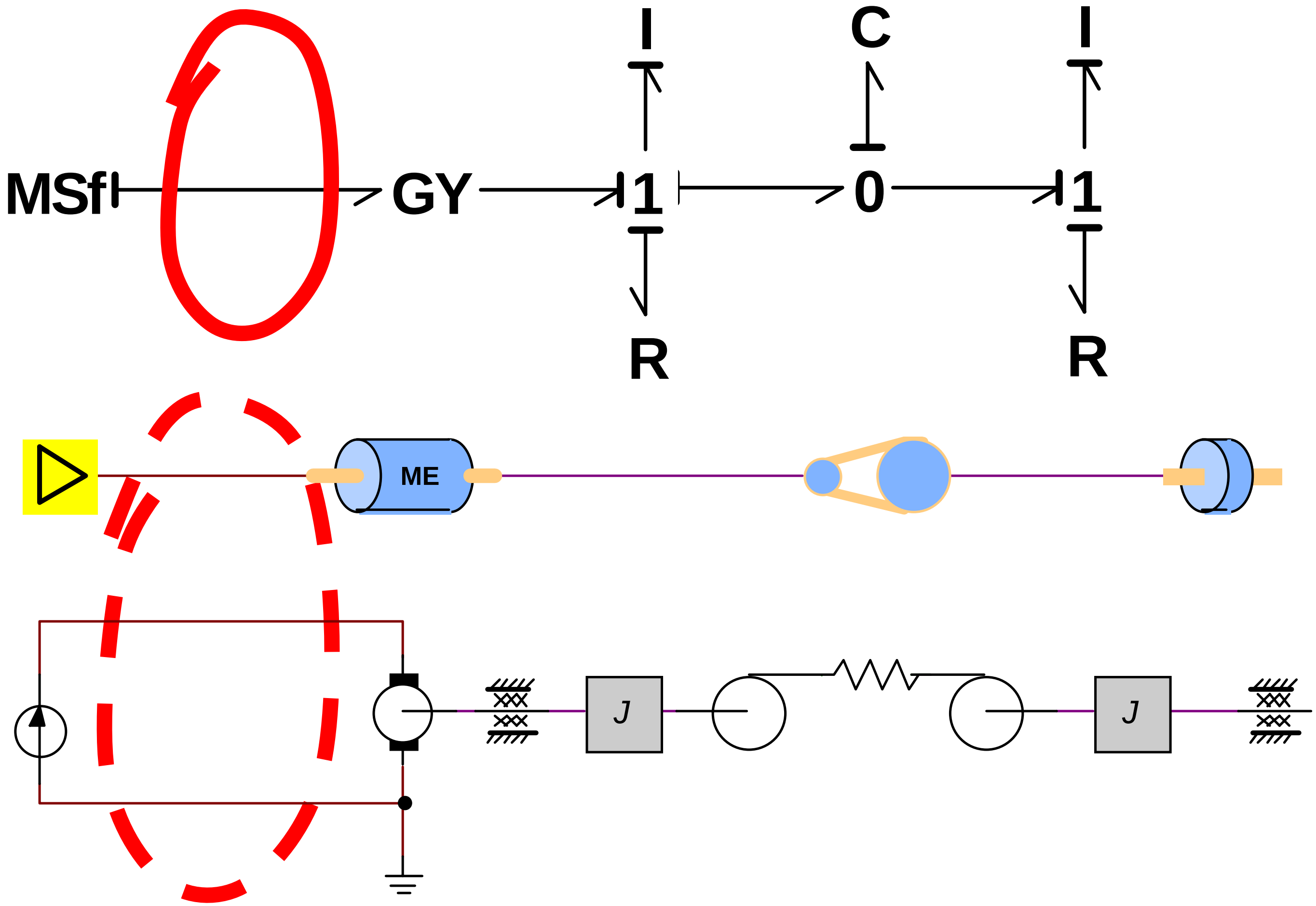
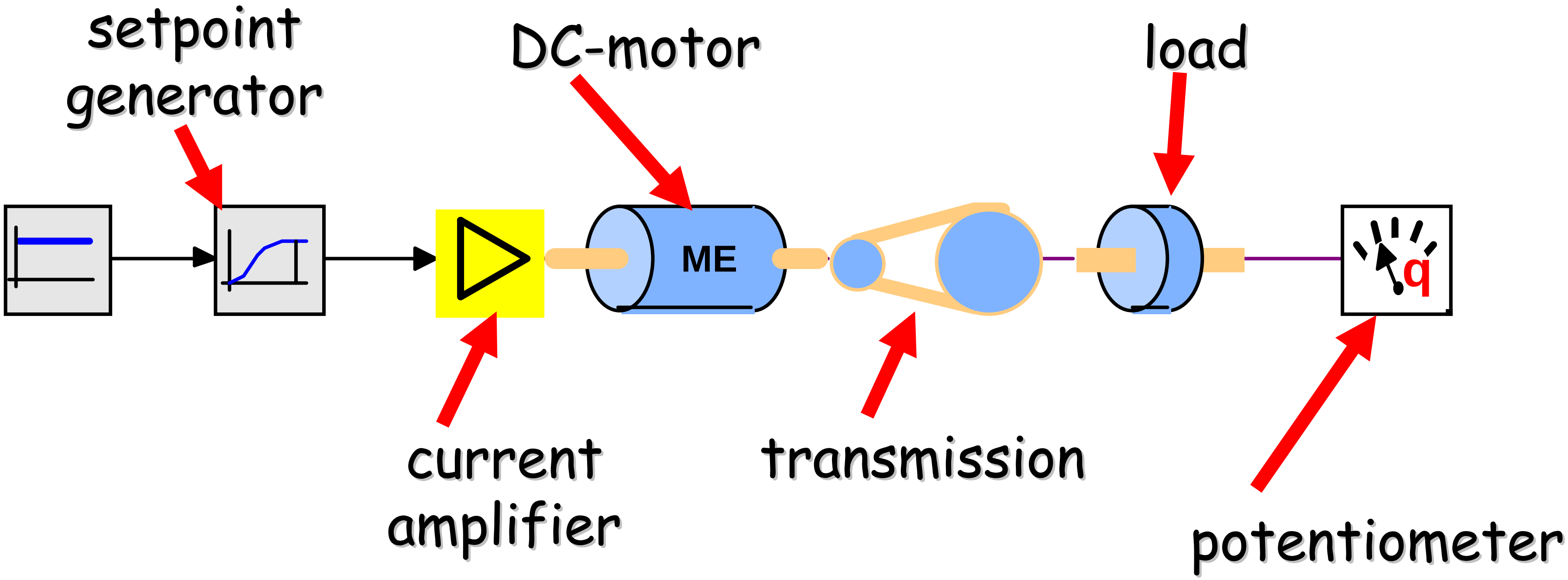
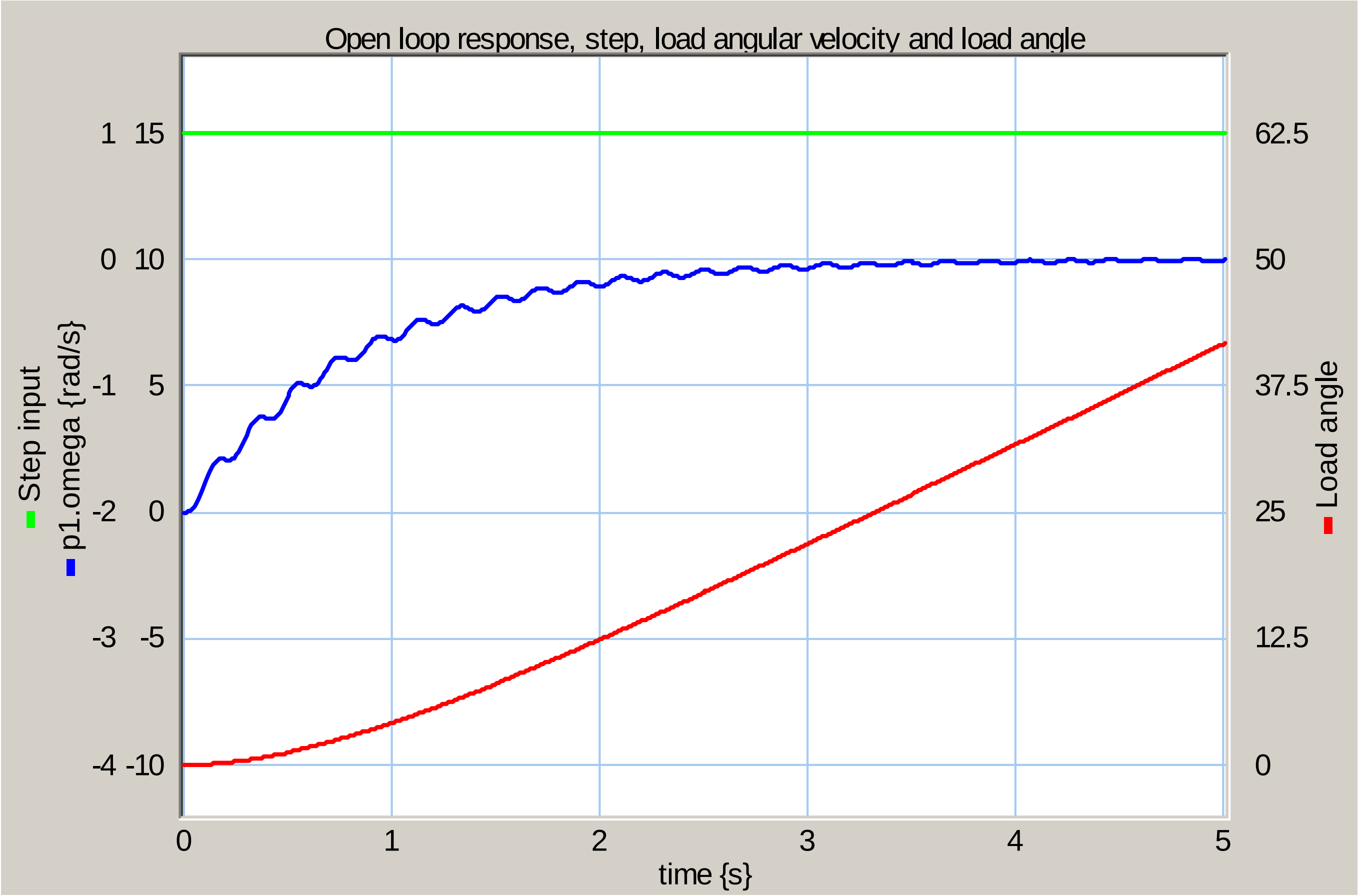


Demo process

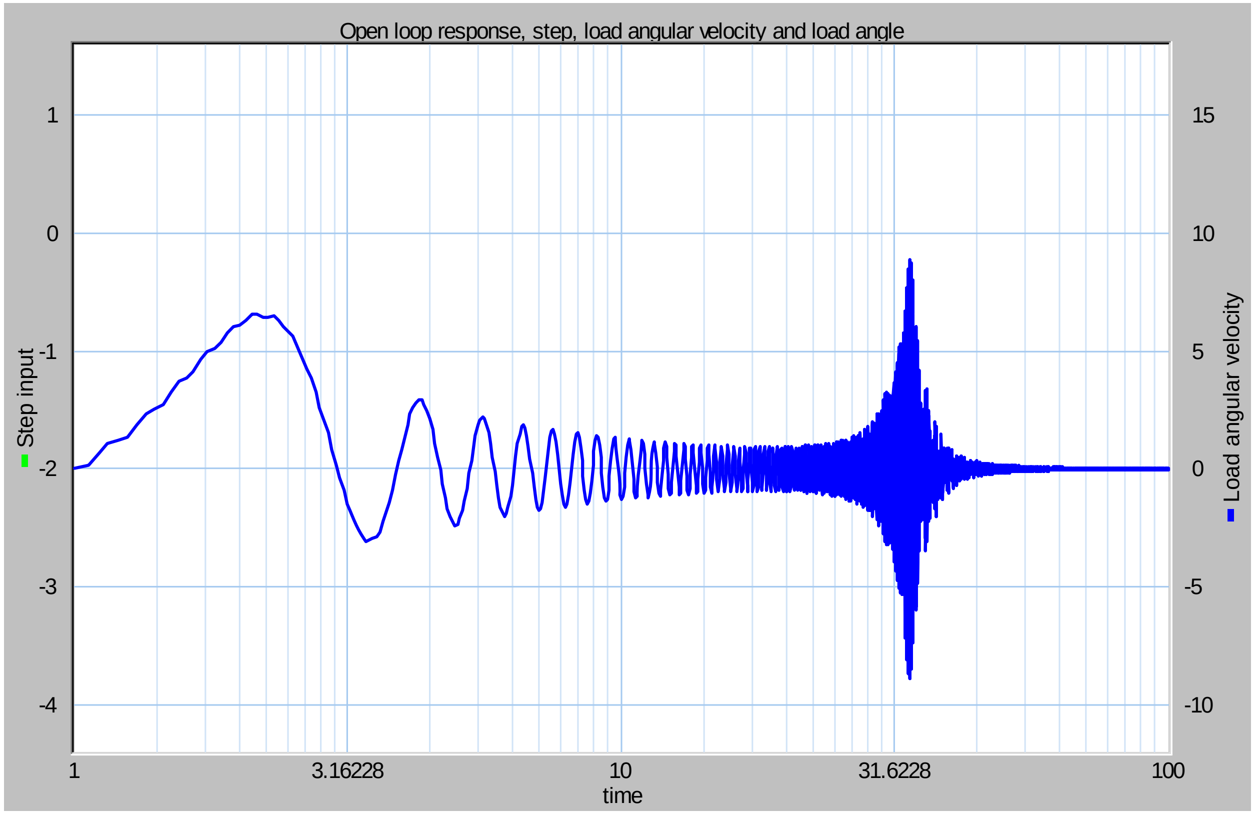


20-sim
demo

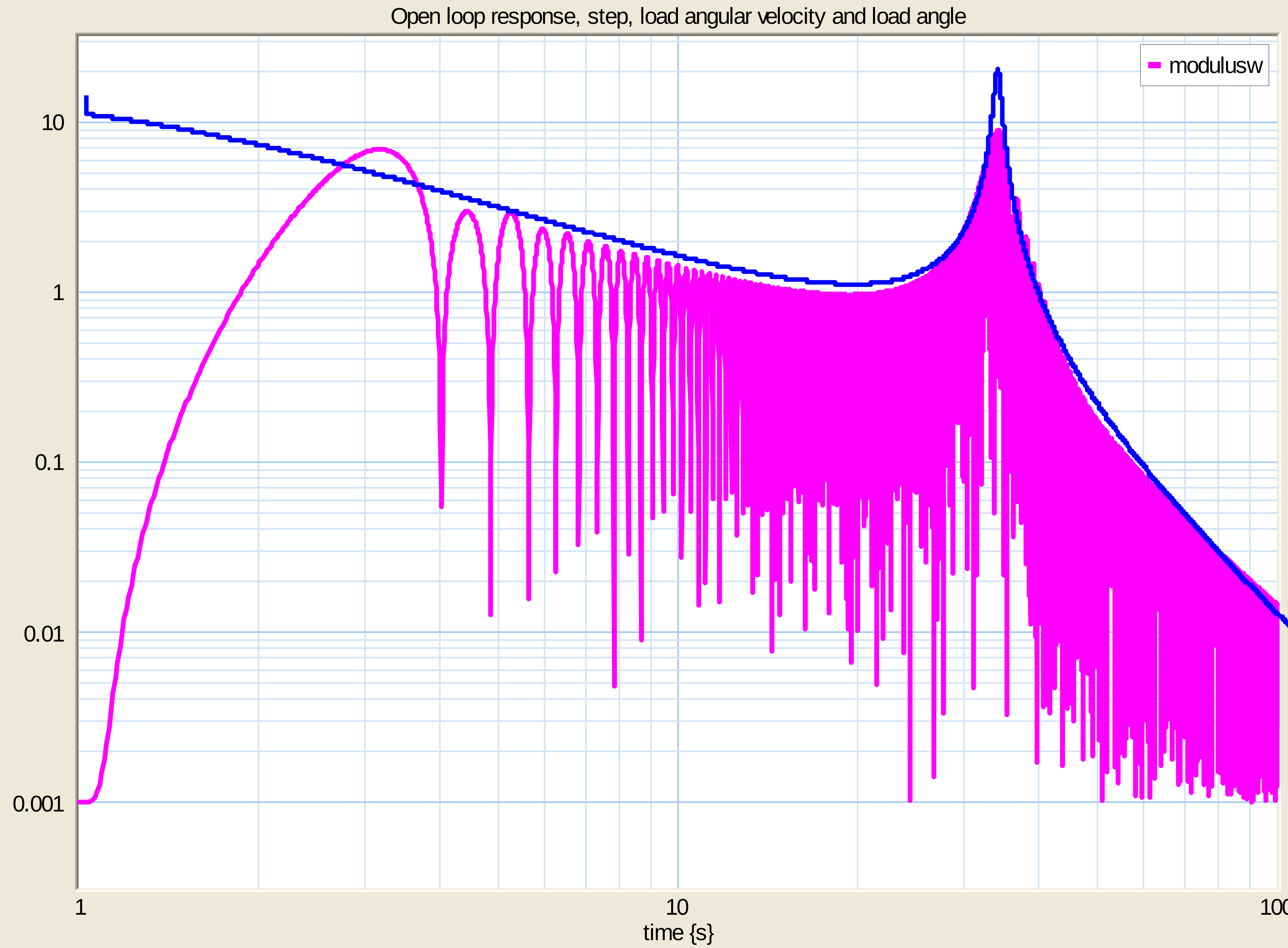
Step response



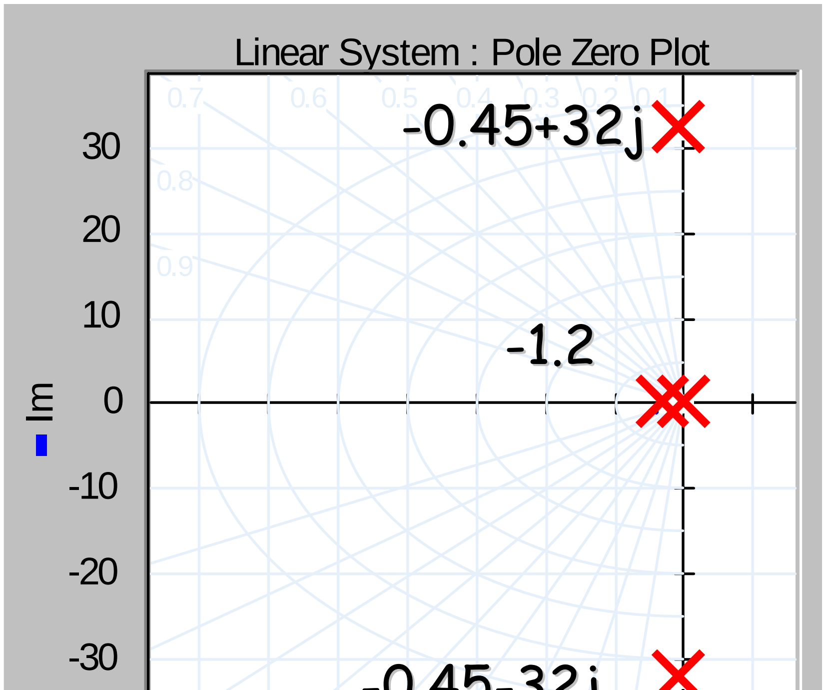
Response to a sweep

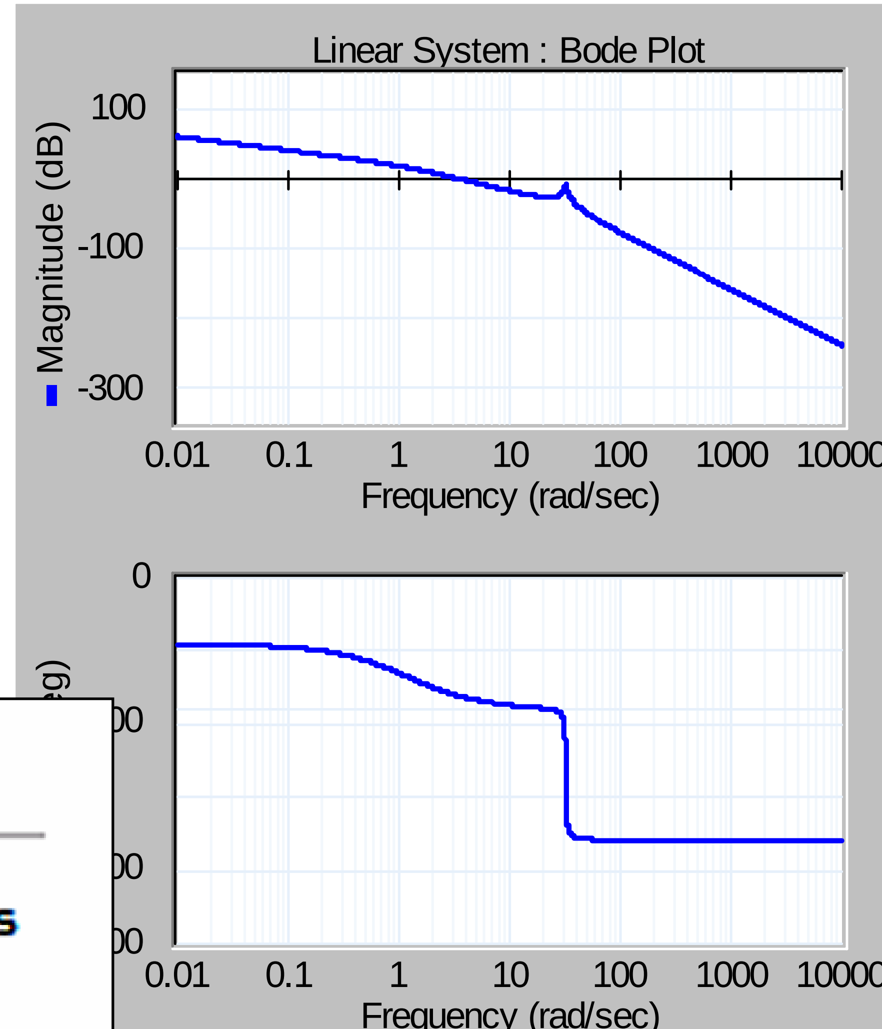


20-sim
demo


Open loop response, step, load angular velocity and load angle
modulusw
10
1
0.1
0.01
0.001
1
10
100
time {s}

After ‘linearisation’ (current)





$$\frac{1.226e{+}004}{s^4 + 2.095\,s^3 + 1028\,s^2 + 1226\,s}$$

system K = 10

After simplifcation (current)

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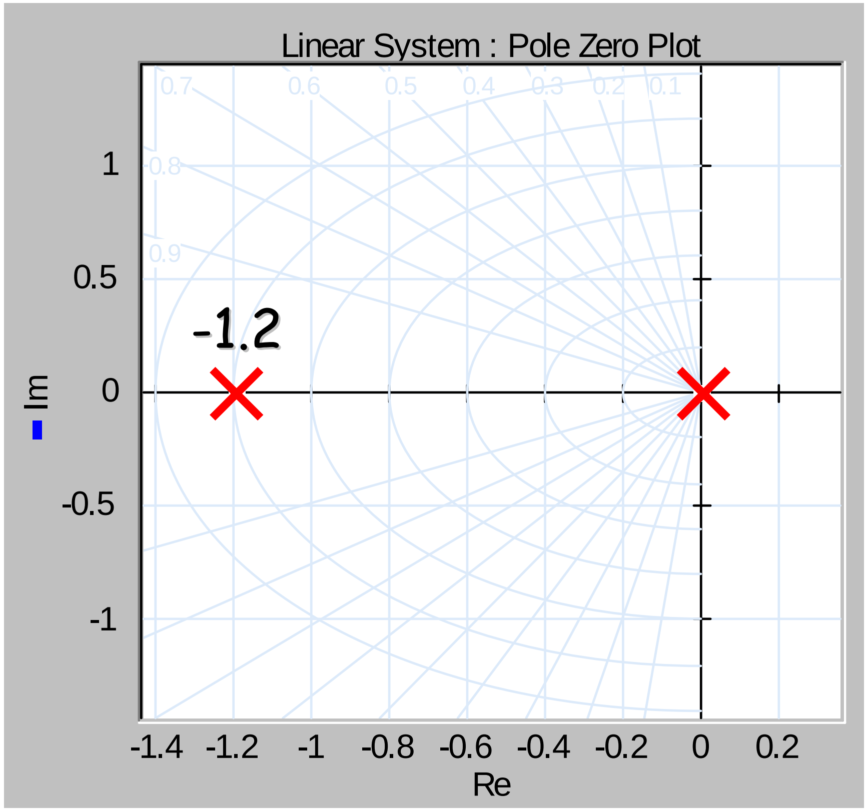
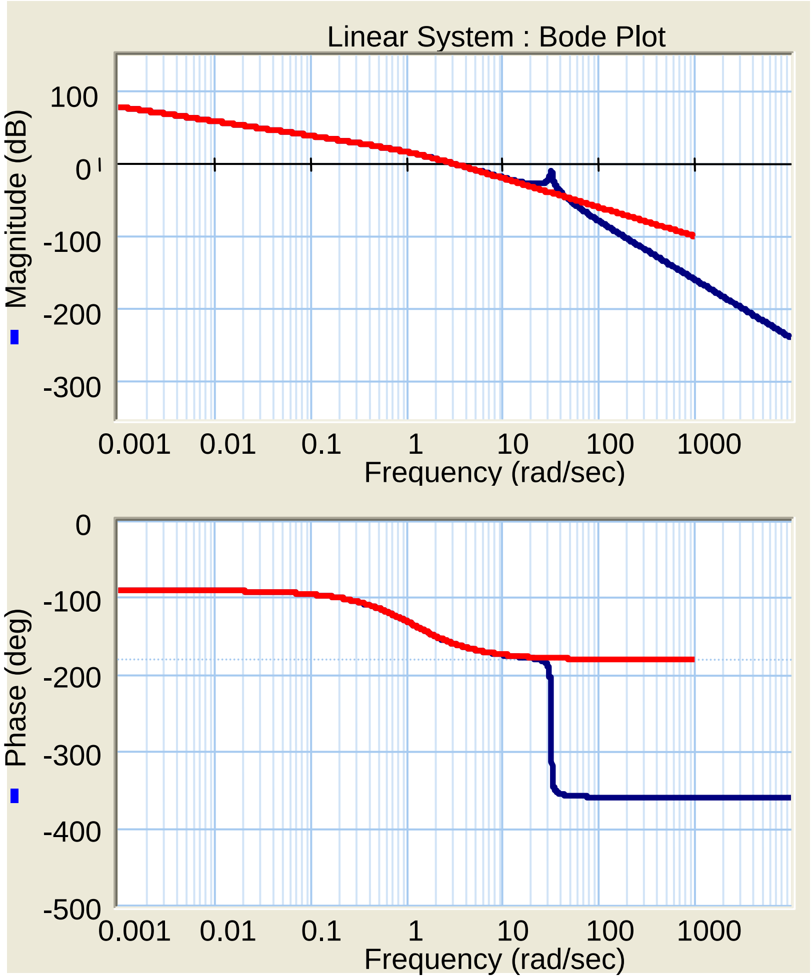
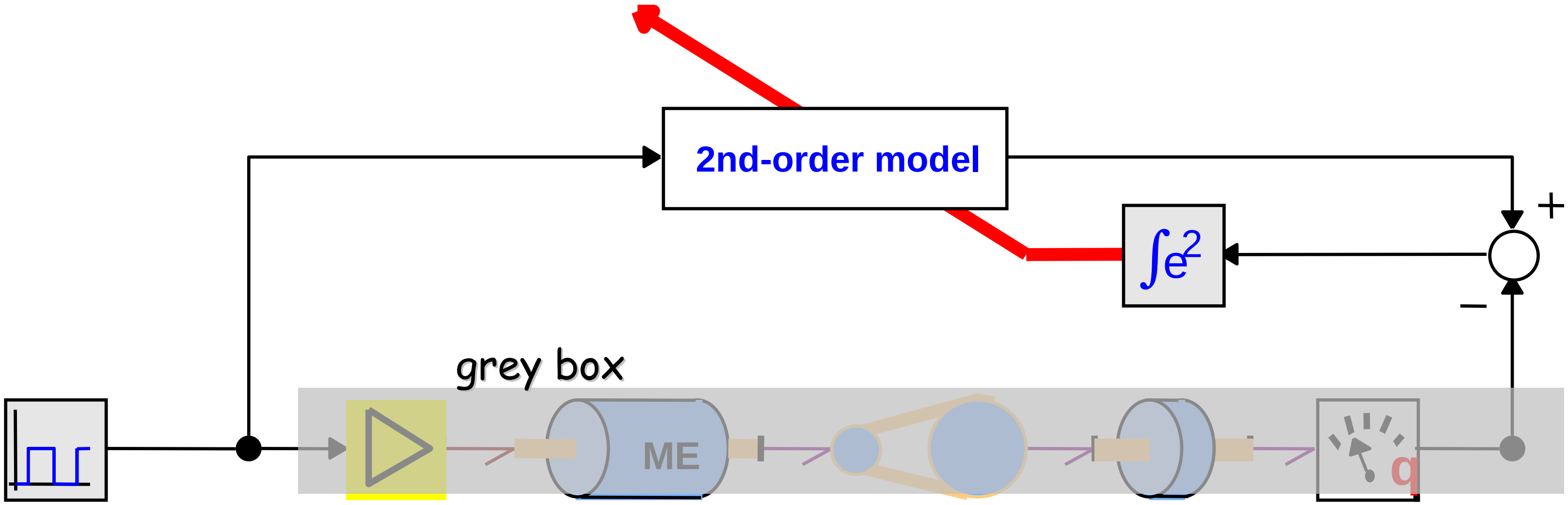




Black (grey) box modelling

- Starts with measurements of an existing system
- Fits a (simple) dynamic model through the response

Identification (current)

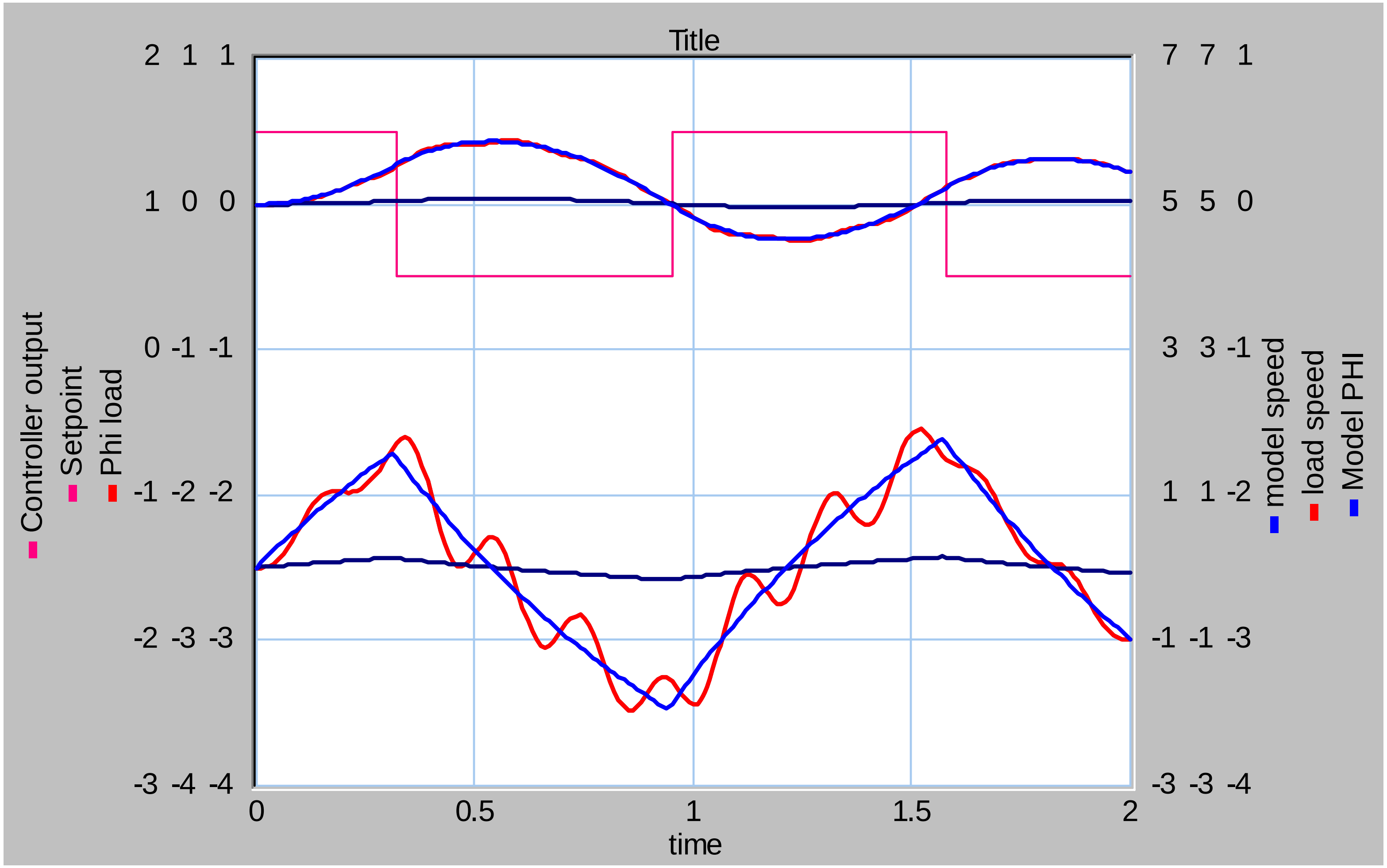


This model demonstrates how a 4th-order system can be approximated by a 2nd-order system

Run the experiment for identification

20-sim

Hill climbing



Hill climbing (noise)

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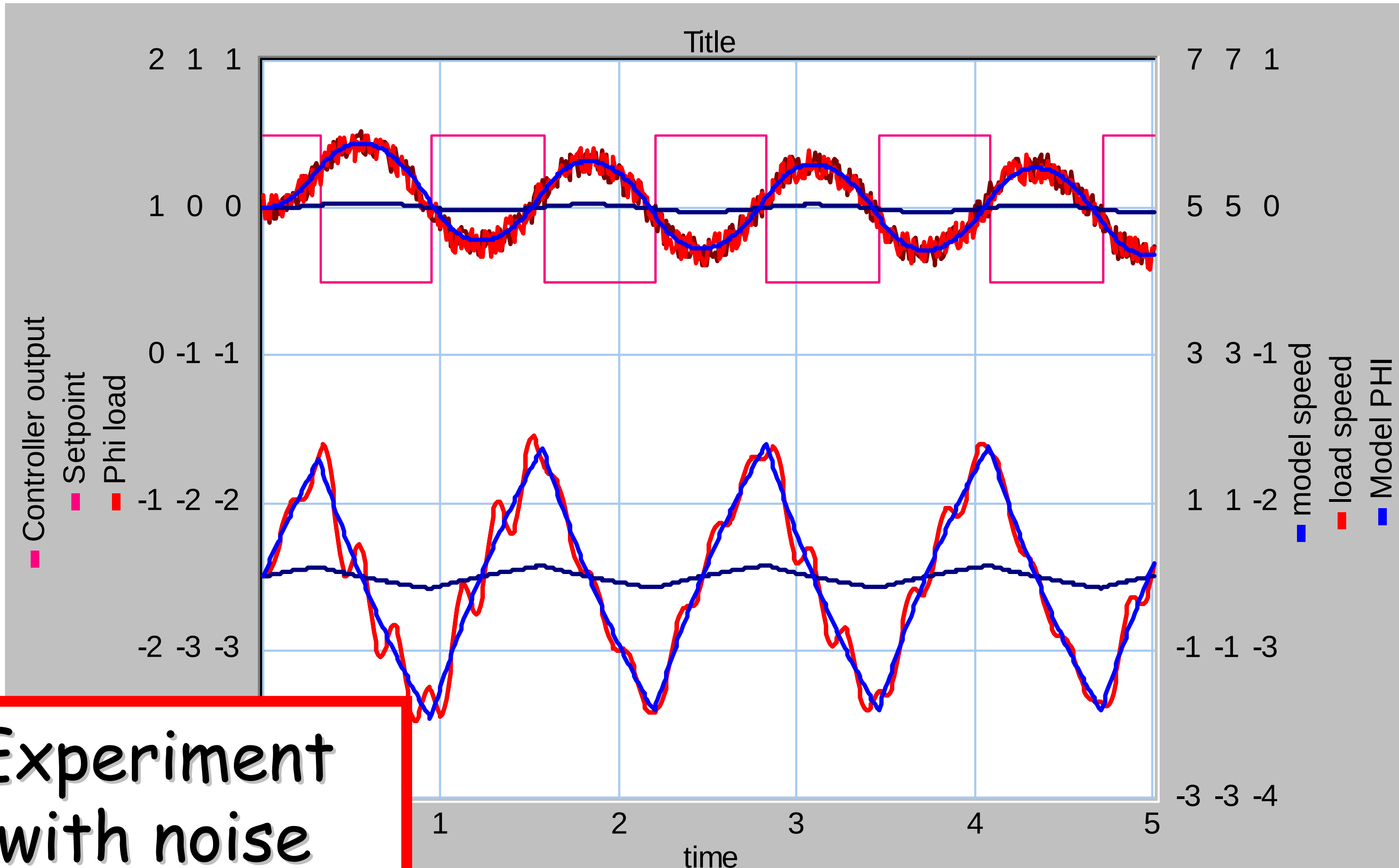
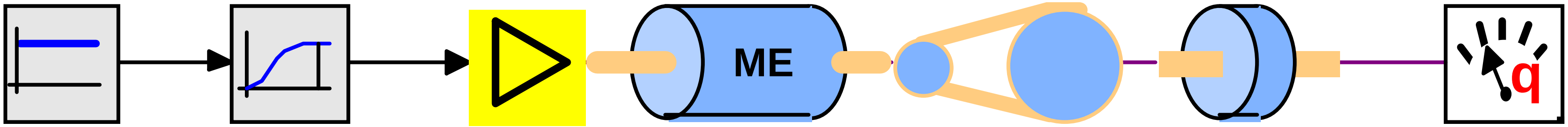


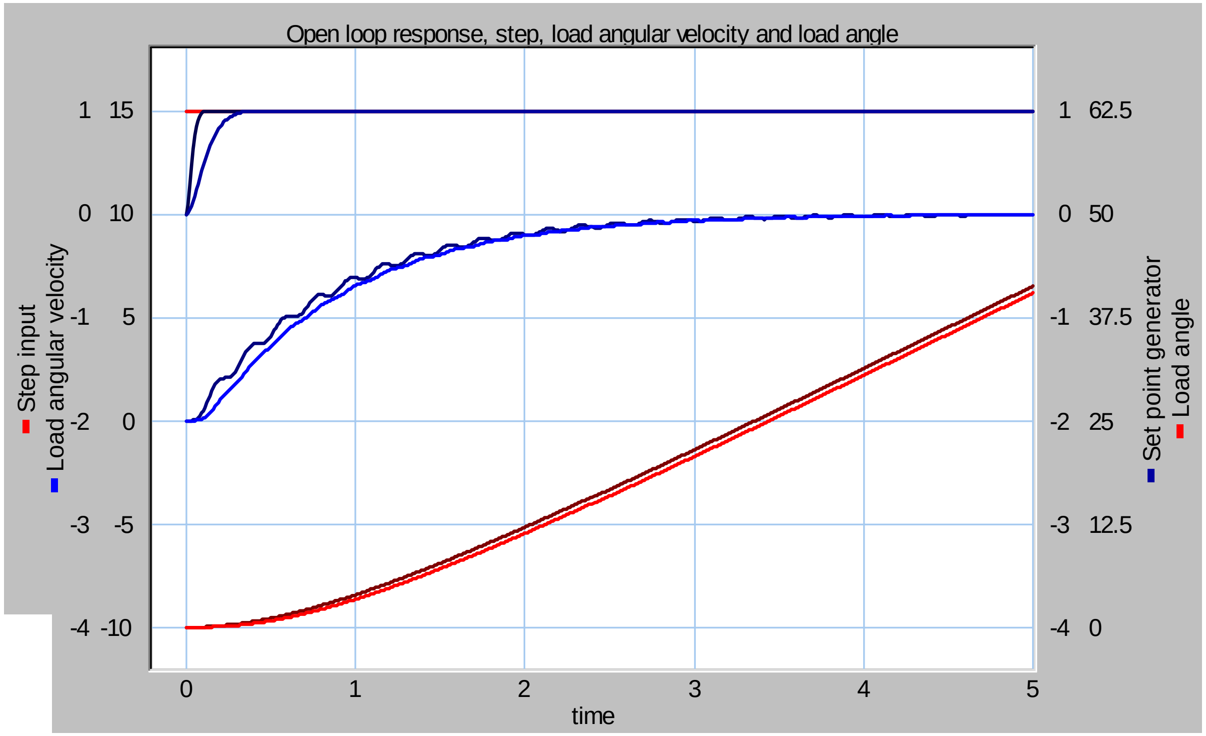
Experiment
with noise

LF behaviour



$\omega = 50$

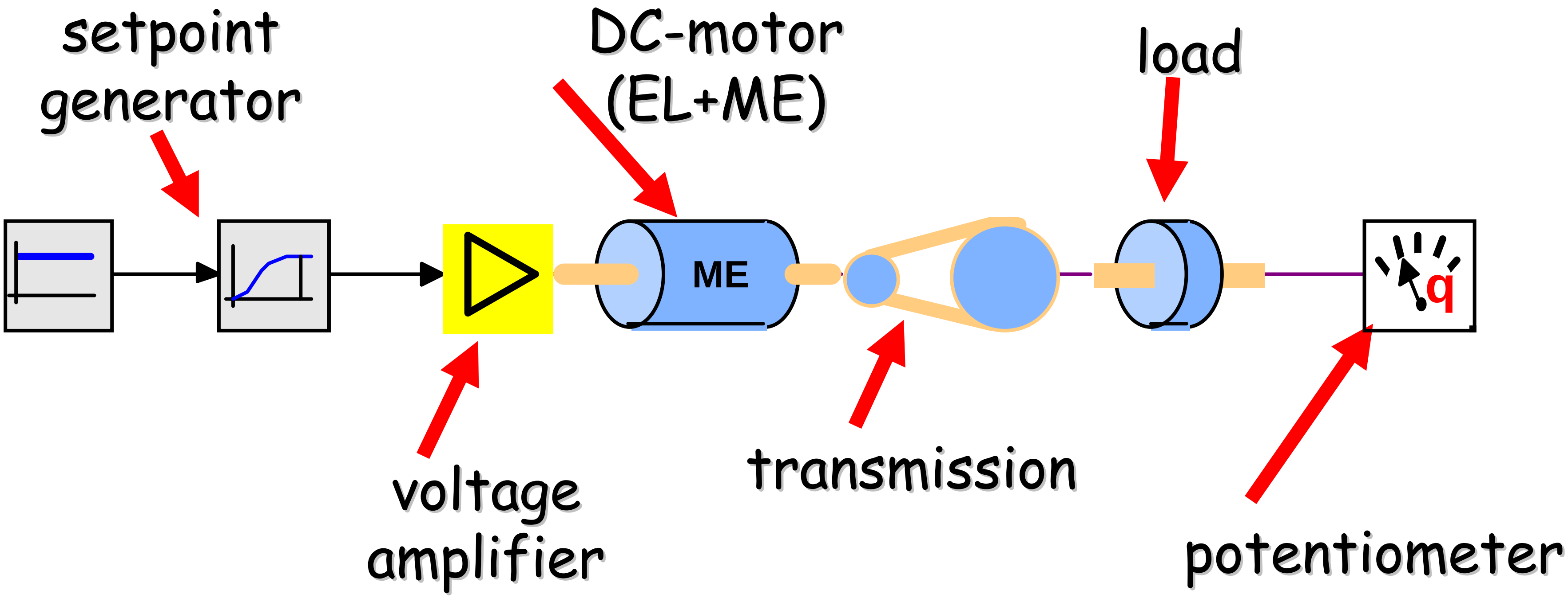
$\omega = 15$



demo

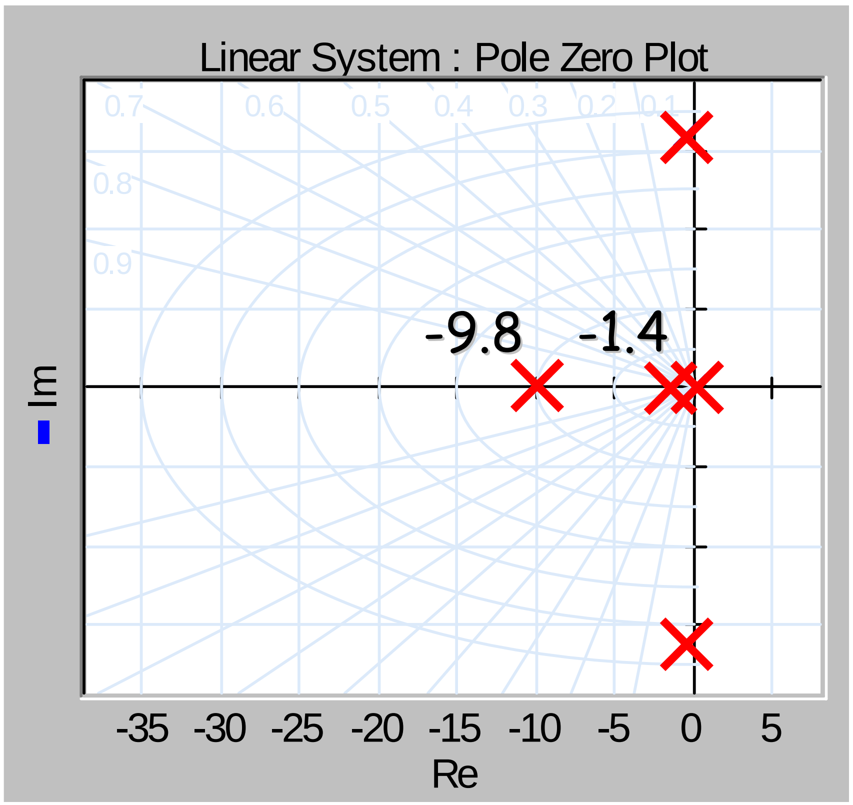
20-sim

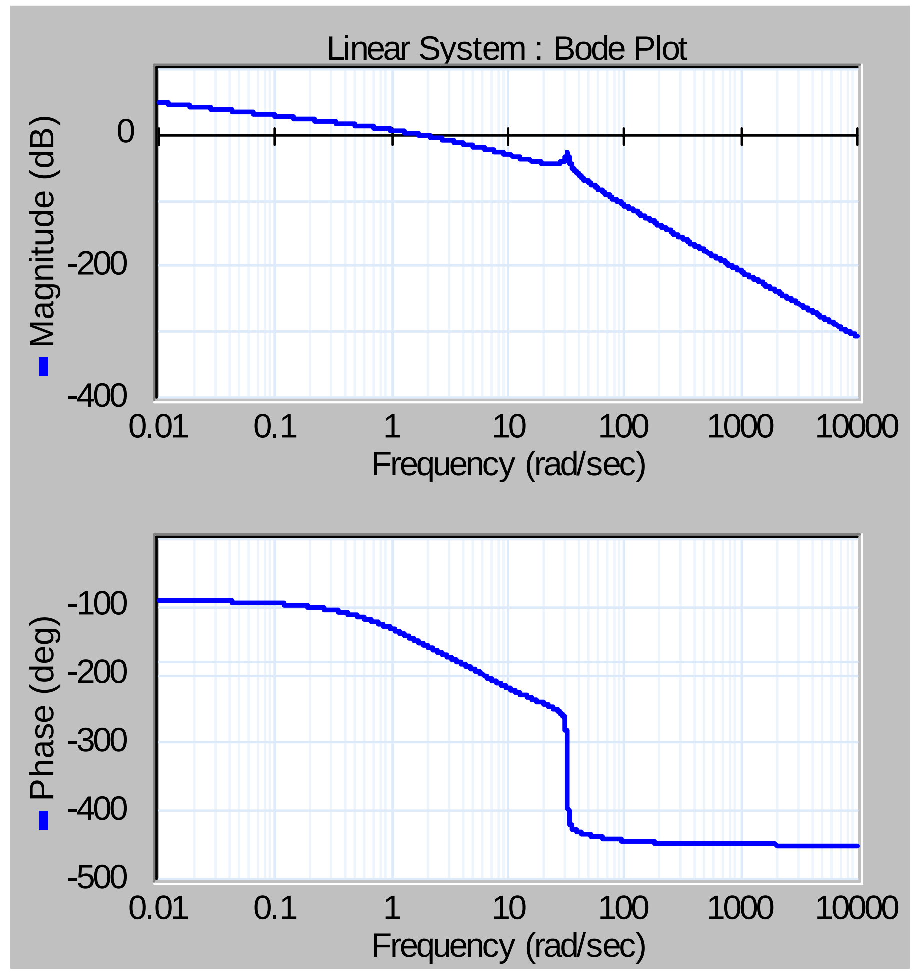
Demo process (volt)



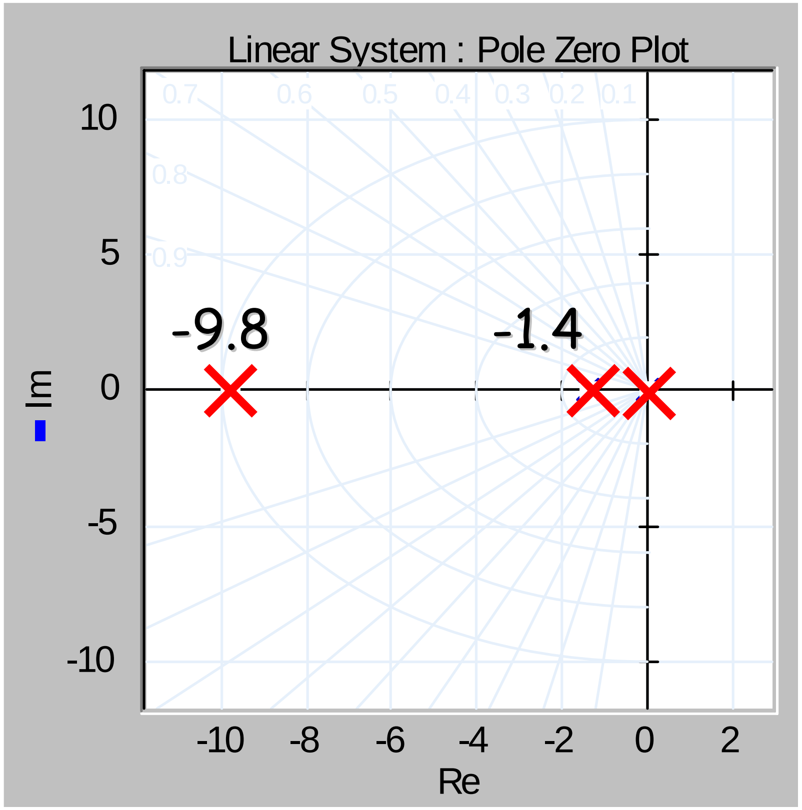
Demo
20-sim

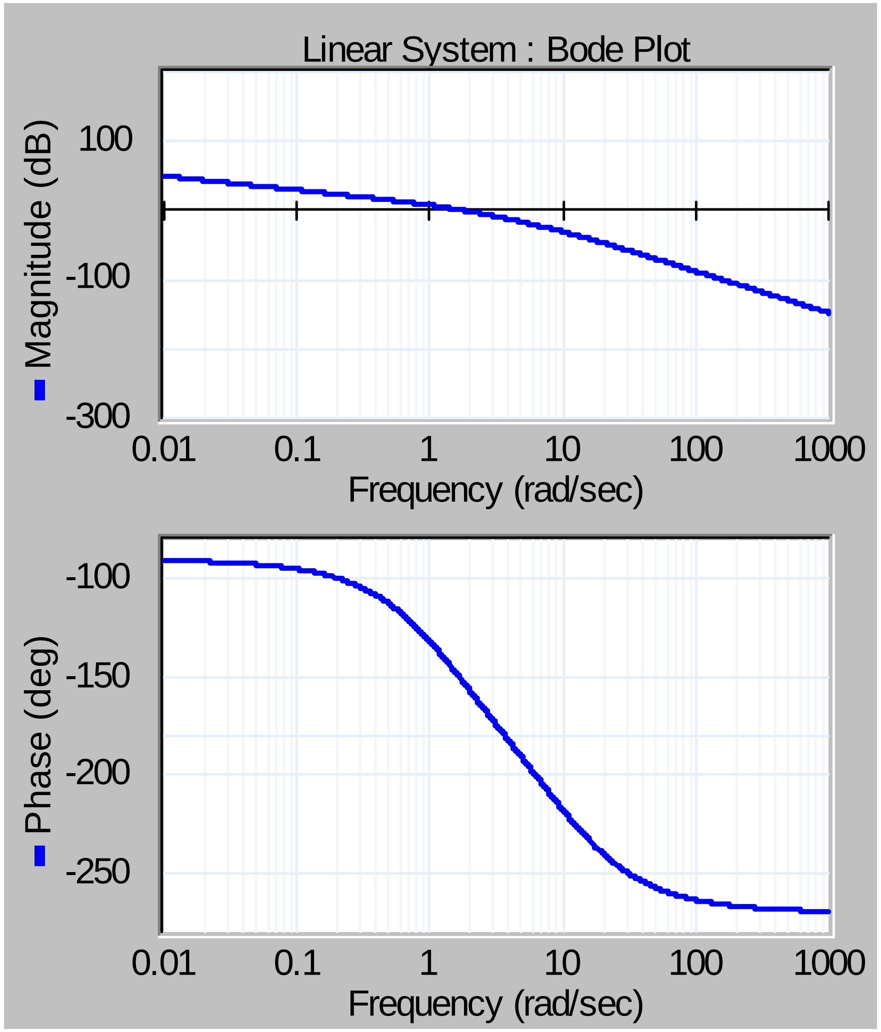
After 'linearisation' (volt.)





After simplification (volt.)

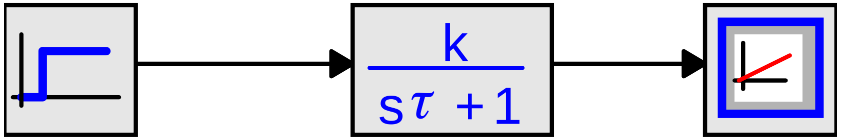


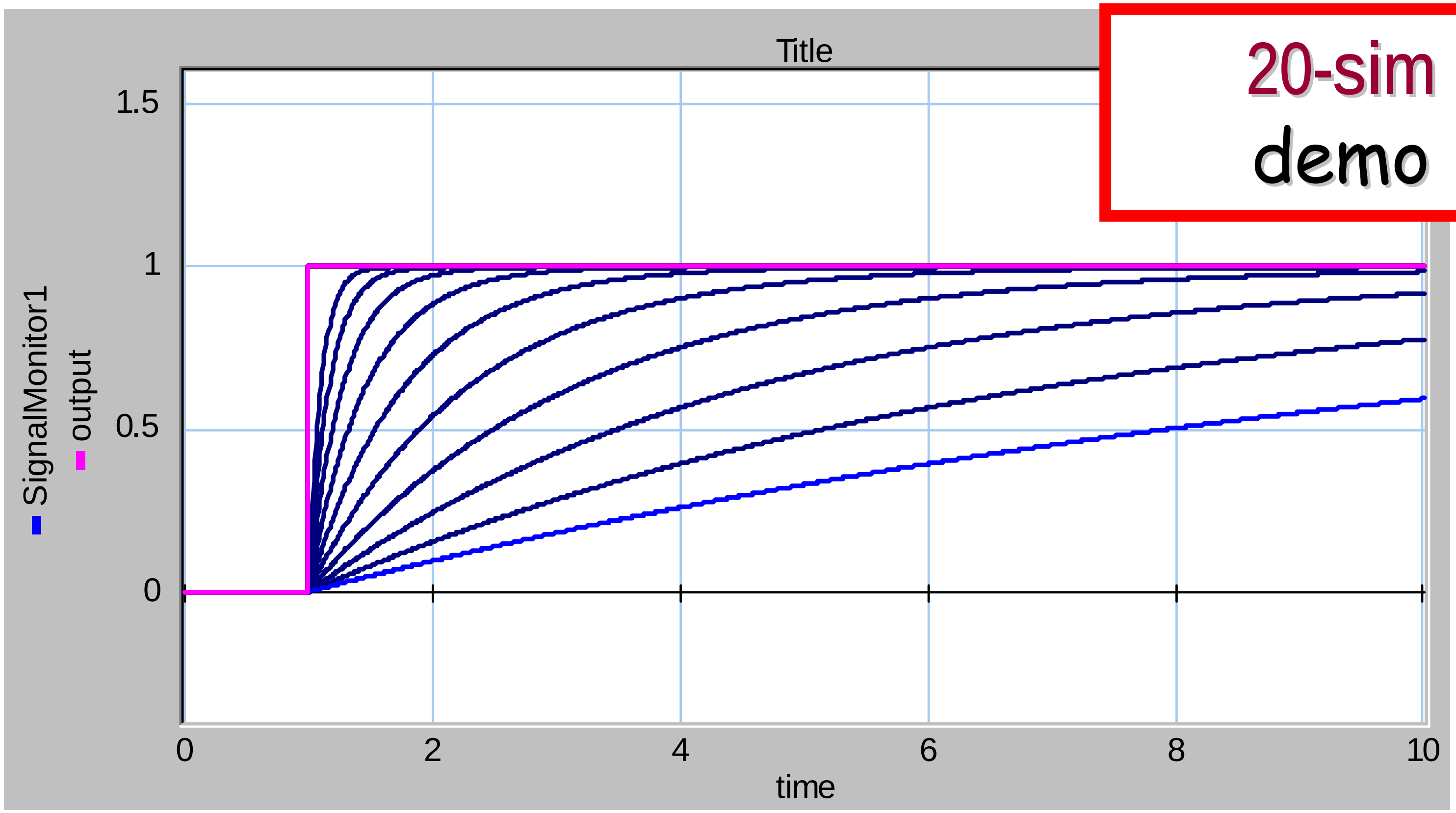


Relation s-domain ↔ time domain

First-order system

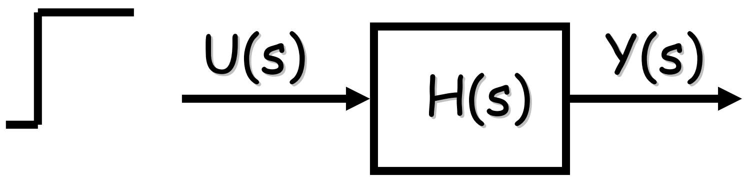
0.1 <tau <10





20-sim demo

Solving the Differential Eq.



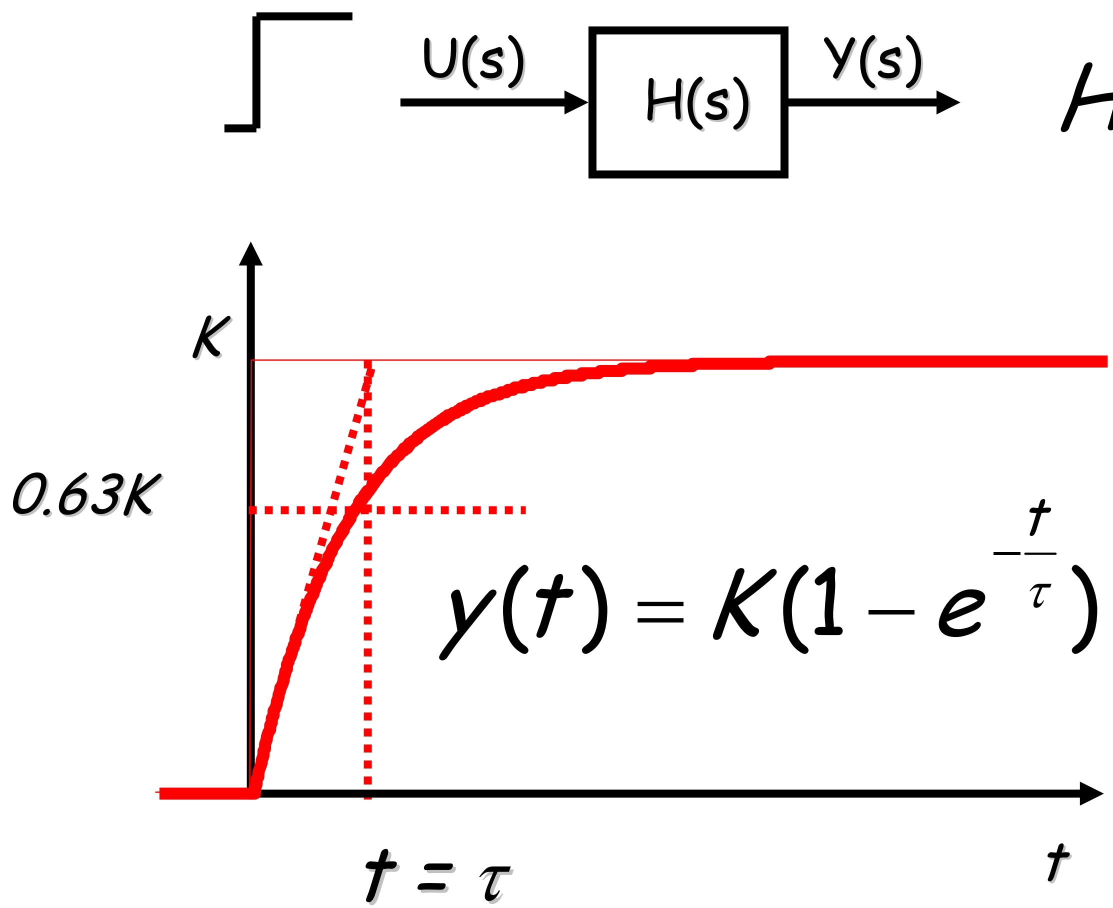
$$H(s)=\frac{K}{s\tau+1}$$

$$Y(s)=H(s)\frac{1}{s}=\frac{K}{s(s\tau+1)}$$

$$=\frac{Ka}{s(s+a)}=K\left(\frac{1}{s}-\frac{1}{s+a}\right)$$

$$\rightarrow y(t)=K(1-e^{-at})=K(1-e^{-\frac{t}{\tau}})$$

Solving the Differential Eq.



$$H(s)=\frac{K}{s\tau+1}$$

$$y(t)=K(1-e^{-\frac{t}{\tau}})$$

$$e^{-1}=0.37$$

representations

system gain

root locus gain

$$\frac{K}{s\tau+1}=\frac{K/\tau}{s+1/\tau}=\frac{Ka}{s+a}=\frac{K'}{s+a}$$

lowest power of s = 1

highest power of s = 1

A polynomium with the highest power of s = 1, is called a monic polynomium

representations (2)

system gain

root locus gain

$$\frac{K}{s(s\tau+1)} = \frac{K/\tau}{s(s+1/\tau)} = \frac{Ka}{s(s+a)} = \frac{K'}{s^2+as}$$

lowest power of $s = 1$

highest power of $s = 1$

DC-gain versus system gain

$$\frac{K}{s\tau + 1}$$

system gain = K

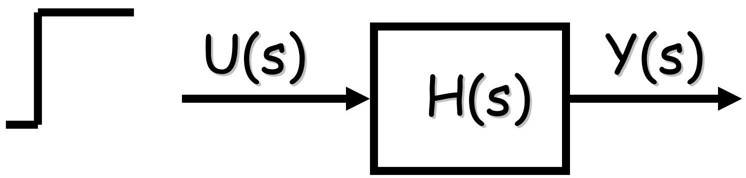
DC-gain = K

$$\frac{K}{s(s\tau + 1)}$$

system gain = K

DC-gain = ∞

Final value theorems



$$H(s) = \frac{K}{s\tau + 1}$$

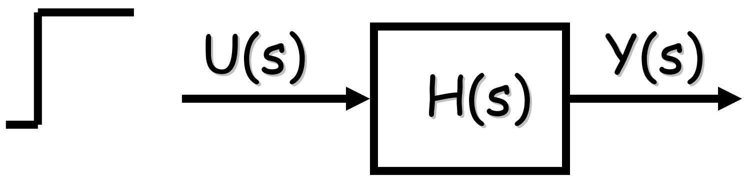
$$Y(s) = H(s)\frac{1}{s}$$

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} H(s) =$$

$$\lim_{s \to 0} \frac{K}{s\tau + 1} = K$$

final value theorem

Initial value theorems



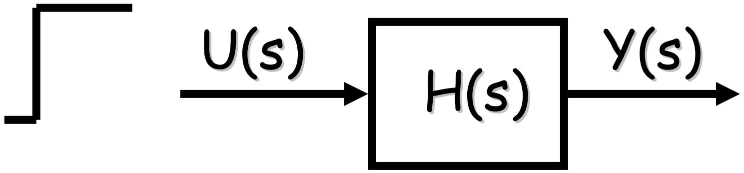
$$H(s) = \frac{K}{s\tau + 1}$$

$$\lim_{t\to 0} y(t) = \lim_{s\to\infty} sY(s) = \lim_{s\to\infty} \frac{K}{s\tau + 1} = 0$$

initial value theorem

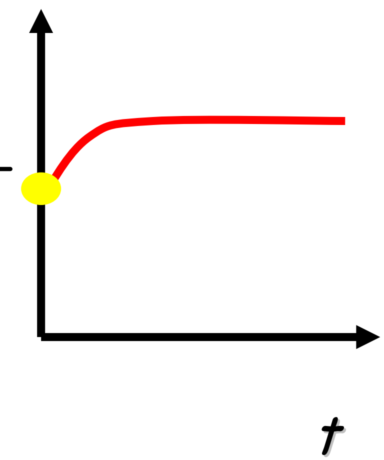
$$\lim_{t\to 0} \frac{d}{dt}\left(y(t)\right) = \lim_{s\to\infty} s\left(sY(s)\right) = \lim_{s\to\infty} \frac{Ks}{s\tau + 1} = \frac{K}{\tau}$$

First-order plus zero



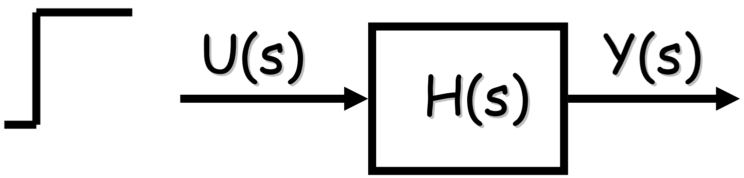
$$H(s) = K\frac{s\tau_2 + 1}{s\tau_1 + 1}$$

$$\lim_{t \to 0} y(t) = \lim_{s \to \infty} sY(s) = K\frac{s\tau_2 + 1}{s\tau_1 + 1} = K\frac{\tau_2}{\tau_1}$$



$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = K\frac{s\tau_2 + 1}{s\tau_1 + 1} = K$$

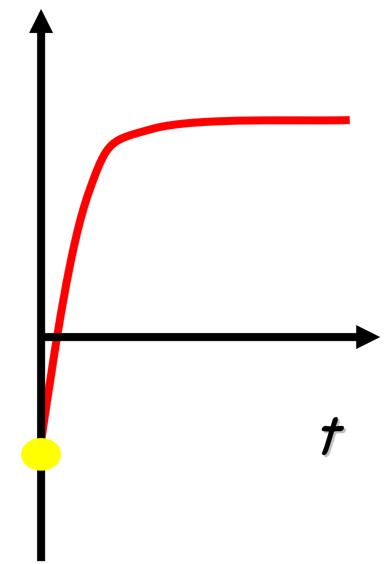
First-order plus zero



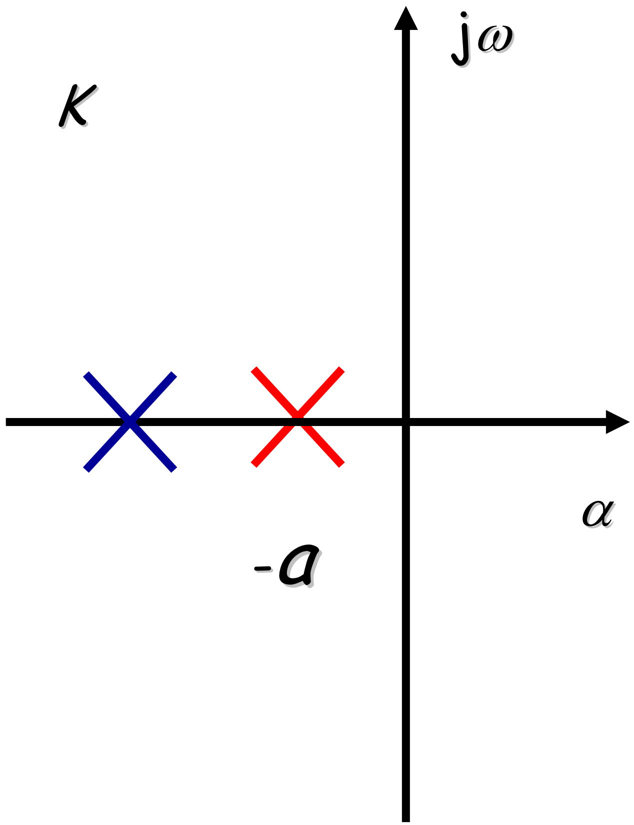
$$H(s) = K\frac{-s\tau_2 + 1}{s\tau_1 + 1}$$

$$\lim_{t\to 0} y(t) = \lim_{s\to\infty} sY(s) = K\frac{-s\tau_2 + 1}{s\tau_1 + 1} = -K\frac{\tau_2}{\tau_1}$$

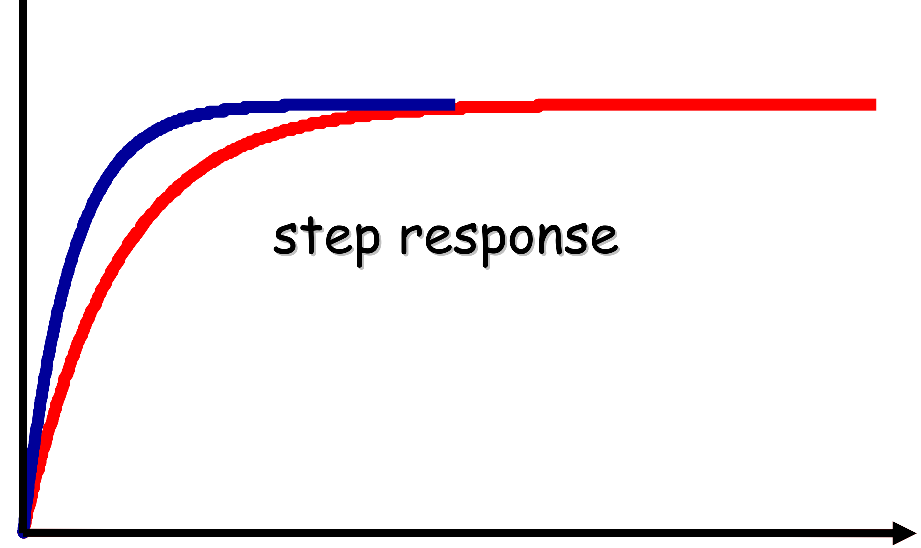
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = K\frac{-s\tau_2 + 1}{s\tau_1 + 1} = K$$



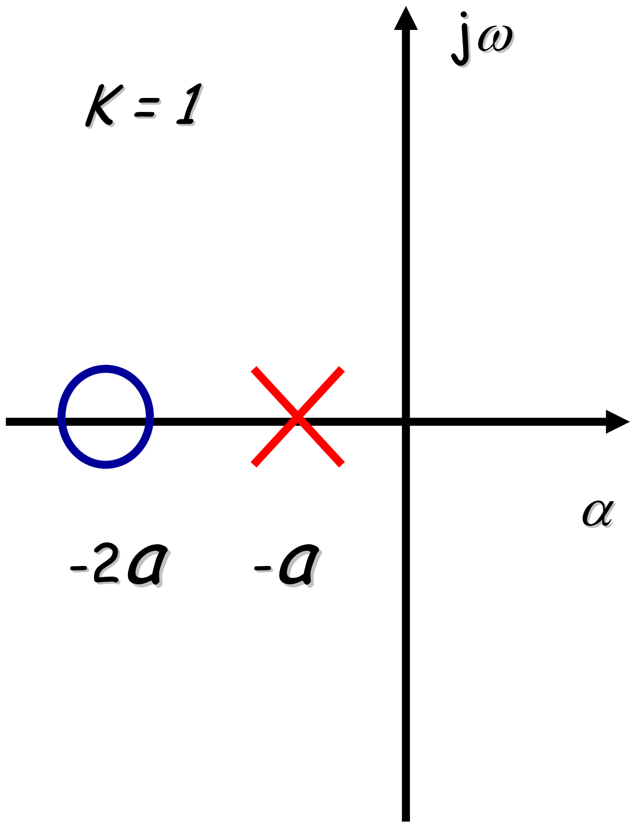
s-plane ←→ time

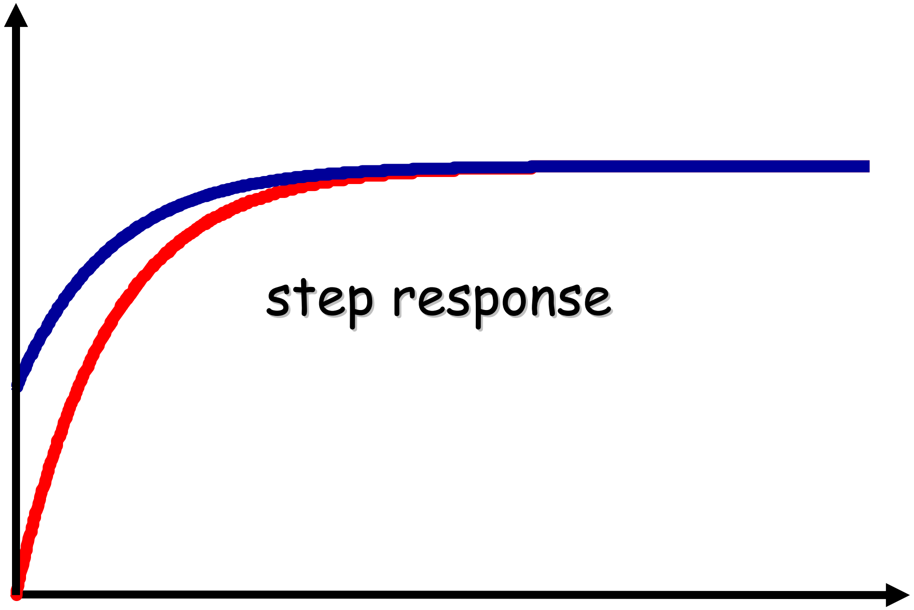


$$\frac{K}{s\tau+1}=\frac{K'}{s+a}$$

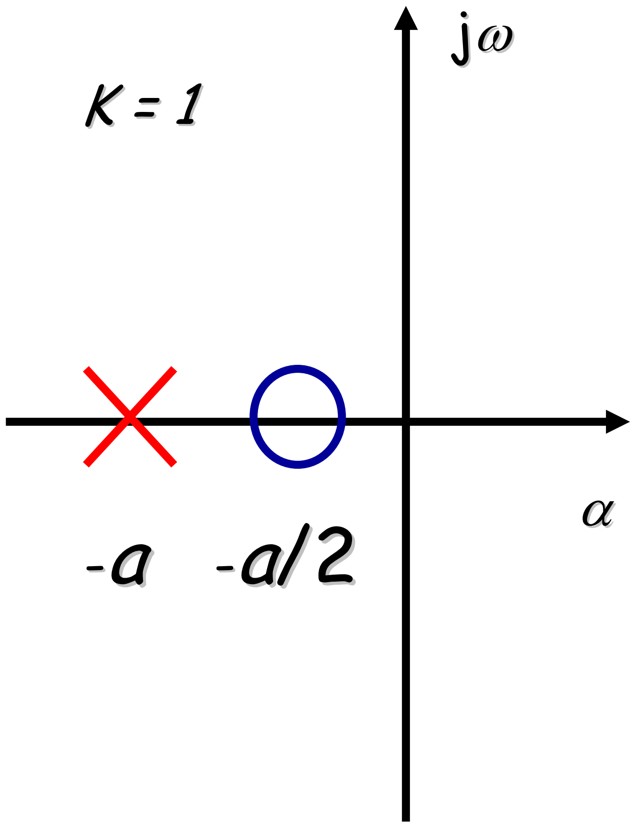


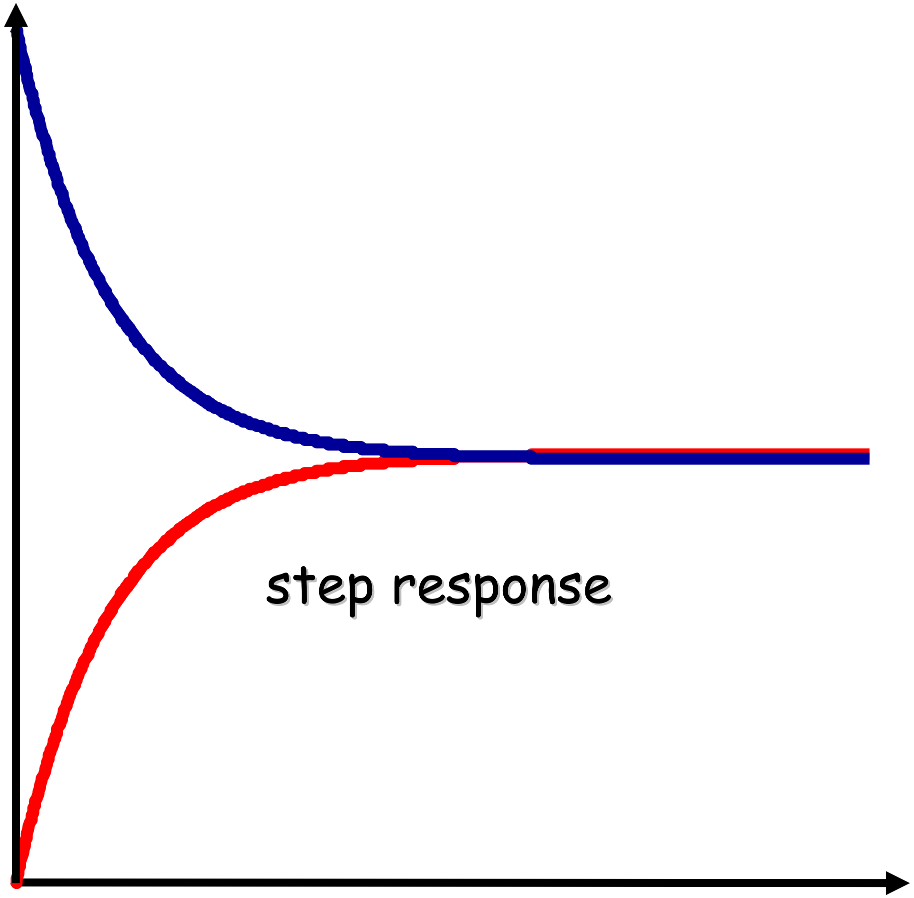
s-plane ⟷ time



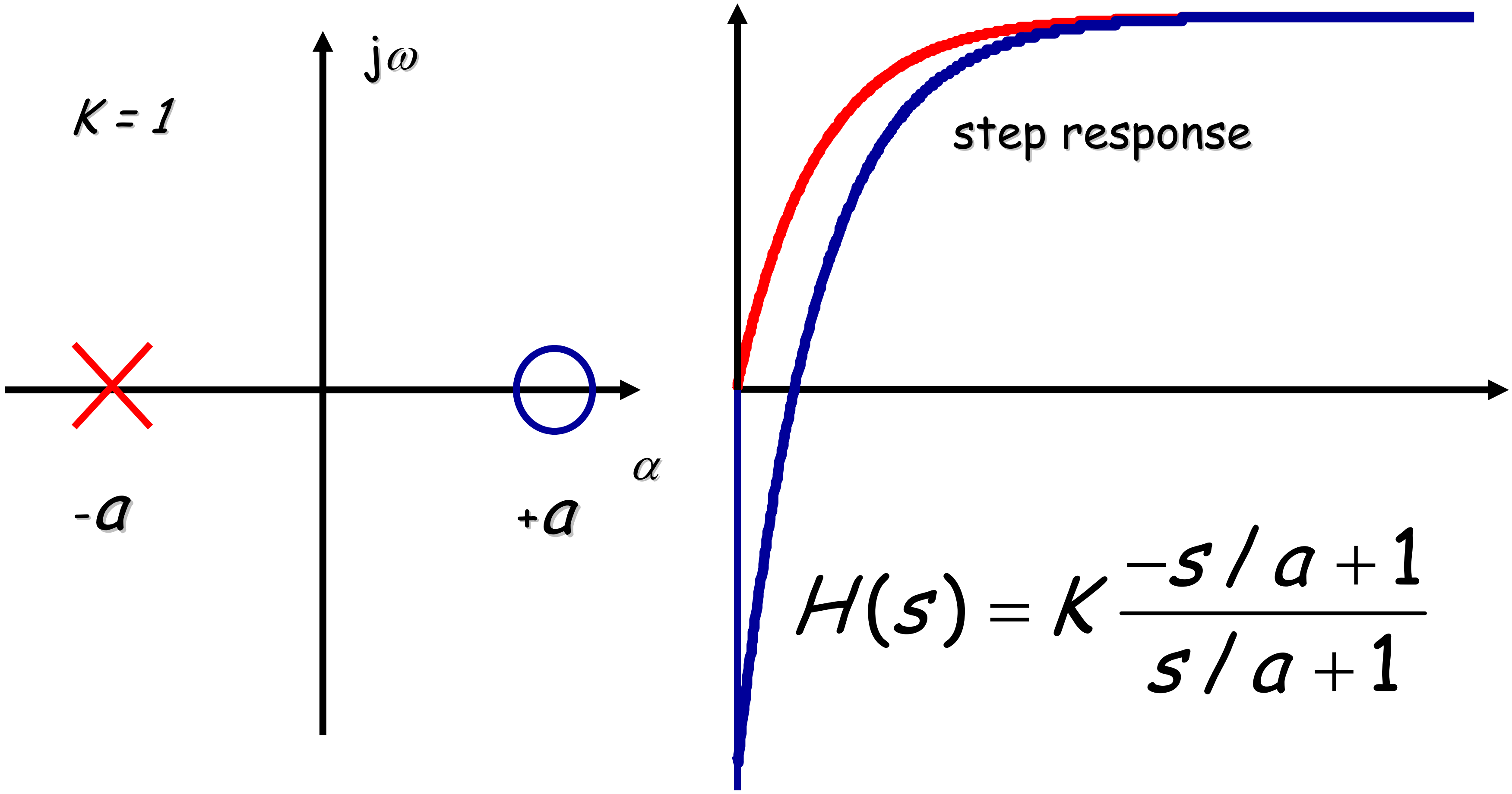


s-plane ←→ time



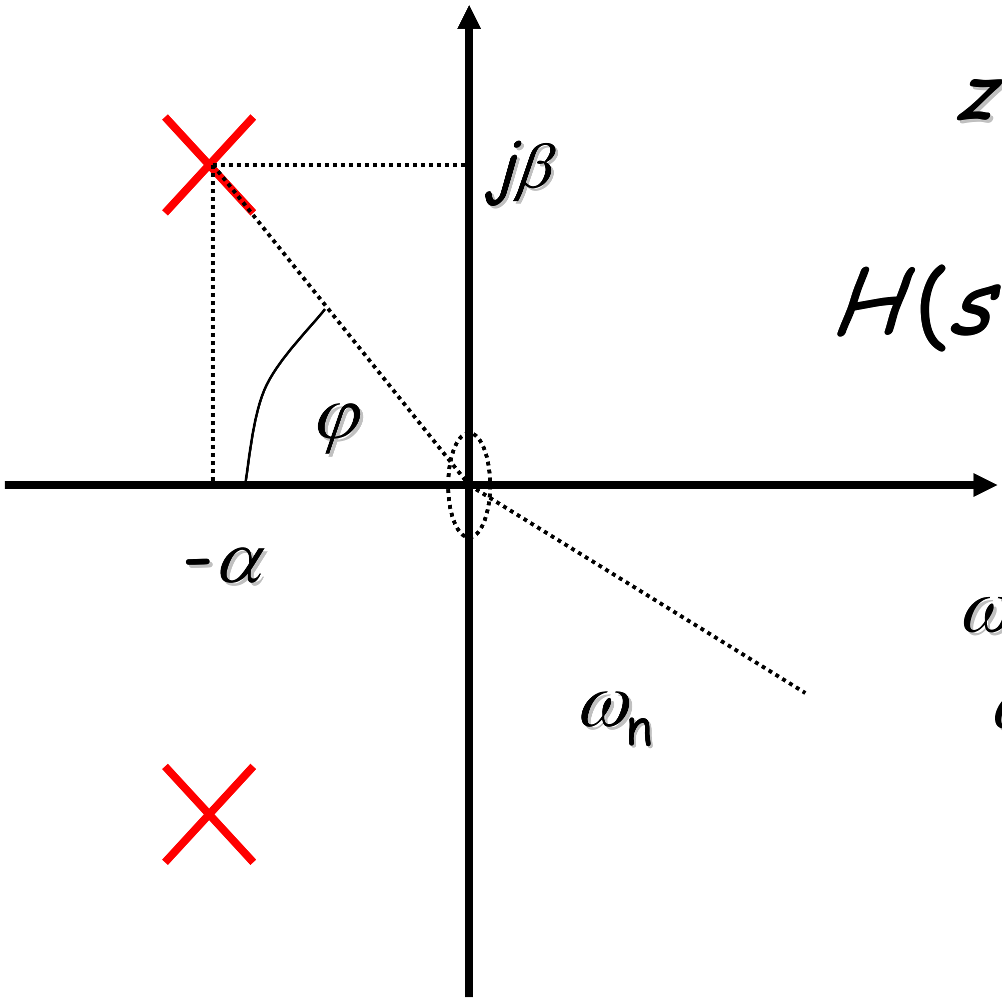


s-plane ⟷ time



$$H(s) = K\frac{-s/a+1}{s/a+1}$$

Second-order system



$z = \cos\varphi$ damping ratio

$$H(s) = \frac{\omega_n^2}{s^2 + 2z\omega_n s + \omega_n^2}$$

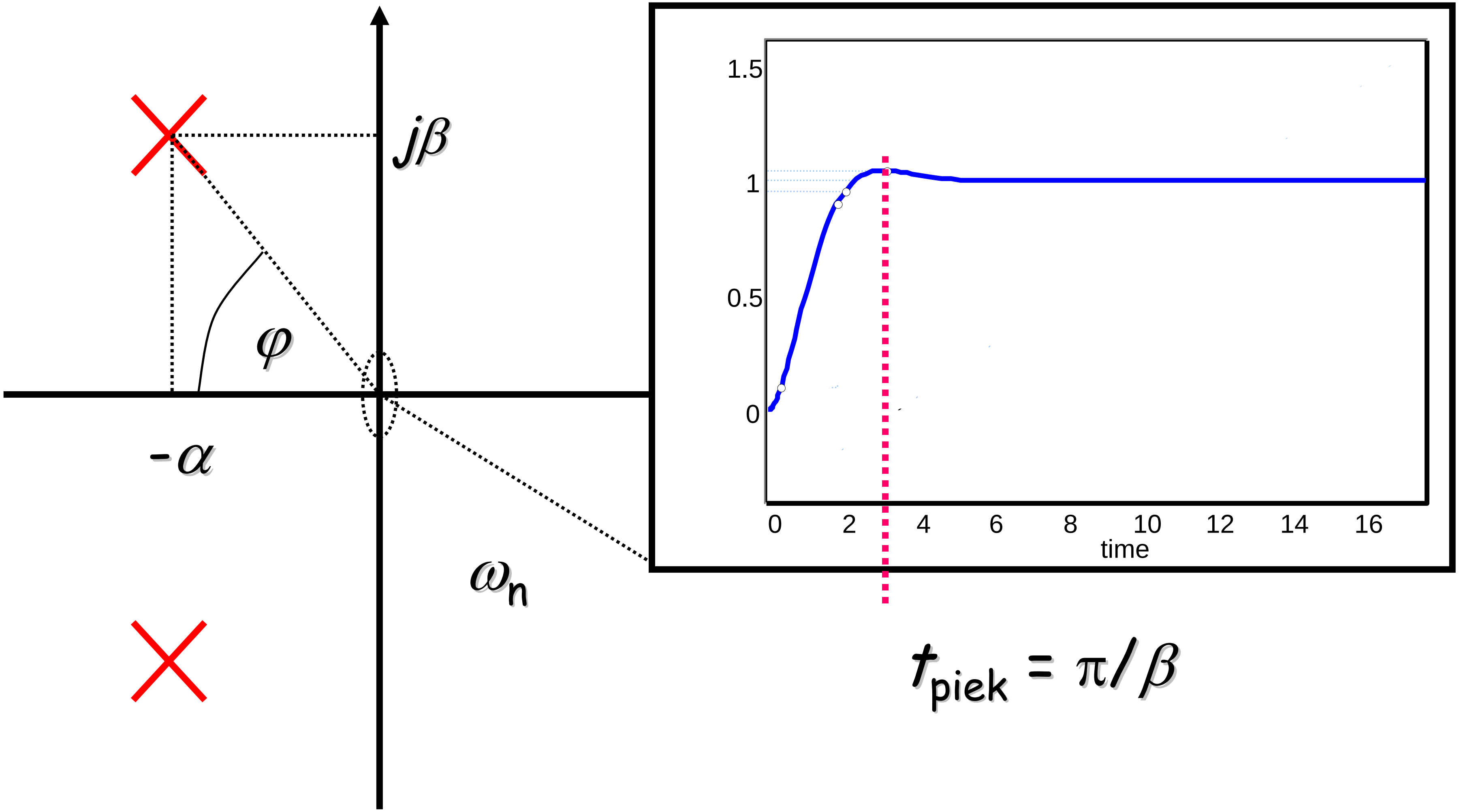
ω_n = natural frequency

α = absolute damping

$$\alpha = \omega_n \cos(\varphi) = \omega_n z$$

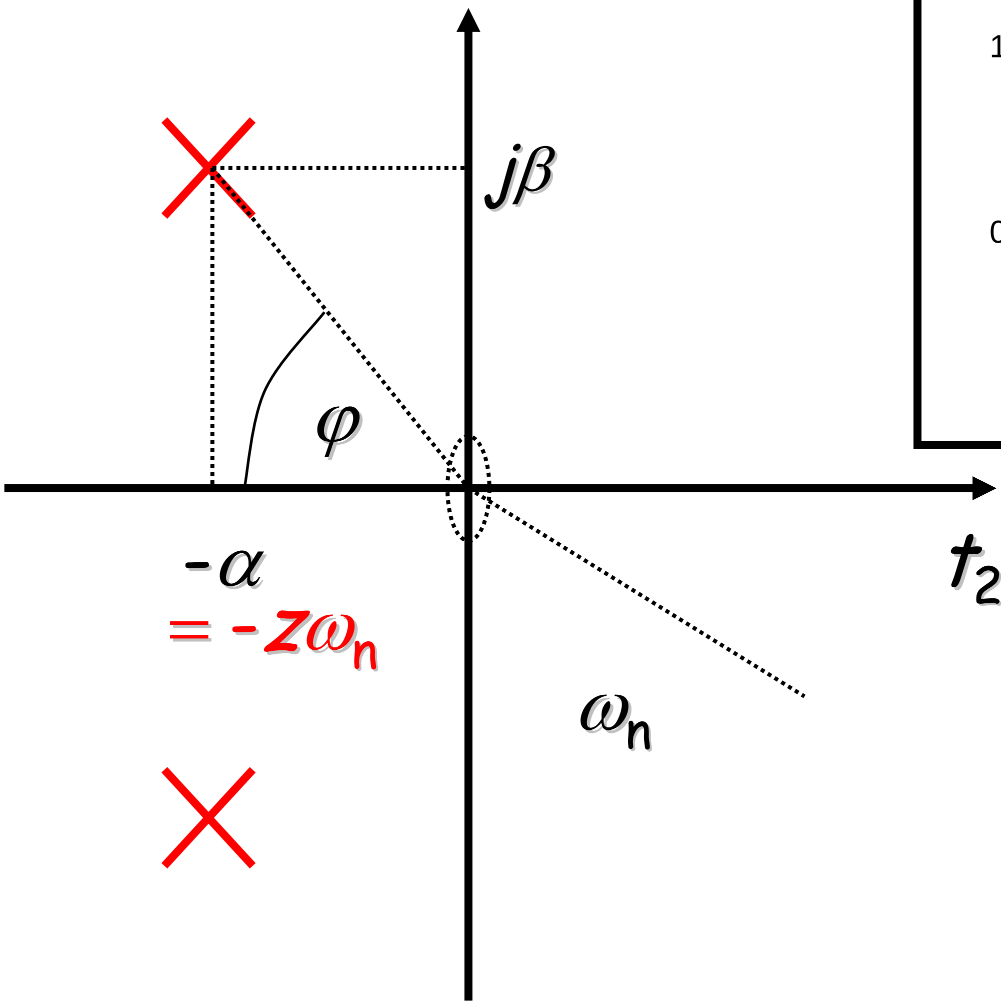
$$\beta = \omega_n \sin(\varphi)$$

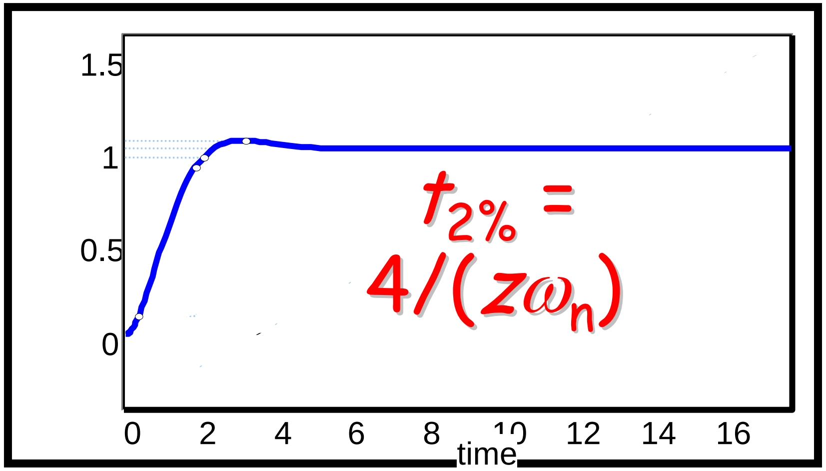
Second-order system



$$t_{piek} = \pi / \beta$$

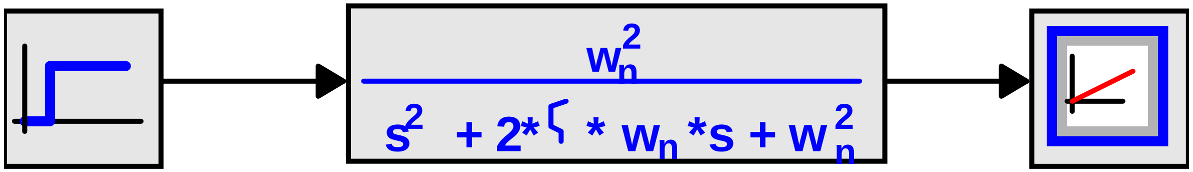
Second-order system

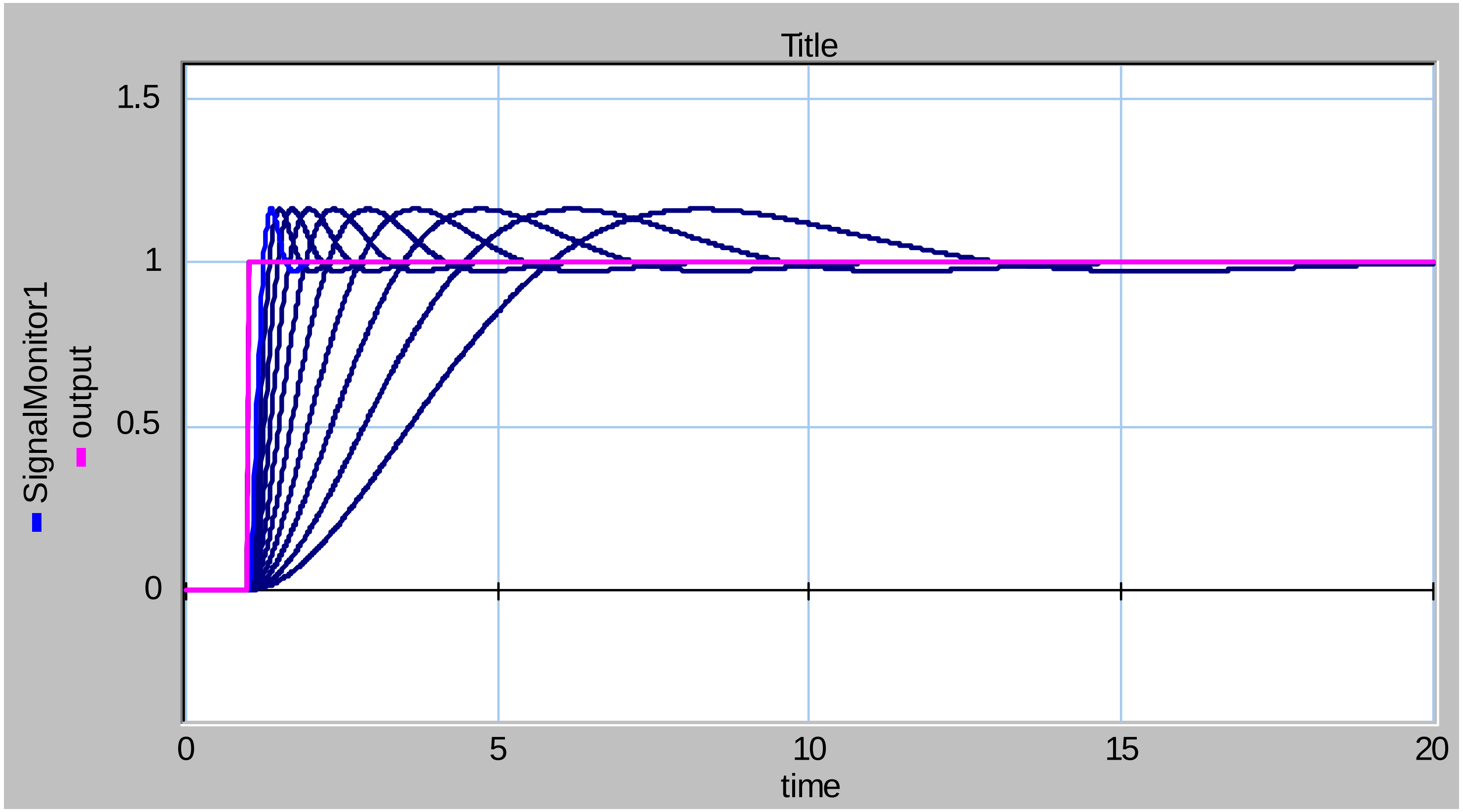




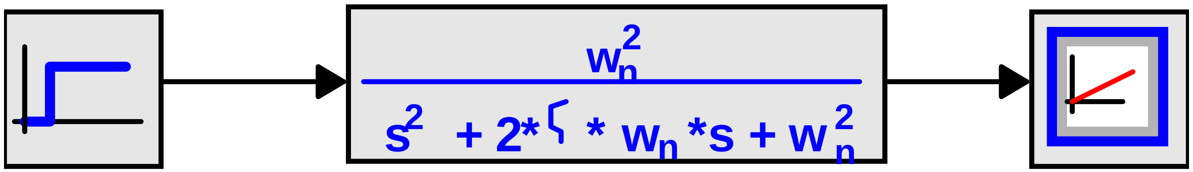
$t_{2\%}$ = time where response remains within 2% of final value

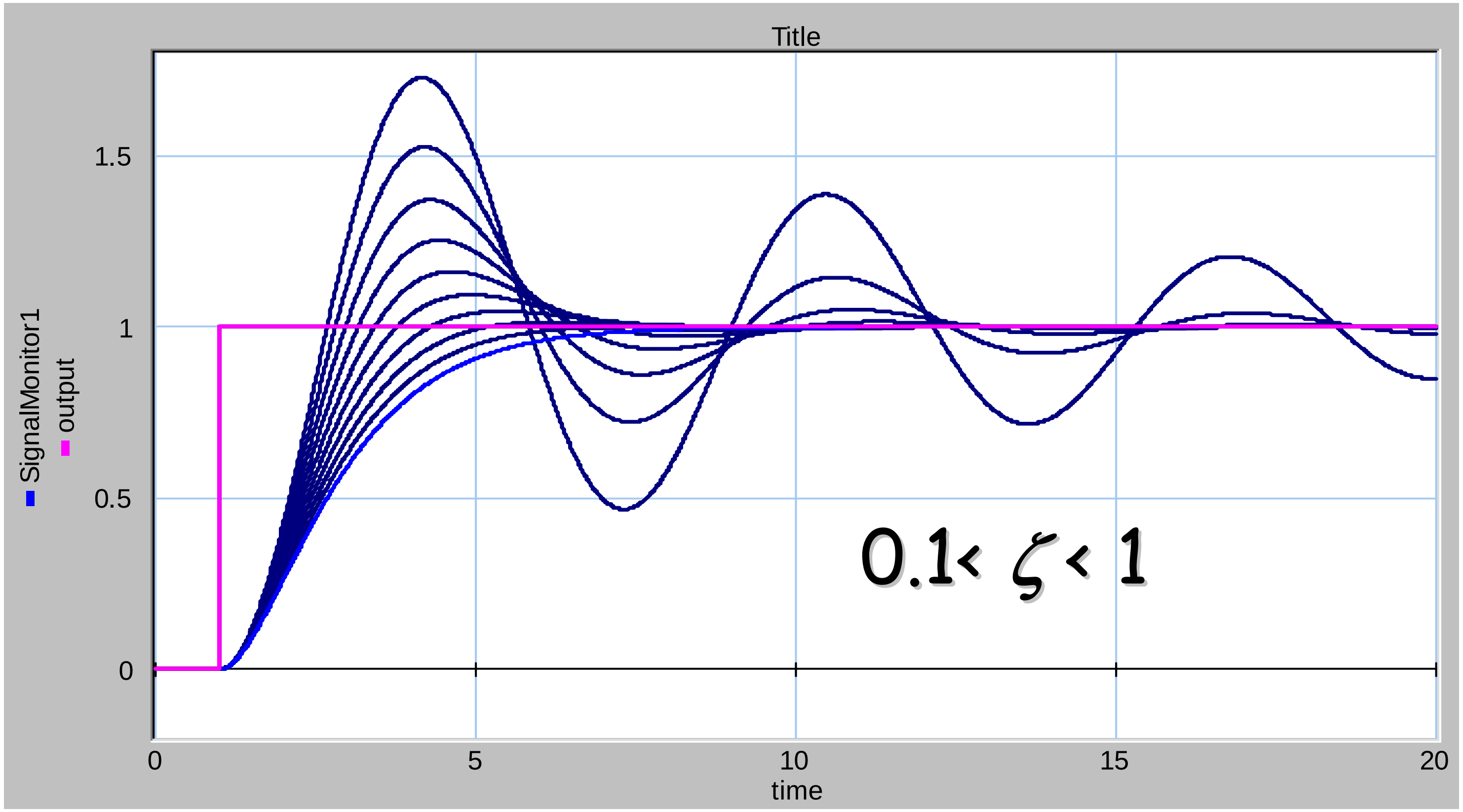
Second-order system





Second-order system





Exercise (1)

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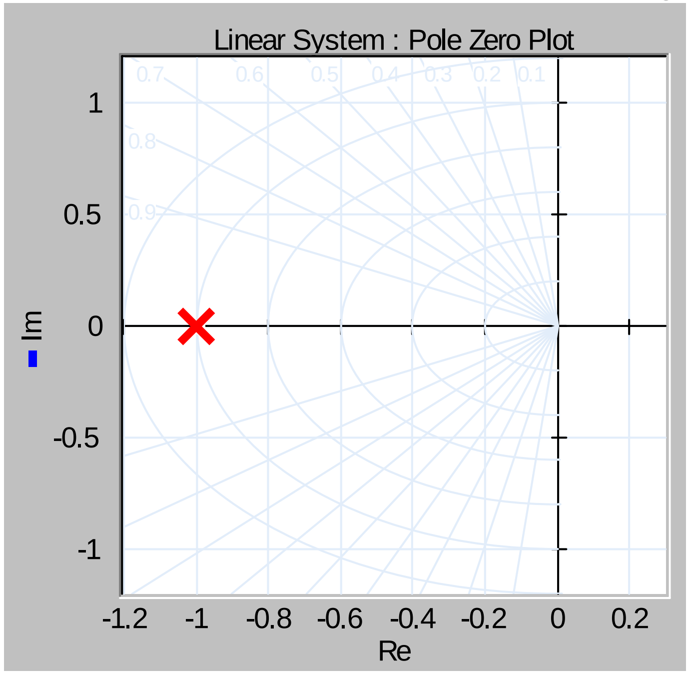
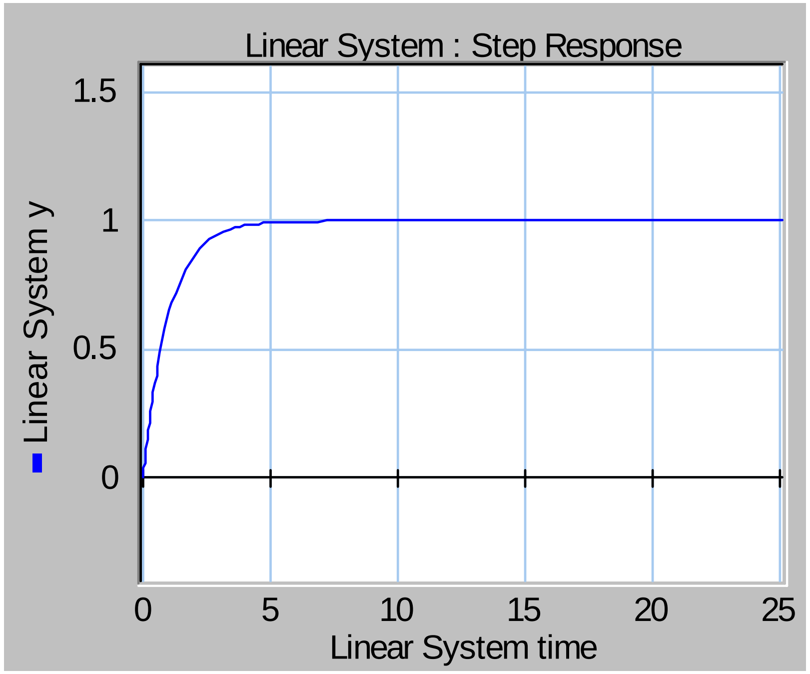
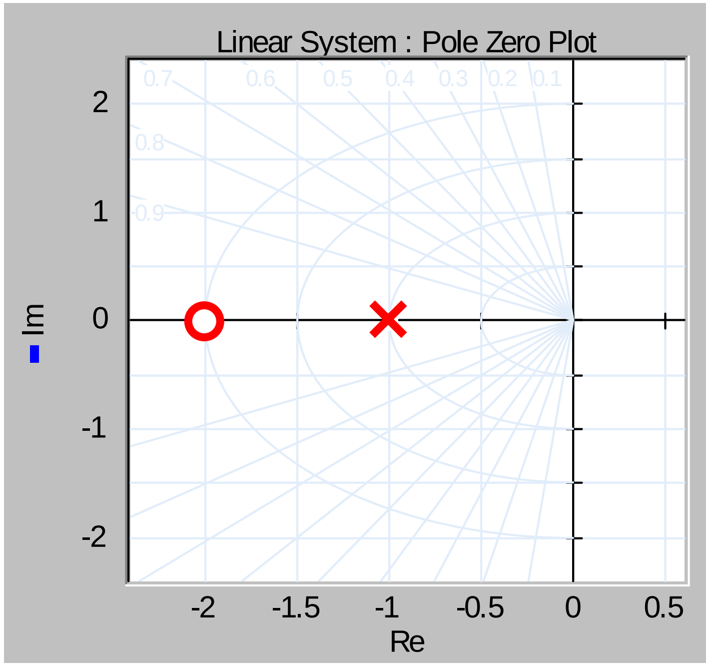
Use the linear systems editor of 20-sim to simulate a first-order system and investigate the influence of a varying gain and time constant on the step response

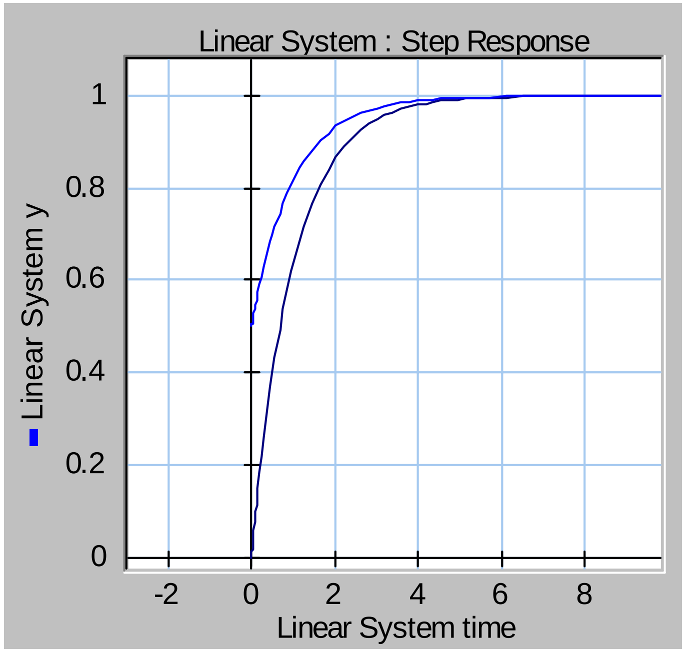




Exercise (2)

Do the same for a first-order system with a zero and investigate the influence of a varying zero on the step response





Exercise (3)

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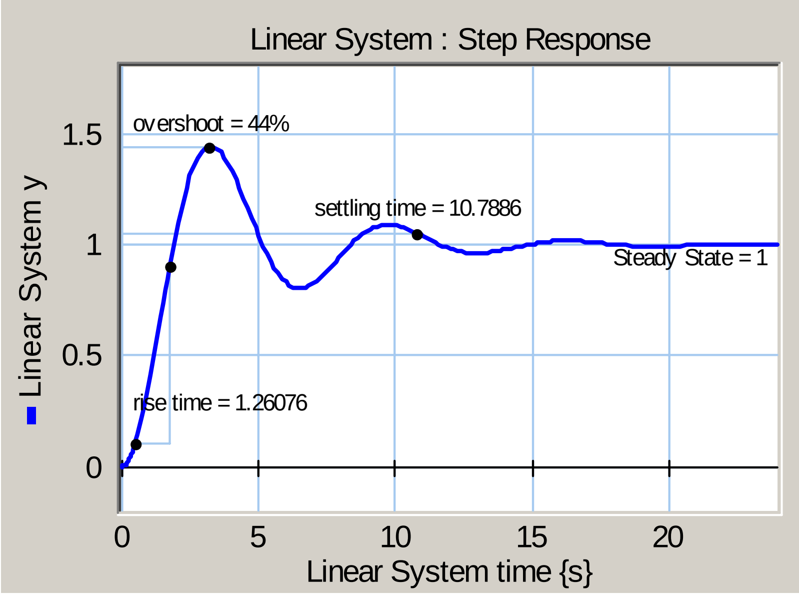
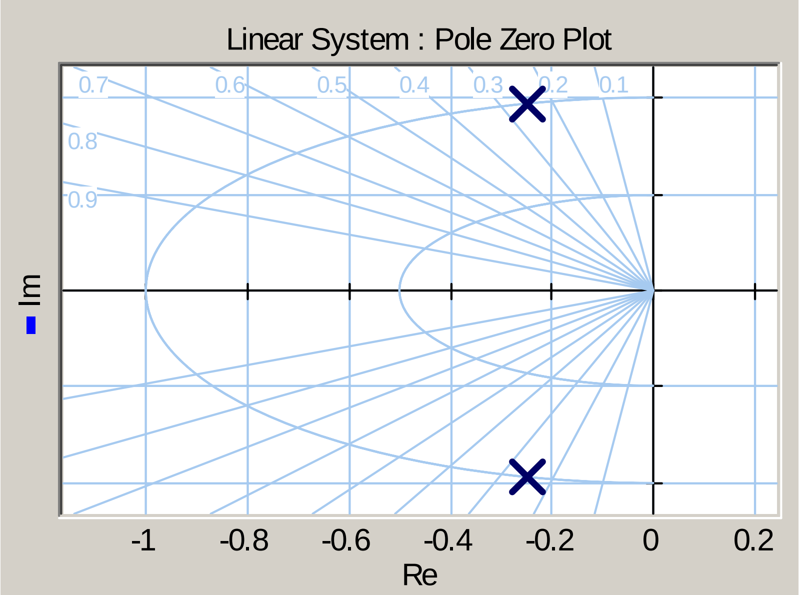
Investigate the influence of varying ω_n en z for a second-order system on the step response.

Investigate the effect of an extra pole or zero





Exercise (4)

Check the relation between peak time and settling time and the location of the poles of a second order system.

Generate the responses with 20-sim